FRIDAY 15:40 - 17:30

Section:

Name & Surname: SAMPLE

ID Number: SOLUTION

Grade:

Math 119 Spring 2021-2022
Quiz no.: 03
Date: 29.04.2022
Time Limit: ~10 Minutes

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1. Using logarithmic differentiation, find $f'\left(\frac{\pi}{4}\right)$ if $f(x) = (\sin x)^{x^3}$ for $x \in (0,1)$. Explain why $f(x)$ is defined for $0 < x < 1$.

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4 Note that the logarithmic function has domain $(0, \infty)$.
To use logarithmic differentiation, $f(x) > 0$ on its domain.
For $x \in (0,1)$, $\sin x > 0$ and $(\sin x)^{x^3} > 0$.

4 Now, take the natural logarithm both sides of $f(x) = (\sin x)^{x^3}$

$$\ln f(x) = x^3 \ln(\sin x)$$

Take derivative w.r.t. x,

$$\frac{f'(x)}{f(x)} = 3x^2 \ln(\sin x) + x^3 \frac{\cos x}{\sin x}$$

Thus, we have

$$f'(x) = (\sin x)^{x^3}\left[3x^2 \ln(\sin x) + x^3 \cot x\right]$$

2 Putting $x = \frac{\pi}{4}$,

$$f'\left(\frac{\pi}{4}\right) = \left[\sin\left(\frac{\pi}{4}\right)\right]^{\left(\frac{\pi}{4}\right)^3}\left[3\left(\frac{\pi}{4}\right)^2 \ln\left(\sin\frac{\pi}{4}\right) + \left(\frac{\pi}{4}\right)^3 \underbrace{\cot\left(\frac{\pi}{4}\right)}_{1}\right]$$

$$= \left(\frac{\sqrt{2}}{2}\right)^{\left(\frac{\pi}{4}\right)^3}\left[3\left(\frac{\pi}{4}\right)^2 \ln\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\pi}{4}\right)^3\right]$$