arge as possible ?

Sketching Graph:

a) $y = \dfrac{(x+1)^3}{(x-1)^2}$

i. Domain: $\mathbb{R} - \{1\}$

ii. Intercept pts: $x=0 \Rightarrow y=1$

$y=0 \Rightarrow x=-1$

iii. Asymptotes:

- Oblique:

$$\frac{(x+1)^3}{(x-1)^2} = x+5+\frac{12x-4}{(x-1)^2}$$

$$\lim_{x\to\pm\infty}\left(\frac{(x+1)^3}{(x-1)^2} - (x+5)\right) = 0$$

$\Rightarrow x+5=y$ is oblique asymptote.

- $\lim_{x\to-\infty} y(x) = -\infty$

$\lim_{x\to+\infty} y(x) = +\infty$

HOW? Explain it!

- $\lim_{x\to1^+} y(x) = +\infty$

$\lim_{x\to1^-} y(x) = -\infty$

$x=1$ is vertical asymptote

iv. $y'(x) = \dfrac{3\cdot(x+1)^2(x-1)^2 - 2(x-1)\cdot(x+1)^3}{(x-1)^4}$

$= \dfrac{(x+1)^2\cdot(x-1)\cdot(3(x-1)-2(x+1))}{(x-1)^4}$

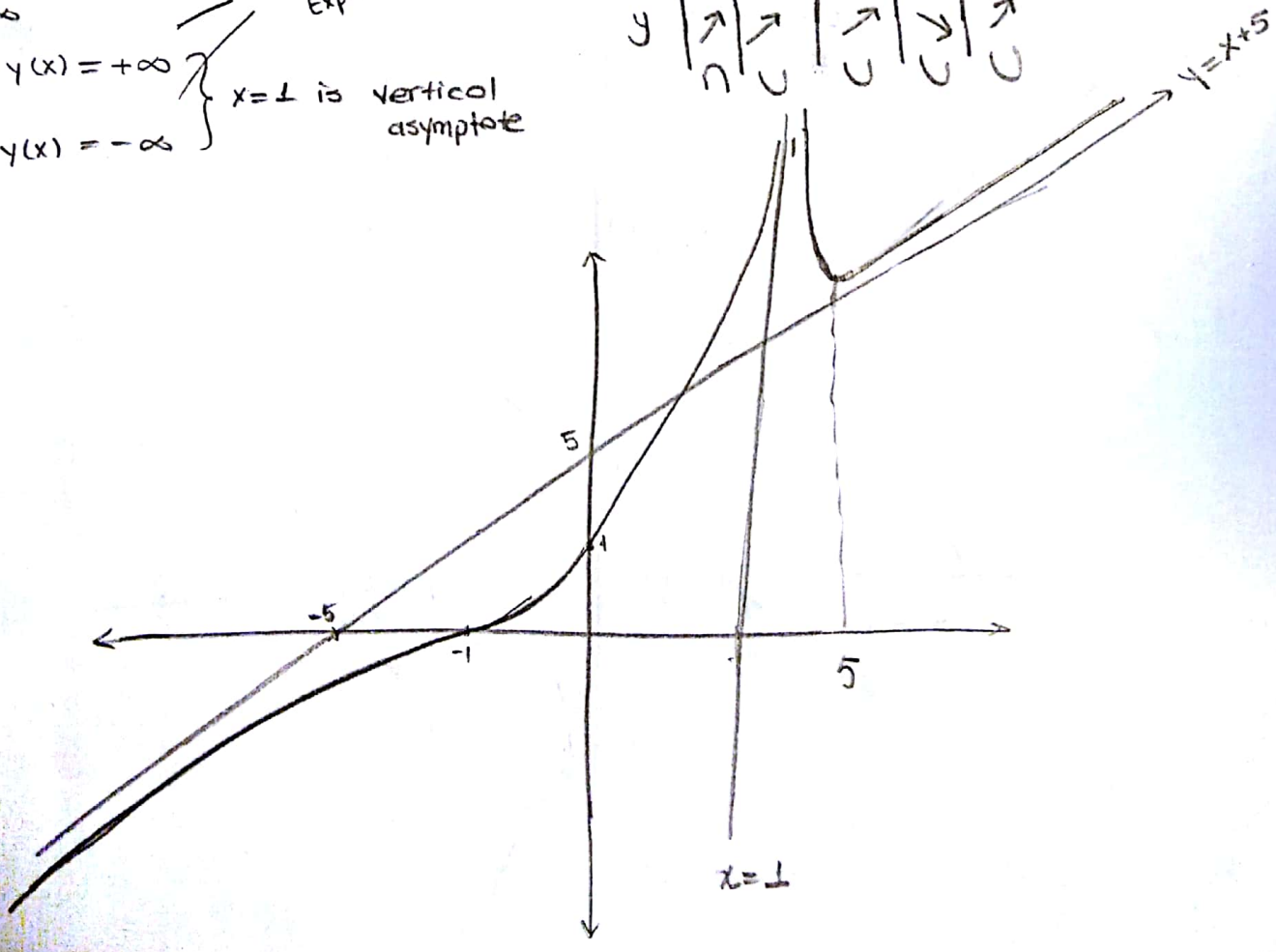
$= \dfrac{(x+1)^2\cdot(x-5)}{(x-1)^3}$

