MATH119 – Recit. Prb. – Week01

① Solve the inequalities and express your answer in terms of intervals

(a) $\dfrac{3}{x-1} < \dfrac{2}{x+1}$ (b) $|2x+5| < 1$ (c) $x^2 - x < 2$ (d) $|x-3| < 2|x|$

Solution: a) $\dfrac{3}{x-1} < \dfrac{2}{x+1} \Rightarrow \dfrac{3}{x-1} - \dfrac{2}{x+1} < 0 \Rightarrow \dfrac{3(x+1)-2(x-1)}{(x-1)(x+1)} < 0$

$\Rightarrow \boxed{\dfrac{x+5}{(x-1)(x+1)}} < 0$, $\dfrac{x+5}{(x-1)(x+1)} = f(x)$

		-5		-1		1	
$f(x)$	$-$	0	$+$	$\Vert$	$-$	$\Vert$	$+$

So, the solution set is

$(-\infty, -5) \cup (-1, 1)$

d) $|x-3| < 2|x| \Rightarrow |x-3| - 2|x| < 0$, $x = 3$, $x = 0$.

If $x < 0$; $|x-3| - 2|x| < 0 \Rightarrow -(x-3) - 2(-x) < 0 \Rightarrow x + 3 < 0$
$\Rightarrow x < -3$

So, we get $x < -3$

If $0 \leq x < 3$; $|x-3| - 2|x| < 0 \Rightarrow -(x-3) - 2x < 0$
$\Rightarrow -3x + 3 < 0 \Rightarrow 3x > 3$
$\Rightarrow x > 1$

So, we get $1 < x < 3$

If $x \geq 3$; $|x-3| - 2|x| < 0 \Rightarrow x - 3 - 2x < 0 \Rightarrow -x - 3 < 0$
$\Rightarrow x > -3$

So, we get $x \geq 3$.

Thus, the solution set will be

$(-\infty, -3) \cup (1, \infty)$

2) Write the following expressions in interval form:

(a) $x \geq 0$ and $x \leq 5$ (b) $x > -5$ or $x < -6$

Sol: a) $[0,5]$ (b) $(-\infty,-6) \cup (-5,\infty)$

3) Find an equation for the given line

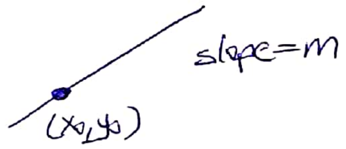
(a) through the points $(3,-5)$ with slope -2

(b) " " $(1,2)$ and $(2,5)$

(c) " " $(1,2)$ and $(2,2)$

(d) " " $(1,2)$ and $(1,5)$

Soln: An equation for the straight line passing through the point (x_0,y_0) with slope m is



$$y-y_0=m(x-x_0)$$

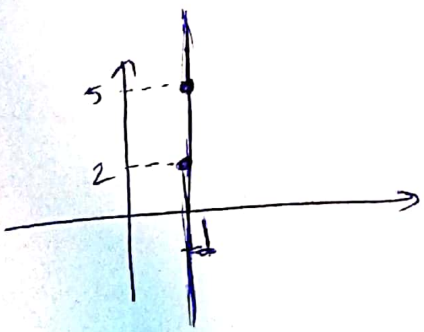
a) Point $=(3,-5)$, $m=-2$

$$y-(-5)=(-2).(x-3) \Rightarrow y+5=-2x+6 \Rightarrow y=-2x+1$$

b) $m=\frac{5-2}{2-1}=3$, let the point be $(1,2)$. (May choose any one of them)

$$y-2=3.(x-1) \Rightarrow y=3x-1$$

d)



$x=1$.

④ Find the point of intersection of the lines $2x-y=5$ and $3x+y=10$

Soln: $\left.\begin{array}{l}2x-y=5\\3x+y=10\end{array}\right\} \Rightarrow y=2x-5$, put this into 2nd eqn, we get

$3x+2x-5=10 \Rightarrow x=3$

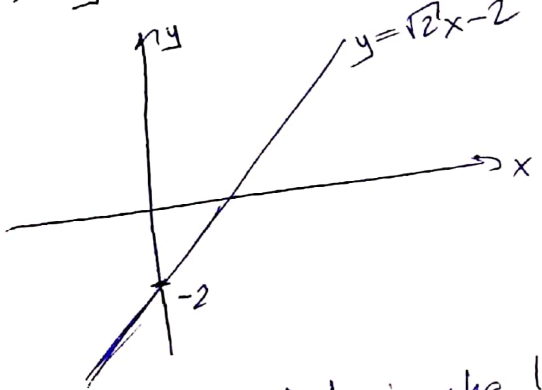
$\Rightarrow y=1$, so the point of intersection is $(3,1)$.

