Spring 2018
Quiz 6
11.05.18
Time Limit: 15 Minutes
Section: 24

Name: Solution Key

ID number:

Grade:

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1. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a function given by $f(x,y) = \begin{cases} \dfrac{x^2+xy+y^2}{x+y} & if\ y \neq -x \\ 0 & if\ y = -x \end{cases}$.

$a)$ Calculate $\dfrac{\partial f}{\partial x}$ at the point $(0,0)$ by using definition of derivatives.

$b)$ Calculate $\lim\limits_{(x,y)\to(0,0)} f(x,y)$ on the path $y = x^2 - x$. Decide whether f is continuous or not at $(0,0)$.

a) $$\lim_{h\to 0} \frac{f(0+h,0)-f(0,0)}{h} = \lim_{h\to 0} \frac{(h^2-0)/h}{h} = 1.$$

b) $$\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{\substack{x\to 0 \\ y=x^2-x}} \frac{x^2+x(x^2-x)+(x^2-x)^2}{x+\cancel{x^2}-\cancel{x}}$$

$$= \lim_{x\to 0} \frac{\cancel{x^2}+x^3-\cancel{x^2}+x^4-2x^3+x^2}{x^2} = \lim_{x\to 0} \frac{x^4-x^3+x^2}{x^2}$$

$$= \lim_{x\to 0} \frac{\cancel{x^2}(x^2-x+1)}{\cancel{x^2}} = \lim_{x\to 0}(x^2-x+1) = 1.$$

Along $y=0$.

$$\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{x\to 0} \frac{x^2}{x} = 0$$

Limit does not exis. Therefore, f is not continuous at $(0,0)$