$y'(x)=0 \iff x=-1$ and $x=5$

v. $y''(x) = \dfrac{25\cdot(x+1)}{(x-1)^4}$

$y''(x)=0 \iff x=-1$

x	-1		0		1		5	
y''	$-$		$+$		$+$		$+$	$+$
y'	$+$		$+$		$+$		$-$	$+$
y	↗	∩	↗	∪	↗	∪	↘	↗ ∪



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b. $y = \dfrac{x^2+1}{x-1}$

i. Domain: $\mathbb{R} - \{1\}$

ii. Intercept pts:

$x = 0 \Rightarrow y = -1$

$y = 0 \Rightarrow$ no x-intercept

iii, Asymptotes:

• oblique asymptote:

$$\frac{x^2+1}{x-1} = x+1 + \frac{2}{x-1}$$

$$\lim_{x\to\pm\infty}\left(x+1 - \frac{x^2+1}{x-1}\right) = 0$$

$\Rightarrow y = x+1$ is oblique asymptote.

• $\lim_{x\to\infty} y(x) = \infty$

$\lim_{x\to-\infty} y(x) = -\infty$ — HOW??

• $\lim_{x\to1^+} y(x) = \infty$

$\lim_{x\to1^-} y(x) = -\infty$ } $x=1$ is vertical asymptote.