⑤ Find the slope, y-intercept and sketch the line $\sqrt{2}\,x-y=2$.

Soln: $\sqrt{2}\,x-y=2 \Rightarrow y=\sqrt{2}\,x-2$

The slope is $m=\sqrt{2}$.

$x=0 \Rightarrow y=-2$., So, y-intercept is the point $(0,-2)$.



⑥ For what value of k is the line $2x+ky=3$ perpendicular to the line $4x+y=1$? For what value of k are the lines parallel?

Soln: The slope m_1 of the line $y=\frac{-2x}{k}+\frac{3}{k}$ is $\left(\frac{-2}{k}\right)$. $(k\neq 0)$.

The slope m_2 " $y=-4x+1$ is (-4).

$\left(-\frac{2}{k}\right)\cdot(-4)=-1 \Rightarrow k=-8$. So, if $k=-8$ these lines are perp.

If $\left(-\frac{2}{k}\right)=-4 \Rightarrow k=\frac{1}{2}$, then these lines are parallel.

(7) Sketch a rough graph

(a) $x^2+y^2-2x-y+1=0$, (b) $x^2+y^2-4x+2y>4$

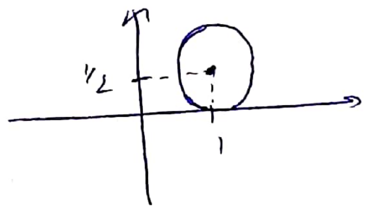
$x+y>1$

(c) $y=6x^2+13x+5$, (d) $x=-y^2+2y+3$

Soln: a) $x^2+y^2-2x-y+1=0 \Rightarrow (x-1)^2+(y-1/2)^2-1-\frac{1}{4}+1=0$

$\Rightarrow (x-1)^2+(y-1/2)^2=1/4$

Circle with radius $r=1/2$ and center $=(1,1/2)$



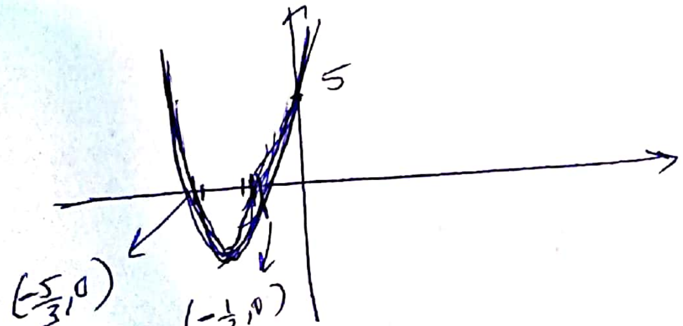
c) $y=6x^2+13x+5 \Rightarrow \Delta = 169-4.5.6=169-120=49$

the coefficient of x^2 is positive.

It is a parabola which is upward sine

Roots of $6x^2+13x+5$ are $\frac{-13\pm\sqrt{49}}{12} = \left(-\frac{13}{12}\right)\pm\frac{7}{12}$

$\Rightarrow x_1=-\frac{1}{2}$, $x_2=-5/3$. And $x=0 \Rightarrow y=5$



⑧ Find the domain of the function $f(x) = \frac{1}{1-\sqrt{x-2}}$

Soln: $x-2 \geq 0 \Rightarrow x \geq 2$ and

$\sqrt{x-2} \neq 1 \Rightarrow x \neq 3$. So, the domain is

$Dom(f(x)) = [2, \infty) - \{3\}$

⑨ Sketch a rough graph of the given func. and determine whether it is an even/odd func.

