Math 501 Homework 4

Due Date January 17, 2022 Monday

January 10, 2022

Chapter 10

Problem 1

Suppose μ is a finite measure. For measurable f and g define

$$d(f,g) = \int \frac{|f-g|}{1+|f-g|} d\mu.$$

Prove that d is a metric on the space of measurable functions except for the fact that $d(f,g) = 0$ only implies $f = g$ a.e. Prove also that $f_n \to f$ in measure if and only if $d(f_n, f) \to 0$.
Hint: Let $g(x) = \frac{x}{1+x}$. Show that g is an increasing function on $[0,\infty)$.

Chapter 11

Problem 1

Suppose f is a measurable function. Prove the equality

$$\int_{-\infty}^{\infty} |f(x)|\ dx = \int_{0}^{\infty} m(\{x : |f(x)| \geq y\})\ dy.$$

Where m is the Lebesque measure.
Hint: Show that the set $E = \{\ (x,y) \mid 0 \leq y \leq |f(x)|\ \}$ is in $\mathcal{L} \times \mathcal{L}$.

Chapter 12

Problem 1

Suppose μ is a signed measure. Prove that A is a null set with respect to μ if and only if $|\mu|(A) = 0$, where $|\mu| = \mu^+ + \mu^-$.

Chapter 13

Problem 1

Suppose μ and ρ are two measures with $\mu \ll \rho$. Let f be a Radon-Nikodym derivative of μ with respect to ρ. Show that $f > 0$ μ-a.e.