iv. $y' = \dfrac{2x\cdot(x-1) - (x^2+1)}{(x-1)^2} = \dfrac{x^2-2x-1}{(x-1)^2}$

$y'(x) = 0 \iff x^2 - 2x - 1 = 0.$

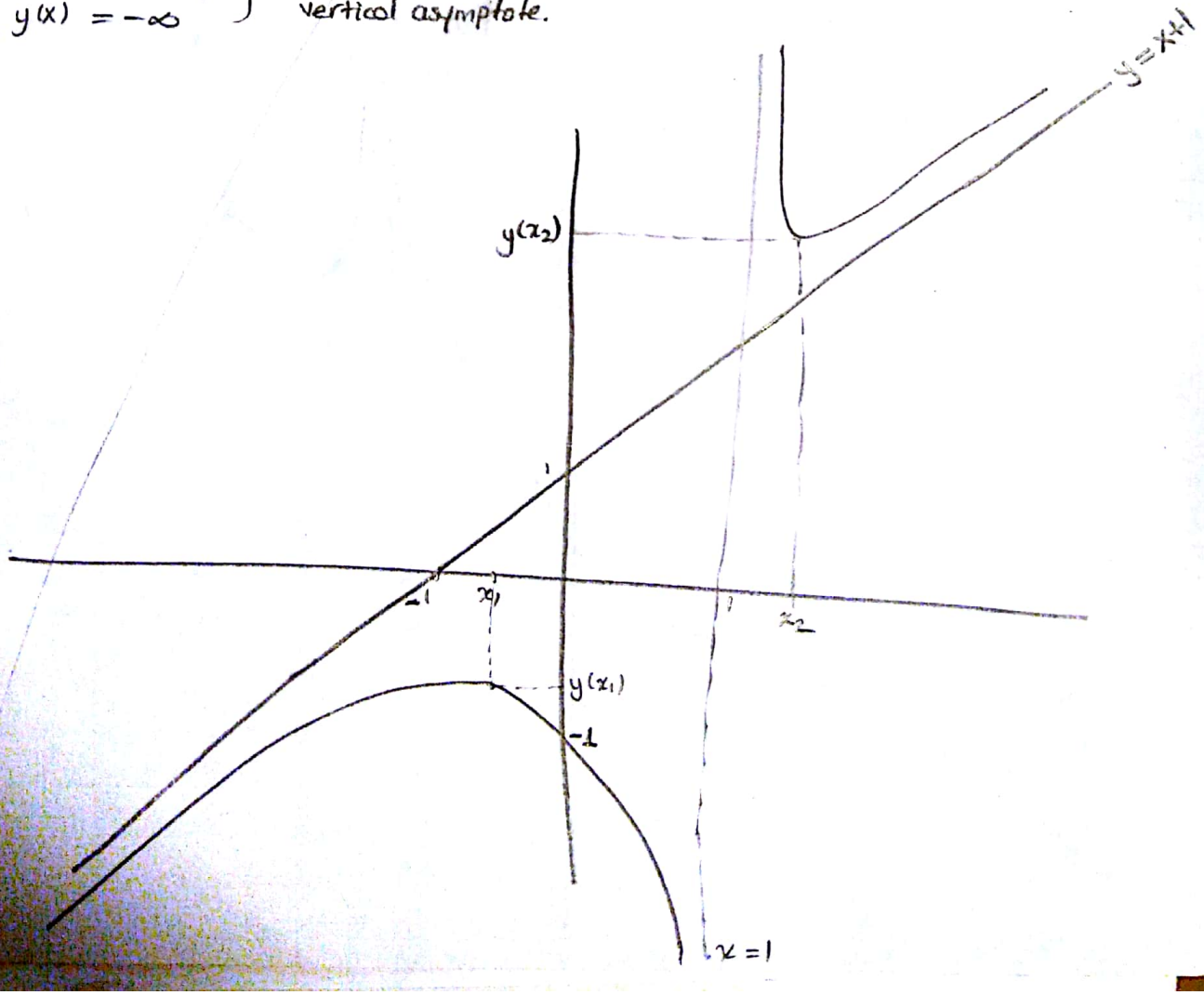
$\Delta = 4+4 = 8 > 0.$

$x_1 \simeq -0.414 \quad x_2 \simeq 2.414$

v. $y''(x) = \dfrac{(2x-2)(x-1)^2 - 2(x-1)(x^2-2x-1)}{(x-1)^4}$

$= \dfrac{2x^2 - 4x + 2 - 2x^2 + 4x + 2}{(x-1)^3} = \dfrac{4}{(x-1)^3}$

x		x_1		1		x_2	
y''	$-$		$-$		$+$		$+$
y'	$+$		$-$		$-$		$+$
y	↗		↘		↘		↗
	∩		∩		∪		∪