(a) $f(x) = x^{2/3}$, (b) $f(x) = (x+2)^3$, (c) $f(x) = |x^2 - 1|$

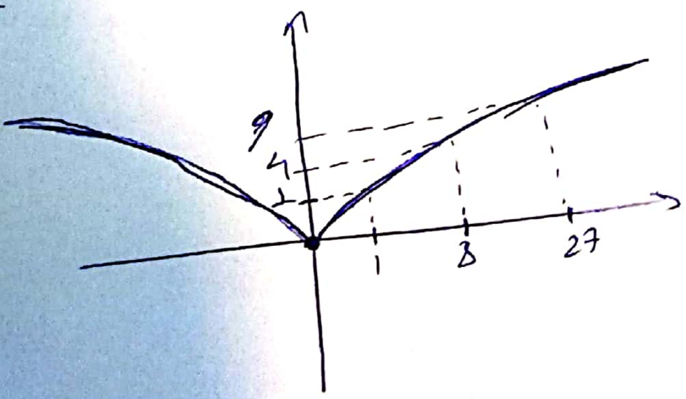
(d) $f(x) = \sqrt{x+1}$, (e) $f(x) = \frac{x}{1-x}$ (Write it as $-\left(1+\frac{1}{x-1}\right)$ and use the graph of $y = 1/x$)

(f) $f(x) = \begin{cases} 2x+1 & \text{if } x < -2, \\ 3 & \text{if } -2 \leq x \leq 1, \\ x^2 & \text{if } x > 1 \end{cases}$

Soln: $y = x^{2/3} = \sqrt[3]{x^2} = f(x)$,

since $f(-x) = \sqrt[3]{(-x)^2} = \sqrt[3]{x^2} = f(x)$ for all $x \in \mathbb{R}$, $f(x)$ is an even func. So, the graph will be symmetric with respect to the y-axis. Try familiar values; e.g. $f(1) = 1$, $f(8) = (8)^{2/3} = 4$, $f(27) = (3^3)^{2/3} = 9$, $f(64) = (4^3)^{2/3} = 16$, ...

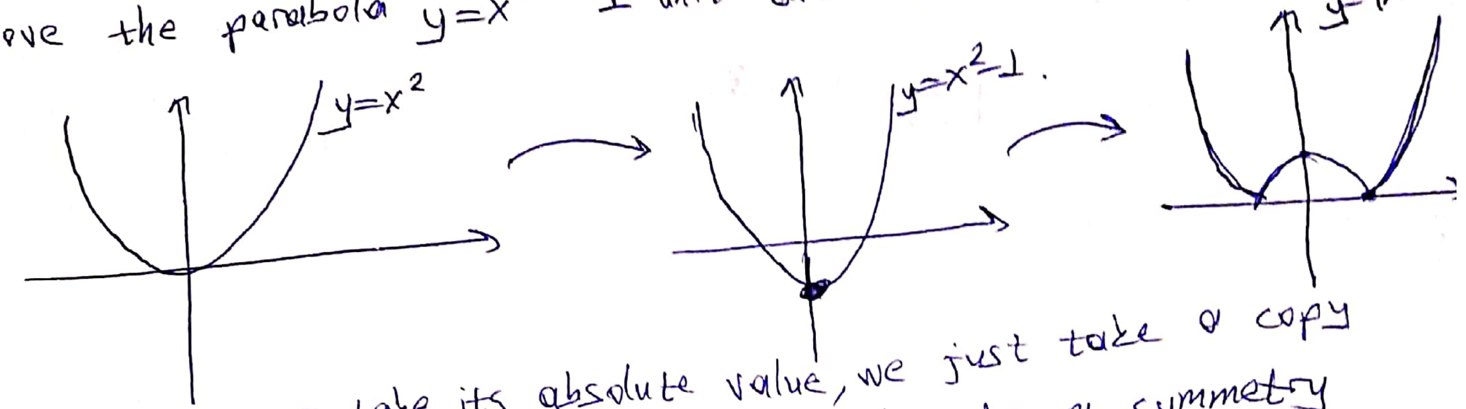
Note that $f(x) = \sqrt[3]{x^2} \geq 0$, and when $x \to \infty$, $f(x) \to \infty$.



