

F2 Quiz 1

Let $f(x) = \begin{cases} \frac{2x^2+3x+1}{x+1} & \text{if } -1 < x \leq 1 \\ ax+3b & \text{if } 1 < x \leq 3 \\ 4bx & \text{if } x \geq 3 \end{cases}$

What must be the value(s) of a and b so that $f(x)$ is continuous at $x=1$ and $x=3$?

Solution

$f(x)$ is cont. at $x=1 \Rightarrow \lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^+} f(x) = f(1)$

$$\lim_{x \rightarrow 1^-} \frac{2x^2+3x+1}{x+1} = \lim_{x \rightarrow 1^+} (ax+3b) = 3$$

$$\lim_{x \rightarrow 1^-} \frac{2x^2+3x+1}{x+1} = 3$$

$$\lim_{x \rightarrow 1^+} (ax+3b) = a+3b$$

$$\Rightarrow \boxed{a+3b=3}$$

$f(x)$ is cont. at $x=3 \Rightarrow \lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^+} f(x) = f(3)$

$$\lim_{x \rightarrow 3^-} (ax+3b) = \lim_{x \rightarrow 3^+} 4bx = 12b$$

$$\lim_{x \rightarrow 3^-} (ax+3b) = 3a+3b$$

$$\lim_{x \rightarrow 3^+} 4bx = 12b$$

$$\Rightarrow 3a+3b = 12b$$

$$\Rightarrow \boxed{a = 3b}$$

$$a+3b=3 \Rightarrow 6b=3 \Rightarrow \boxed{\begin{matrix} b = \frac{1}{2} \\ a = \frac{3}{2} \end{matrix}}$$