Section:

Name & Surname: SAMPLE SOLUTION

Math 119 Spring 2021-2022
Quiz no.: 01
Date: 01.04.22
Time Limit: ~10 Minutes

ID Number:

Grade:

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1. **If they exist, find the values of** a and b that make f continuous everywhere on $\mathbb{R}$.

$$f(x) = \begin{cases} \dfrac{x^2-4}{x+2} & \text{if } x < -2 \\ ax^2 - bx + 1 & \text{if } -2 \le x < 2 \\ ax - 3b & \text{if } x \ge 2 \end{cases}$$

To be continuous everywhere, we need to check all endpoints of the subdomains.

For $x=-2$, $\lim_{x\to -2^-} f(x) = \lim_{x\to -2^-} \frac{x^2-4}{x+2} = \lim_{x\to -2^-} \frac{(x-2)(x+2)}{x+2} = -4$

$\lim_{x\to -2^+} f(x) = f(-2) = 4a+2b+1$

$\Rightarrow$ (1) $4a+2b+1 = -4$

For $x=2$, $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} (ax^2+bx+1) = 4a-2b+1$

$\lim_{x\to 2^+} f(x) = f(2) = 2a-3b$

$\Rightarrow$ (2) $4a-2b+1 = 2a-3b$

$2a+b = -1$

By (1) & (2), $4a+2b=-5$

$-2a+b=-1$ (+)

$0 = -1$ $\therefore$ This is impossible.

Thus, there is no such a & b that make f continuous everywhere on $\mathbb{R}$.