FRIDAY – 15:40-17:30

Section:

Name & Surname: SAMPLE

Math 119 Spring 2021-2022
Quiz no.: 02
Date: 15.04.2022
Time Limit: ~10 Minutes

ID Number: SOLUTION

Grade:

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1. Consider the curve defined implicitly by the equation

$$\cos(xy) + y\sin(x-1) = 0.$$

Show that the tangent line to this curve at the point $\mathbf{P}\left(1, \frac{\pi}{2}\right)$ is horizontal.

Differentiate both sides of the equation with respect to x, considering $y = y(x)$

$$-\sin(xy)\left[y + x\frac{dy}{dx}\right] + \left[\frac{dy}{dx}\sin(x-1) + y\cos(x-1)\right] = 0$$

Check that $P\left(1, \frac{\pi}{2}\right)$ is on the curve

$$\cos\left(1\cdot\frac{\pi}{2}\right) + \frac{\pi}{2}\sin(1-1) = 0 \Rightarrow 0 = 0 \quad \checkmark \qquad y(1) = \frac{\pi}{2}$$

We need to find the slope of the tangent line $\left.\frac{dy}{dx}\right|_{x=1}$

$$-\sin\left(\frac{\pi}{2}\right)\left[\frac{\pi}{2} + \left.\frac{dy}{dx}\right|_{x=1}\right] + \left[\left.\frac{dy}{dx}\right|_{x=1}\underbrace{\sin(1-1)}_{=0} + \frac{\pi}{2}\underbrace{\cos(1-1)}_{=1}\right] = 0$$

$$-1\left[\frac{\pi}{2} + \left.\frac{dy}{dx}\right|_{x=1}\right] + \frac{\pi}{2} = 0$$

$$\left.\frac{dy}{dx}\right|_{x=1} = 0$$

Thus, the tangent line at the point $P\left(1, \frac{\pi}{2}\right)$ is horizontal.