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c. $y = e^{-x}\sin x$ on $[0, 5\pi]$

i. intercept

$x = 0 \Rightarrow y = 0$

$y = 0 \Rightarrow x = 0, \pi, 2\pi, 3\pi, 4\pi, 5\pi$

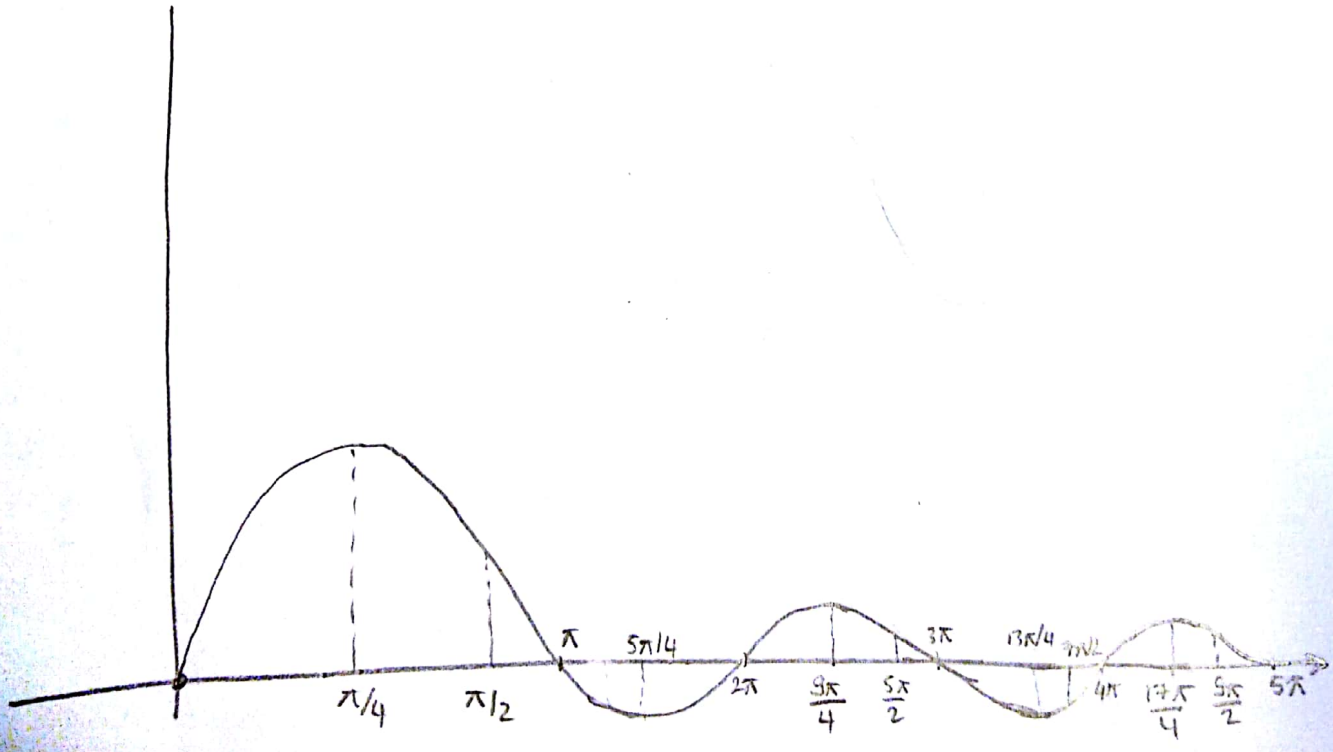
ii. $y' = e^{-x}(\cos x - \sin x)$

$y' = 0 \Leftrightarrow \cos x = \sin x \Rightarrow x = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4}, \frac{17\pi}{4}$

iii. $y'' = -2e^{-x}\cos x$

$y'' = 0 \Leftrightarrow \cos x = 0 \Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \frac{9\pi}{2}$

	0		$\pi/4$		$\pi/2$		$5\pi/4$		$3\pi/2$		$9\pi/4$		$5\pi/2$		$13\pi/4$		$7\pi/2$		$17\pi/4$		$9\pi/2$		5π
y''		−		−		+		+		−		−		+		+		−		−		+	
y'	+		−		−		+		+		−		−		+		+		−		−		
	↗		↘		↘		↗		↗		↘		↘		↗		↗		↘		↘		
	∩		∩		∪		∪		∩		∩		∪		∪		∩		∩		∪		



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