MATH 153 4th Jan 2019 - QUIZ 6

Surname, Name, Student ID and Section: Name of assistant :) _____

1. (10 points) Sketch the graph of

$$f(x) = \frac{-3x^2+2}{x-1} = -3\cdot(x+1) - \frac{1}{x-1}$$

i domain, range, and symmetry

$\text{Dom} f = \mathbb{R} - \{1\}$

$\text{Range} f = \mathbb{R}$

$f(-x) = \frac{-3x^2+2}{x-1} \neq f(x)$, $\neq -f(x)$ No symm.

ii intercepts and asymptotes (HA,VA,OA if any)

x-inter: $-3x^2+2=0 \Rightarrow x = \mp\frac{\sqrt{2}}{\sqrt{3}}$

y-inter: $f(0) = -2 \Rightarrow y = -2$

$\# \lim_{x\to+\infty} f(x) = \lim_{x\to\infty} \frac{x^2\cdot(-3+2/x^2)}{x(1-1/x)} = -\infty$

$* \wedge \# \Rightarrow$ no h.a.

$* \lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{x^2\cdot(-3+2/x^2)}{x\cdot(1-1/x)} = +\infty$

$\lim_{x\to1^-} f(x) = +\infty$ and $\lim_{x\to1^+} f(x) = -\infty$

so $x=1$ is V.A.

$\lim_{x\to\mp\infty} f(x) - (-3x-3) = 0 \Rightarrow y = -3x-3$ is O.A.

iii derivatives and tangents

$f'(x) = -3 - 1\cdot\frac{(-1)}{(x-1)^2}$

$f'(x) = \frac{-3x^2+6x-2}{(x-1)^2}$ so $f'(x)=0 \Leftrightarrow x = \frac{-6\mp\sqrt{12}}{-6}$, $a = \frac{-6-\sqrt{12}}{-6} > 0$, $b = \frac{-6+\sqrt{12}}{-6} > 0$

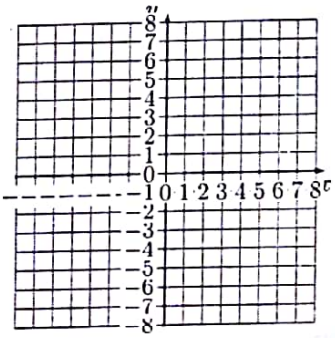
iv table of extreme values, intervals of increase and decrease, concavity, and points of inflection

$f''(x) = \frac{-2}{(x-1)^3}$ $f''(x) \neq 0$

- inflection pt → concavity changes at $x=1$ but $1 \notin \text{Dom}$

x	a		1		b	
f''	+		+		−	−
f'	−		+		+	−
conc	∪		∪		∩	∩
inc. dec.	↘		↗		↗	↘

v Sketch

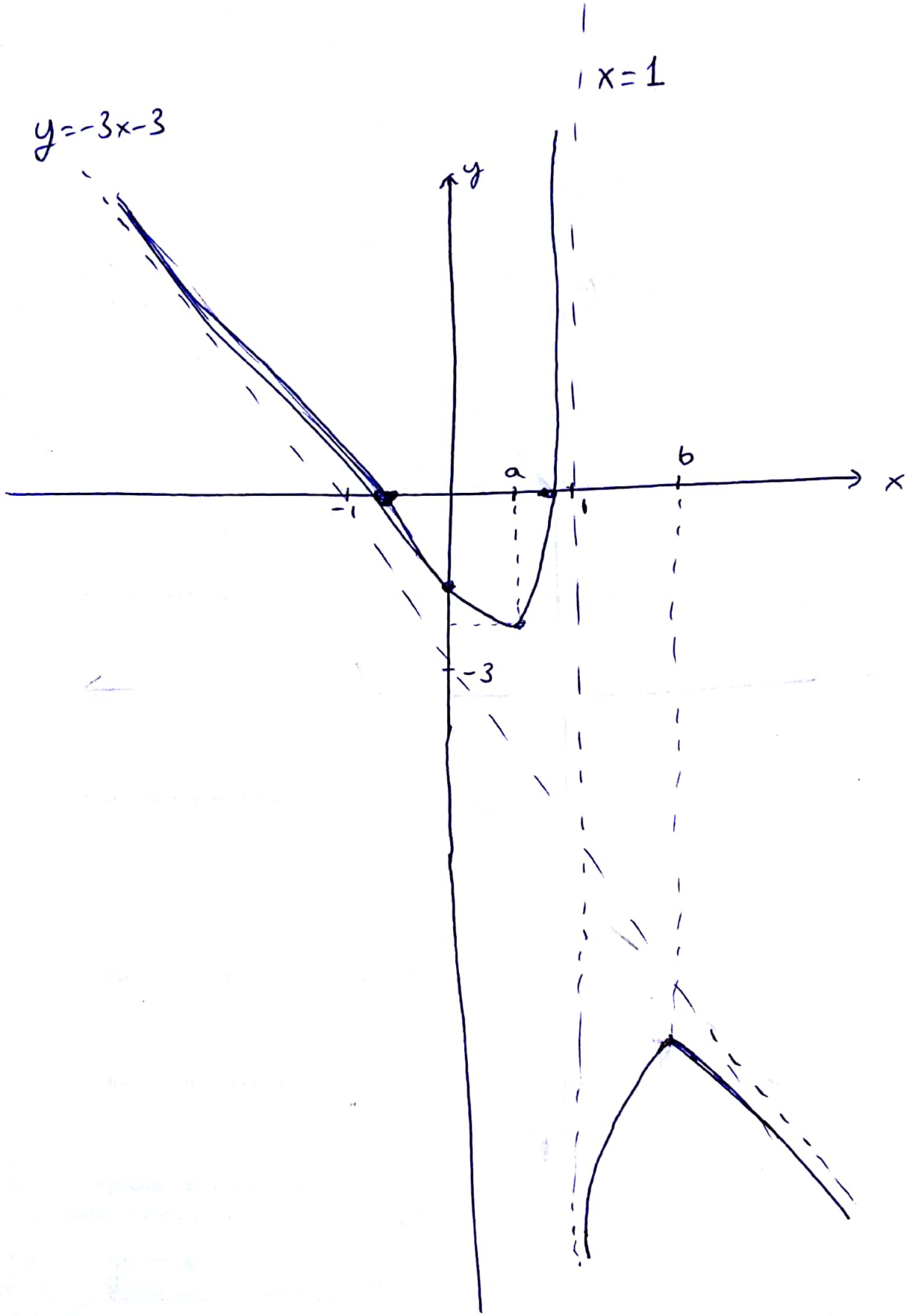


increasing intervals: $(a,1)\cup(1,b)$

decreasing int: $(-\infty,a)\cup(b,\infty)$

concave up int: $(-\infty,1)$

concave down int: $(1,\infty)$



sketched by H.S.