1. F2-Quiz2

Show your work and explain your claims.

Let f and g be differentiable functions on $\mathbb{R}$ such that the tangent line to $y = f(x)$ at $x = 1$ is given by the equation $y = 2x + 1$, and the tangent line to $y = g(x)$ at $x = 1$ is given by the equation $y = 3x - 2$. Find an equation for the tangent line to $y = f(x)g(x)$ at $x = 1$.

* tangent line to f at $x=1$ is $y = 2x+1 \Rightarrow f'(1) = 2$ (Slope of tangent)
($y = 2x+1$: slope $= 2$)

$(1, f(1))$ is intersection of tangent with the curve
So we can find $f(1)$ by putting $x=1$ in the tangent line, $x=1 \Rightarrow y = 2+1 = 3 \Rightarrow f(1) = 3$

* tangent line to g at $x=1$ is $y = 3x-2 \Rightarrow g'(1) = 3$
(slope of tangent is 3)

Similarly put $x=1$ in the tangent of g, $y=3x-2$
$y = 3-2 = 1 \Rightarrow g(1) = 1$

* Let $F(x) = f(x) \cdot g(x)$ tangent of $F(x)$ at $x=1$ has slope $F'(1)$

$$F'(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x) \Rightarrow F'(1) = \underbrace{f'(1)}_{2} \cdot \underbrace{g(1)}_{1} + \underbrace{f(1)}_{3} \underbrace{g'(1)}_{3}$$

$= 2 + 9 = 11 \rightarrow$ slope of tangent of $F(x)$

$F(1) = f(1) \cdot g(1) = 3 \cdot 1 = 3$

eqn. of tangent line: $y - 3 = 11(x-1)$

$y = 11x - 8$