

1. T2-Quiz1

Show your work and explain your claims.

Let $f(x) = \begin{cases} 2, & \text{if } x \leq 0 \\ \frac{x^2+2x}{x}, & \text{if } 0 < x < 2 \\ 5, & \text{if } x \geq 2 \end{cases}$. Evaluate the following limits, if they exist:

$$\lim_{x \rightarrow 0} f(x),$$

$$\lim_{x \rightarrow 1} f(x),$$

$$\lim_{x \rightarrow 2} f(x).$$

Determine whether $f(x)$ is continuous at $x = 0$ and $x = 2$.

* $\lim_{x \rightarrow 0} f(x) = ?$ i) $\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} \frac{x^2+2x}{x} = \lim_{x \rightarrow 0^+} \frac{x(x+2)}{x} = \lim_{x \rightarrow 0^+} x+2 = 2$
($\begin{matrix} x \rightarrow 0^+ \\ x > 0 \end{matrix}$)

ii) $\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} 2 = 2$
($\begin{matrix} x \rightarrow 0^- \\ x < 0 \end{matrix}$)

$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^-} f(x) = 2$, hence

$\lim_{x \rightarrow 0} f(x) = 2$

* $\lim_{x \rightarrow 1} f(x) = ?$ $\lim_{x \rightarrow 1} f(x) = \lim_{x \rightarrow 1} \frac{x^2+2x}{x} = 3$
(as $x \rightarrow 1$)
 $f(x) = \frac{x^2+2x}{2}$

* $\lim_{x \rightarrow 2} f(x) = ?$ i) $\lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^+} 5 = 5$
($\begin{matrix} x \rightarrow 2^+ \\ x > 2 \\ f(x) = 5 \end{matrix}$)

ii) $\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^-} \frac{x^2+2x}{2} = \frac{8}{2} = 4$
($\begin{matrix} x \rightarrow 2^- \\ x < 2 \end{matrix}$)

$\lim_{x \rightarrow 2^+} f(x) \neq \lim_{x \rightarrow 2^-} f(x)$
 then

$\lim_{x \rightarrow 2} f(x) \text{ dne}$

* $f(0) = 2 = \lim_{x \rightarrow 0} f(x)$ so $f(x)$ is continuous at $x=0$

* $f(2) = 5 \neq \lim_{x \rightarrow 2} f(x)$ (because this limit does not exist)

so $f(x)$ is not continuous at $x=2$