1. F1-Quiz1

Show your work and explain your claims.

Let $f(x) = \begin{cases} b+3, & \text{if } x \le 0 \\ \frac{x^2+3x}{x}, & \text{if } 0 < x < 2 \\ a+2x, & \text{if } x \ge 2 \end{cases}$. What must be the value(s) of a and b so that $f(x)$ is continuous at $x = 0$ and $f(x)$ is **not** continuous at $x = 2$.

* $f(x)$ to be continuous at $x=0$ the equality $\lim_{x\to 0} f(x) = f(0)$ must hold.

For $\lim_{x\to 0} f(x)$,

- $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{x^2+3x}{x} = \lim_{x\to 0^+} \frac{\cancel{x}(x+3)}{\cancel{x}} = \lim_{x\to 0^+}(x+3) = 3$

$\left(\begin{matrix} x\to 0^+ \\ x>0 \end{matrix}\right)$

- $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} b+3 = b+3$

$\left(\begin{matrix} x\to 0^- \\ x<0 \end{matrix}\right)$

$\lim_{x\to 0^+} f(x) = 3 = b+3 = \lim_{x\to 0^-} f(x)$ (for existence of the limit)
$= f(0)$ (for continuity)

$b+3=3 \Rightarrow \boxed{b=0}$

* $f(2) = a+4$

$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} a+2x = a+4$

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} \frac{x^2+3x}{x} = \frac{10}{2} = 5$

If these three are equal then f is continuous at $x=2$

If $f(x)$ is not continuous at $x=2$, then $\lim_{x\to 2} f(x) \neq f(2)$

$a+4 \neq 5$

(for this question since the limit dne. it cannot be equal to $f(2)$ when $a \neq 1$)

$\Rightarrow \boxed{a \neq 1}$

$b=0$, a can be any real number except for 1. !