

F-1 Quiz-2 Solution

Let $f(x)$ be a differentiable function on \mathbb{R} with $f'(x) \leq 5$ for all x , and $f(2) = -1$. Show that $f(5) \leq 14$.

Since f is differentiable on \mathbb{R} , we can deduce that

f is continuous on $[2, 5]$ and

f is differentiable on $(2, 5)$.

So by using Mean Value Theorem, $\exists c \in (2, 5)$ s.t

$$\frac{f(5) - f(2)}{5 - 2} = f'(c)$$

since $f(2) = -1$ and $f'(x) \leq 5$

$$\frac{f(5) + 1}{3} = f'(c) \leq 5$$

$$f(5) + 1 \leq 15$$

$$f(5) \leq 14 .$$