**MATH 353 Complex Calculus**

**Credit:** (4-0) 4

**Catalog description:** Algebra of complex numbers. Polar representation. Analyticity. Cauchy-Riemann equations. Power series. Elementary functions. Mapping by elementary functions. Linear fractional transformations. Line integral. Cauchy-Theorem. Cauchy integral formula. Taylor`s Series. Laurent series. Residues. Residue theorem. Improper integrals.

**Course Objectives:** By the end of this course, a student will:

- Understand in detail the complex number system and the complex plane,
- Use functions of a complex variable and explore their properties
- Understand the concepts of the derivative of a complex function and analyticity of a function,
- Make calculations with elementary functions of a complex variable
- Evaluate line integrals and prove results using the Cauchy-Goursat theorem and the Cauchy integral formulas,
- Master the concepts of complex sequences and infinite series, Laurent series, residues and the residue theorem and apply them to various problems
- Understand the basics of conformal mappings and Möbius transformations

**Course Website:** https://odtuclass.metu.edu.tr/

**Instructor:** Özcan Yazıcı.
**Assistant:** M.Yiğit Demir.

**Office Hours:** To be announced.

**Textbook:** Complex Analysis, Bak, J., Newman D. J., Third Edition.

**Reference Material:** Complex Analysis, Gamelin, T. W.
Complex Variables and Applications, Brown, J. W., Churchill, R. V., 8th ed.

**Exams and Grading:**

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|---|---|
| Midterm 1 | : 30 % - Nov 7, 2024 |
| Midterm 2 | : 30 % - Dec 12, 2024 |
| Final | : 40 % (During Finals' week, date TBA) |

**Attendance:** Students must attend to the lectures regularly.

**Suggested Problems**: Suggested problems from the textbook will be posted on ODTUCLASS. Students are encouraged to attempt to solve all of these problems. Some of the exam problems will be chosen among these problems.

**NA Policy:** A student who misses all exams will receive a grade of NA for the course.

**Make-up Policy:** In order to be eligible to enter a make-up examination for a missed examination, a student should have a documented or verifiable, and officially acceptable excuse. A student cannot get make-up examinations for two missed exams. The make-up examination for all exams will be after the final exam, and will include all topics.

The table below is a rough guideline for the content of course lectures. Chapters and problems are from the textbook, "Complex Analysis", Bak, J., Newman D. J., Third Edition.

| | | | Suggested Problems |
|---|---|---|---|
| **Week 1:** Sep.30-Oct.4 | 1 | **Chapter 1. The Complex Numbers** Complex Numbers, Polar Representation, Regions in the Complex Plane. | **Ch1:** 1-a,b,2,4-a,c 5,10,12-a,c,13, 15-b,c,g, 20,21,25. |
| | 2 | Roots of Complex Numbers. Stereographic Projection. | |
| **Week 2:** Oct.7-11 | 3 | Sequences and Series of Complex Numbers. | |
| | 4 | Continuous Functions. | |
| **Week 3:** Oct.14-18 | 5 | **Chapter 2. Functions of the Complex Variable z** Polynomials, Power Series. | **Ch2:** 2,3,4,8,9-b,10, 14-a,d,16,23-a,c. |
| | 6 | Differentiability of Power Series | |
| **Week 4:** Oct.21-25 | 7 | **Chapter 3. Analytic Functions** Complex Differentiation, Analyticity. | **Ch3:** 2,5,6,7,8,9,10, 14-a,d,15,16,23. |
| | 8 | Cauchy-Riemann Equations. | |
| **Week 5:** Oct.28-Nov1 | 9 | Exponential and Trigonometric functions. | |
| | 10 | The Analytic Function log z **(Section 8.2)** | |
| **Week 6:** Nov.4-8 | 11 | **Chapter 4. Line Integrals and Entire Functions** Properties of Line Integral. | **Ch4:**2,3,5,6,8,9,10,11. |
| | 12 | Fundamental Theorem of Calculus. | |
| **Week 7:** Nov.11-15 | 13 | Line Integral over Rectangles. Antiderivatives (Integral Theorem ) | |
| | 14 | Cauchy-Goursat Theorem. | |
| **Week 8:** Nov.18-22 | 15 | **Chapter 5-6. Properties of Entire Functions and Analytic Functions** Cauchy Integral Formula, Taylor Expansion of Analytic Functions. | **Ch5:** 1,3,6,7,8,9,10,15. |
| | 16 | Liouville Theorem and Applications. | |
| **Week 9:** Nov.25-29 | 17 | Zeros of Analytic Functions, Uniqueness Theorem | **Ch6:** 1,2,4,5,7,8,10,11 12,14. |
| | 18 | Mean Value Property, Maximum Modulus Theorem. | |
| **Week 10:** Dec.2-6 | 19 | **Chapter 7. Further Properties of Analytic Functions** Open Mapping Theorem, Schwarz' Lemma. | **Ch7:** 3,4,5,6,7,8,9, 10,11,12,13,16,19, 20,21. |
| | 20 | Morera's Theorem, Reflection Principle | |
| **Week 11:** Dec.9-13 | 21 | **Chapter 9. Isolated Singularities of an Analytic Function** Classification of Isolated Singularities. | **Ch9:** 1,2,3,4,5,8,9, 11-a,c,12,16,17,18. |
| | 22 | Laurent Series Expansions. | |
| **Week 12:** Dec.16-20 | 23 | **Chapter 10. The Residue Theorem** Residue Theorem. | **Ch10:**1-a,b,d,f ,2,4, 5,8,9,10,12,13. |
| | 24 | Applications of Residue Theorem: Argument Principle, Rouche's Theorem | |
| **Week 13:** Dec.23-27 | 25 | **Chapter 11. Applications of Residue Theorem to the Evaluation of Integrals and Sums** Evaluation of Definite Integrals by Contour Integrals. | **Ch11:**1-a,e,g,j,3,4-a, 8,9-a,13. |
| | 26 | Evaluation and Estimation of Sums by Contour Integrals. | |
| **Week 14:** Dec.30-Jan.3 | 27 | **Chapter 13. Introduction to Conformal Mappings** Conformal Equivalence. | **Ch13:** 2,3,5,6,10, 11,15,19,20. |
| | 28 | Linear Fractional Transformations. | |