

# Sample Solution

**Section: 153**

Name & Surname: \_\_\_\_\_

Math 120 Spring 2019-2020

Quiz no.: 02

ID Number: \_\_\_\_\_

Date: 06.03.20

Time Limit: ~10 Minutes

Grade: \_\_\_\_\_

**Declaration of Honesty:** By signing below, I pledge that I will write this examination as my own work and without the assistance of others or the usage of unauthorized material or information. I understand that possession of any kind of electronic device during the exam is prohibited. I also understand that not obeying the rules of the examination will result in immediate cancellation and disciplinary procedures.

Signature : .....

1. Find the sum of the series  $\sum_{n=1}^{\infty} \frac{(-1)^n}{3^n n}$ .

**1st way**:  $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n, |x| < 1$

$$\Rightarrow \int_0^x \frac{1}{1-t} dt = \sum_{n=0}^{\infty} \int_0^x t^n dt$$

$$\Rightarrow -\ln(1-x) = \sum_{n=0}^{\infty} \frac{x^{n+1}}{n+1} = \sum_{n=1}^{\infty} \frac{x^n}{n}$$

$$x = -\frac{1}{3} \Rightarrow -\ln\left(\frac{4}{3}\right) = \sum_{n=1}^{\infty} \frac{(-1)^n}{3^n n}$$

**2nd way**:  $\frac{1}{1+x} = \sum_{n=0}^{\infty} (-1)^n x^n, |x| < 1$

$$\Rightarrow \int_0^x \frac{1}{1+t} dt = \sum_{n=0}^{\infty} (-1)^n \int_0^x t^n dt = \sum_{n=0}^{\infty} (-1)^n \frac{x^{n+1}}{n+1} = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n}$$

$$x = \frac{1}{3} \Rightarrow -\ln\left(\frac{4}{3}\right) = \sum_{n=1}^{\infty} \frac{(-1)^n}{3^n n}$$

$$= -\sum_{n=1}^{\infty} (-1)^n \frac{x^n}{n}$$

Recall:  $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n, |x| < 1$ ;  $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}, x \in \mathbb{R}$ ;  $\sin(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}, x \in \mathbb{R}$ ;  $\cos(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}, x \in \mathbb{R}$