

F-4 Quiz-1 Solution

$$\text{Let } f(x) = \begin{cases} 2 & \text{if } x \leq 0 \\ \frac{x^2+2x}{x} & \text{if } 0 < x < 2 \\ 4 & \text{if } x \geq 2 \end{cases}$$

Is f a continuous function?

Since $f(x) = 2$ for $x < 0$ and $f(x) = 4$ for $x > 2$ are polynomials, f is continuous for $x < 0$ and for $x > 2$.

$f(x) = \frac{x^2+2x}{x} = x+2$ is also continuous for $0 < x < 2$.

We need to check continuity at $x=0$ and $x=2$.

For $x=0$

$$\lim_{x \rightarrow 0^-} f(x) = 2 \quad \lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} x+2 = 2$$

$$\text{Then } \lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^+} f(x) = f(0) = 2$$

So, f is continuous at $x=0$.

For $x=2$

$$\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^-} x+2 = 4 \quad \lim_{x \rightarrow 2^+} f(x) = 4$$

$$\text{Then } \lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^+} f(x) = f(2) = 4$$

So, f is continuous at $x=2$.