

FRIDAY 15:40 - 17:30

Section:

Name & Surname:

SAMPLE

Math 119 Spring 2021-2022

Quiz No: 04

Date: 27.05.2022

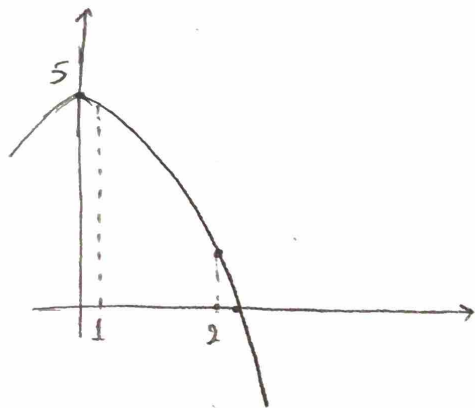
Time Limit: ~12 Minutes

ID Number:

SOLUTION

Grade:

1. Calculate the lower and upper Riemann sums for the function  $f(x) = 5 - x^2$  on the interval  $[1, 2]$ , corresponding to the partition  $P$  of  $[1, 2]$  into four subintervals of equal length.



$P = \left\{ 1, \frac{5}{4}, \frac{3}{2}, \frac{7}{4}, 2 \right\}$  is the equal length partition of  $[1, 2]$  with  $\Delta x_i = \frac{1}{4} = \left( \frac{b-a}{4} \right)$

On the interval  $[1, 2]$ ,  $f$  is decreasing

Lower Riemann Sum:

$$\left[ 1, \frac{5}{4} \right]: \text{Min } f\left(\frac{5}{4}\right) = \frac{55}{16}$$

$$\left[ \frac{5}{4}, \frac{3}{2} \right]: \text{Min } f\left(\frac{3}{2}\right) = \frac{11}{4}$$

$$\left[ \frac{3}{2}, \frac{7}{4} \right]: \text{Min } f\left(\frac{7}{4}\right) = \frac{31}{16}$$

$$\left[ \frac{7}{4}, 2 \right]: \text{Min } f(2) = 1$$

$$\begin{aligned} L(f, P) &= f\left(\frac{5}{4}\right)\Delta x_1 + f\left(\frac{3}{2}\right)\Delta x_2 + f\left(\frac{7}{4}\right)\Delta x_3 + f(2)\Delta x_4 \\ &= \frac{55}{16} \cdot \frac{1}{4} + \frac{11}{4} \cdot \frac{1}{4} + \frac{31}{16} \cdot \frac{1}{4} + 1 \cdot \frac{1}{4} \\ &= \frac{146}{64} = \frac{73}{32} \end{aligned}$$

Upper Riemann Sum:

$$\left[ 1, \frac{5}{4} \right]: \text{Max } f(1) = 4$$

$$\left[ \frac{5}{4}, \frac{3}{2} \right]: \text{Max } f\left(\frac{5}{4}\right) = \frac{55}{16}$$

$$\left[ \frac{3}{2}, \frac{7}{4} \right]: \text{Max } f\left(\frac{3}{2}\right) = \frac{11}{4}$$

$$\left[ \frac{7}{4}, 2 \right]: \text{Max } f\left(\frac{7}{4}\right) = \frac{21}{16}$$

$$\begin{aligned} U(f, P) &= f(1)\Delta x_1 + f\left(\frac{5}{4}\right)\Delta x_2 + f\left(\frac{3}{2}\right)\Delta x_3 + f\left(\frac{7}{4}\right)\Delta x_4 \\ &= 4 \cdot \frac{1}{4} + \frac{55}{16} \cdot \frac{1}{4} + \frac{11}{4} \cdot \frac{1}{4} + \frac{21}{16} \cdot \frac{1}{4} \\ &= \frac{194}{64} = \frac{97}{32} \end{aligned}$$