T-1 Quiz - 1 - Solution Key

11.11.2021

1) Evaluate the limit $\lim_{x \to 0^+} \dfrac{\frac{1}{x} + \sin\frac{1}{x}}{\frac{1}{x}}$ or explain why it does not exist.

Solution: $\lim_{x \to 0^+} \dfrac{\frac{1}{x} + \sin\frac{1}{x}}{\frac{1}{x}} = \lim_{x \to 0^+} \left(\overbrace{\dfrac{\cancel{\frac{1}{x}}}{\cancel{\frac{1}{x}}}}^{1} + \overbrace{\dfrac{\sin\frac{1}{x}}{\frac{1}{x}}}^{0} \right) = 1$

Since $\lim_{x \to 0^+} \dfrac{\sin\frac{1}{x}}{\frac{1}{x}} = \lim_{t \to \infty} \dfrac{\sin t}{t} = 0$

let $t = \frac{1}{x}$

when $x \to 0^+$

$t \to +\infty$

$\left(-1 \leq \sin t \leq 1 \right.$

$-\dfrac{1}{t} \leq \dfrac{\sin t}{t} \leq \dfrac{1}{t}$

when $t \to \infty$: $-\dfrac{1}{t} \to 0$, $\dfrac{\sin t}{t} \to 0$, $\dfrac{1}{t} \to 0$

$\left(\lim_{t \to \infty} -\frac{1}{t} = 0 \text{ and } \lim_{t \to \infty} \frac{1}{t} = 0 \right)$

So, $\lim_{t \to \infty} \dfrac{\sin t}{t} = 0$ by Squeeze theorem. $\left.\right)$