(HW-3) Q3.

a) $\lim_{x\to\infty}\left(\sqrt{x^9+x}-x^5\right) = \lim_{x\to\infty}\left(\sqrt{x^{10}\left(\frac{1}{x}+\frac{1}{x^9}\right)}-x^5\right) = \lim_{x\to\infty}\left(\underbrace{|x^5|}_{=x^5 \text{ since } x\to\infty}\cdot\sqrt{\frac{1}{x}+\frac{1}{x^9}}-x^5\right)$

$$= \lim_{x\to\infty} \underbrace{x^5}_{\infty}\cdot\left(\underbrace{\sqrt{\underbrace{\frac{1}{x}}_{0}+\underbrace{\frac{1}{x^9}}_{0}}}_{1}-1\right) = -\infty$$

b) $\lim_{x\to 1^-}\frac{\sqrt{1-x}}{1-\sqrt{x}} = \lim_{x\to 1^-}\frac{\cancel{(1-x)}\cdot\overbrace{(1+\sqrt{x})}^{\to 2}}{\cancel{(1-x)}\cdot\underbrace{(\sqrt{1-x})}_{\to 0^+}} = +\infty$

Multiply by $\frac{1+\sqrt{x}}{1+\sqrt{x}}\cdot\frac{\sqrt{1-x}}{\sqrt{1-x}}=1$

c) $\lim_{x\to-\infty}\frac{2x^3-x+7}{\sqrt{3x^6+x^5-4}} = \lim_{x\to-\infty}\frac{x^3\cdot(2-1/x^2+7/x^3)}{\underbrace{|x^3|}_{=-x^3 \text{ since } x\to-\infty}\cdot\sqrt{3+1/x-4/x^3}}$

$$=\lim_{x\to-\infty}\frac{\cancel{x^3}\,(2-\overbrace{1/x^2}^{0}+\overbrace{7/x^3}^{0})}{-\cancel{x^3}\,\sqrt{3+\underbrace{1/x}_{0}-\underbrace{4/x^3}_{0}}} = -2/\sqrt{3}$$

d) $\frac{1}{x} \underset{-1\le\cos^3 x\le 1}{\le} \frac{2-\cos^3 x}{x+1} \underset{\sin^4 x\le 1}{\le} \frac{2-\cos^3 x}{x+\sin^4 x} \underset{\sin^4 x\ge 0}{\le} \frac{2-\cos^3 x}{x} \underset{-1\le -\cos^3 x\le 1}{\le} \frac{3}{x}$

Since $\lim_{x\to\infty}\frac{1}{x} = 0 = \lim_{x\to\infty}\frac{3}{x}$, by squeeze (sandwich) Thm,

$$\lim_{x\to\infty}\frac{2-\cos^3 x}{x+\sin^4 x} = 0.$$

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e) $\lim_{x\to 2} \frac{|x^3-2x^2+x-2|}{x^2-5x+6} = \lim_{x\to 2} \frac{|x-1||x+1||x-2|}{(x-3)(x-2)}$

$\to \lim_{x\to 2^-} \frac{(x-1).(x+1).[-\cancel{(x-2)}]}{(x-3)\cancel{(x-2)}} = \lim_{x\to 2^-} -\frac{(x-1)(x+1)}{x-3} = 3$

$\to \lim_{x\to 2^+} \frac{(x-1)(x+1)\cancel{(x-2)}}{(x-3)\cancel{(x-2)}} = \lim_{x\to 2^+} \frac{(x-1)(x+1)}{(x-3)} = -3$

Since LHS $\neq$ RHS, this limit dne.

f) $\lim_{x\to 3} \frac{|x^3-26|+|x^3-28|}{|1-x|+1} = \lim_{x\to 3} \frac{x^3-26-(x^3-28)}{-(1-x)+1}$

$= \lim_{x\to 3} \frac{2}{x} = 2/3$

g) since $-1 \le \sin\left(\frac{153}{t}\right) \le 1$, $e^{-1} \le e^{\sin\left(\frac{153}{t}\right)} \le e^{1}$

$\Rightarrow t^2.e^{-1} \le t^2 e^{\sin\left(\frac{153}{t}\right)} \le t^2.e$

Since $\lim_{t\to 0} t^2.e^{-1} = \lim_{t\to 0} t^2 e = 0$, By squeeze $\lim_{t\to 0} t^2.e^{\sin\left(\frac{153}{t}\right)} = 0$

h) $\lim_{x\to 0} \frac{1-\cos x}{x^2} = \lim_{x\to 0} \frac{1-\cos^2 x}{x^2.(1+\cos x)} = \lim_{x\to 0} \frac{\sin^2 x}{x^2.(1+\cos x)}$

Multiply by $\frac{1+\cos x}{1+\cos x} = 1$

$1-\cos^2 x = \sin^2 x$

$= \lim_{x\to 0} \underbrace{\frac{\sin x}{x}}_{1} . \underbrace{\frac{\sin x}{x}}_{1} . \underbrace{\frac{1}{1+\cos x}}_{1/2} = 1/2$

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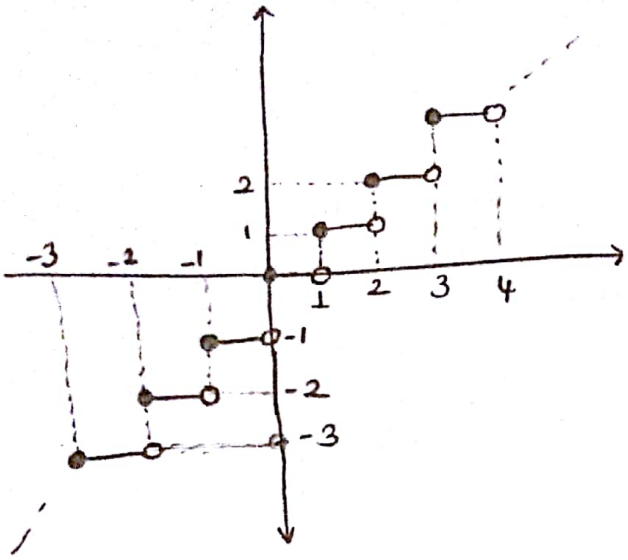
i) Recall $\lfloor x \rfloor = f(x)$ (Greatest integer function)



Note that $x-1 \leq \lfloor x \rfloor \leq x$. $\Rightarrow$ $\dfrac{x-1}{x} \leq \dfrac{\lfloor x \rfloor}{x} \leq \dfrac{x}{x} = 1$

Since $\lim\limits_{x\to\infty} \dfrac{x-1}{x} = \lim\limits_{x\to\infty} \dfrac{x.(1-1/x)}{x} = 1$, and $\lim\limits_{x\to\infty} 1 = 1$

By Squezee, $\lim\limits_{x\to\infty} \dfrac{\lfloor x \rfloor}{x} = 1$.

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