Math 119 Quiz 1 Solution Fall 2020-2021

1. Evaluate $\lim\limits_{x\to-\infty}\left(\sqrt{x^2+4x}-\sqrt{x^2+5}\right)$ if it exists.

Solution

$$\lim_{x\to-\infty}\left(\sqrt{x^2+4x}-\sqrt{x^2+5}\right)=\lim_{x\to-\infty}\frac{\left(\sqrt{x^2+4x}-\sqrt{x^2+5}\right)\cdot\left(\sqrt{x^2+4x}+\sqrt{x^2+5}\right)}{\left(\sqrt{x^2+4x}+\sqrt{x^2+5}\right)}$$

$$=\lim_{x\to-\infty}\frac{x^2+4x-x^2-5}{\left(\sqrt{x^2+4x}+\sqrt{x^2+5}\right)}$$

$$=\lim_{x\to-\infty}\frac{4x-5}{\left(\sqrt{x^2+4x}+\sqrt{x^2+5}\right)}$$

$$=\lim_{x\to-\infty}\frac{x\left(4-\frac{5}{x}\right)}{\left(\sqrt{x^2\left(1+\frac{4}{x}\right)}+\sqrt{x^2\left(1+\frac{5}{x^2}\right)}\right)}$$

$$=\lim_{x\to-\infty}\frac{x\left(4-\frac{5}{x}\right)}{\sqrt{x^2}\left(\sqrt{1+\frac{4}{x}}+\sqrt{1+\frac{5}{x^2}}\right)}$$

$$=\lim_{x\to-\infty}\frac{x\left(4-\frac{5}{x}\right)}{|x|\left(\sqrt{1+\frac{4}{x}}+\sqrt{1+\frac{5}{x^2}}\right)}$$

$$=\lim_{x\to-\infty}\frac{x\left(4-\frac{5}{x}\right)}{-x\left(\sqrt{1+\frac{4}{x}}+\sqrt{1+\frac{5}{x^2}}\right)}$$

$$=\lim_{x\to-\infty}\frac{\left(4-\frac{5}{x}\right)}{-\left(\sqrt{1+\frac{4}{x}}+\sqrt{1+\frac{5}{x^2}}\right)}=-2$$