c) $f(x) = |x^2-1|$

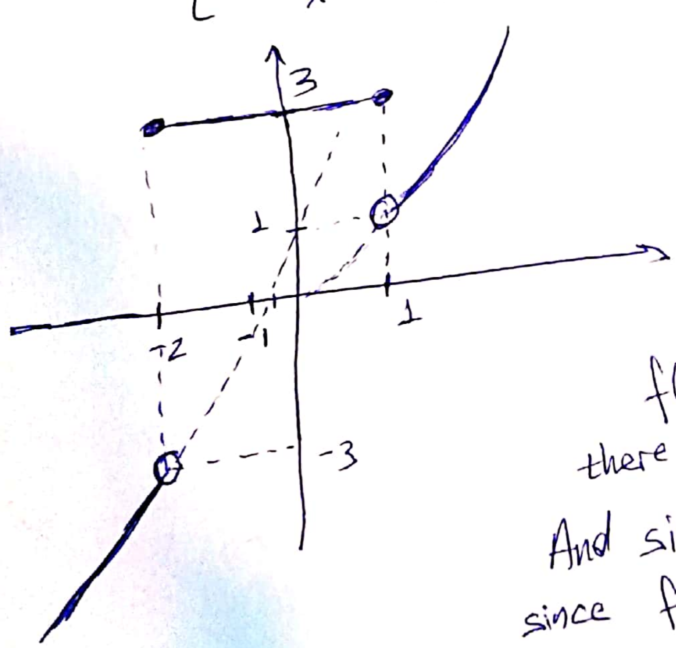
$f(-x) = |(-x)^2-1| = |x^2-1| = f(x)$, so it is an even func.

First, let us sketch the graph of $y=x^2-1$. For this, just move the parabola $y=x^2$ 1 unit downward.



Then, since we take its absolute value, we just take a copy of the negative parts of the graph and take a symmetry w.r.t. the x-axis

(f) $f(x) = \begin{cases} 2x+1 & \text{if } x<-2 \\ 3 & \text{if } -2 \le x \le 1 \\ x^2 & \text{if } x>1 \end{cases}$



$y=2x+1 \Rightarrow$ $x=0 \Rightarrow y=1$

$y=0 \Rightarrow x=-1/2$

$m=2$

f(x) is not an even func. since there is no symmetry w.r.t. the y-axis.

And similary f(x) is not an odd func. since f(x) is not symmetric w.r.t. the origin.

(10) Let $f(x) = \frac{1}{1-x}$ and $g(x) = \sqrt{x-1}$. Find the functions $f \circ f$, $f \circ g$, $g \circ f$, $g \circ g$ and their domains.

<u>Soln:</u> Let us find $(g \circ f)(x)$.

To determine the domain of this func., we need to check the range($f(x)$) as well since $\sqrt{x-1}$ is not defined for the numbers less than 1. So,

$$\frac{1}{1-x} \geq 1 \Rightarrow \frac{1}{1-x} - 1 \geq 0 \Rightarrow \frac{x}{1-x} \geq 0 \Rightarrow x \in [0,1)$$

Also $x \neq 1$.

		0		1	
$\frac{x}{1-x}$	$-$		$+$	$\Vert$	$-$

, So, $\text{Dom}(g \circ f) = [0,1)$

$$(g \circ f)(x) = g(f(x)) = g\left(\frac{1}{1-x}\right) = \sqrt{\frac{1}{1-x} - 1} = \sqrt{\frac{x}{1-x}}$$

(11) Let f be an even func. and g be an odd func. defined on $\mathbb{R}$. Are the following func.s even, odd, neither: $f+g$, $f \cdot g$, $f^2 = f \cdot f$, $g^2 = g \cdot g$, $f \circ f$, $f \circ g$, $g \circ f$, $g \circ g$?

<u>Soln:</u> If f is an even func., $f(-x) = f(x)$ for all x. and

If g is an odd func, $g(-x) = -g(x)$ for all x.

$(f+g)(-x) = f(-x) + g(-x) = f(x) - g(x) = (f-g)(x)$. Neither.

$f^2(x) = f(-x) \cdot f(-x) = f(x) \cdot f(x) = f^2(x)$, even func.

$(f \circ g)(-x) = f(g(-x)) = f(-g(x)) = f(g(x)) = (f \circ g)(x)$, even func

$(g \circ g)(-x) = g(g(-x)) = g(-g(x)) = -g(g(x)) = -(g \circ g)(x)$, odd func.