

Math 501 Homework 3

Due Date December 20, 2021 Monday

December 13, 2021

Chapter 6-7

Problem 1

Let (X, \mathcal{A}, μ) be a measure space and $f : (X, \mathcal{A}, \mu) \rightarrow \mathbb{R}$ be a non-negative measurable function. Show that $\mu(\{x \mid f(x) > 0\}) \geq \frac{(\int f d\mu)^2}{\int f^2 d\mu}$.

Problem 2

Suppose f is integrable and A_n is measurable for each $n = 1, 2, 3, \dots$. Prove that if either $A_n \uparrow A$ or $A_n \downarrow A$, then $\int_{A_n} f d\mu \rightarrow \int_A f d\mu$.

Problem 3

Suppose μ_n is a sequence of measures on (X, \mathcal{A}) such that $\mu_n(X) = 1$ for all n . Suppose μ is a measure on (X, \mathcal{A}) such that $\mu_n(A) \rightarrow \mu(A)$ as $n \rightarrow \infty$ for all $A \in \mathcal{A}$. Show that $\int f d\mu_n \rightarrow \int f d\mu$ whenever f is bounded and measurable.