

MATH 119 - 2021-1 - QUIZ 2

Question: Find the tangent line to $y = \sqrt[3]{x} \cdot g(3x-1)$ at $x=1$, if $g(x)$ is a function such that $g(2)=-2$ and $g'(2)=1$.

Solution: For the slope of the tangent line, we need to find the derivative of y at $x=1$.

$$\frac{dy}{dx} = \frac{1}{3} \cdot x^{-2/3} \cdot g(3x-1) + \sqrt[3]{x} \cdot 3 \cdot g'(3x-1)$$

$$\Rightarrow \left. \frac{dy}{dx} \right|_{x=1} = \frac{1}{3} \cdot g(2) + 3 \cdot g'(2) = \frac{1}{3} \cdot (-2) + 3 = \frac{7}{3}$$

Also, $y(1) = 1 \cdot g(2) = -2$, so the point is $(1, -2)$.

Thus, an equation for the tang. line is

$$y - (-2) = \frac{7}{3} \cdot (x - 1)$$

$$\Rightarrow y = \frac{7}{3}x - \frac{13}{3}$$