# QUIZ-4, SAMPLE SOLUTION

Evaluate

$$\lim_{x\to 0^+} (\ln(x+1))^x. \qquad [0^0]$$

Let $y = (\ln(x+1))^x$

Note that around zero, $\ln(x+1) > 0$ (*)

$\Rightarrow \ln y = x \cdot \underbrace{\ln(\ln(x+1))}$

↳ defined by (*)

We'll find $\lim\limits_{x\to 0^+} (\ln(x+1))^x = \lim\limits_{x\to 0^+} y = ?$

Consider the $\lim\limits_{x\to 0^+} \ln y = \lim\limits_{x\to 0^+} x \ln(\ln(x+1)) \qquad [0.\infty]$

$$= \lim_{x\to 0^+} \frac{\ln(\ln(x+1))}{1/x} \qquad \left[\frac{\infty}{\infty}\right]$$

$$\overset{L'H}{=} \lim_{x\to 0^+} \frac{\frac{1}{(x+1)\ln(x+1)}}{\frac{-1}{x^2}}$$

$$= \lim_{x\to 0^+} \frac{-x^2}{(x+1)\ \ln(x+1)} \qquad \left[\frac{0}{0}\right]$$

$$\overset{L'H}{=} \lim_{x\to 0^+} \frac{-2x}{\ln(x+1) + (x+1)\cdot\frac{1}{x+1}}$$

$$= \lim_{x\to 0^+} \frac{-2x}{\ln(x+1)+1} = 0$$

Hence, we have $\lim_{x \to 0^+} \ln y = 0$

ie,

lnx cont on $(0,\infty)$

$$\lim_{x \to 0^+} \ln y = \ln\left(\lim_{x\to 0^+} y\right) = 0$$

$$\Rightarrow \lim_{x \to 0^+} y = e^0 = 1 = \lim_{x\to 0^+} \left(\ln(x+1)\right)^x$$