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Time: April 6, 12:15
Duration: 15 min.
Weight: 10 points
Score: _____

MATH 118 - 2018 Spring
Section-24 Quiz-4

- a) Using definition of Maclaurin series, find the Maclaurin series of $f(x) = e^x$. (5pts)
b) Determine the interval of convergence of the Maclaurin series of $f(x) = e^x$. (3pts)
c) Find the sum of the series $\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}$. (2pts)

Note: Show all your work as is done in the lectures.

ANSWER

a) Maclaurin series of $f(x)$, $\sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$

$$f(x) = e^x, f'(x) = e^x, \dots, f^{(n)}(x) = e^x$$

$$f(0) = 1, f'(0) = 1, \dots, f^{(n)}(0) = 1$$

$$\Rightarrow \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

b) Let us use the ratio test,

$$\lim_{n \rightarrow \infty} \left| \frac{x^{n+1}}{(n+1)!} \cdot \frac{n!}{x^n} \right| = \lim_{n \rightarrow \infty} \left| \frac{x}{n+1} \right| = 0 < 1 \text{ for all } x \in \mathbb{R}.$$

So, interval of convergence is $(-\infty, \infty)$

c) $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \rightarrow \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} = e^{-1} = \frac{1}{e}$