

Surname, Name, Student ID and Section: _____
 1. Does there exist a differentiable function $f : [0, 2] \rightarrow \mathbb{R}$ satisfying $f(0.5) = -1$, $f(1.8) = 6$ and $f'(x) \leq 3$ for all $x \in [0, 2]$?

NO !!

Assume there is such a f , i.e. $f(0.5) = -1$, $f(1.8) = 6$
 and $f'(x) \leq 3 \quad \forall x \in [0, 2]$

Since f' exists on $[0, 2]$, f is cont on $[0, 2]$ and
 diff'ble on $(0, 2)$ (so is on $[0.5, 1.8]$ and $(0.5, 1.8)$ resp.)

By NVT, $\exists c \in (0.5, 1.8)$ s.t.

$$f'(c) = \frac{f(1.8) - f(0.5)}{1.8 - 0.5} = \frac{6 + 1}{1.3} > 6$$
 contradicts $f'(x) \leq 3$

∴ There is no such a function

Surname, Name, Student ID and Section: _____
 2. Use tangent line approximation to find an approximation of $\sqrt{3.5}$.

Choose $f(x) = \sqrt{x}$, we try to find an approx for $\sqrt{3.5}$

$$(\text{so } f(3.5) = \sqrt{3.5}).$$

$$f(x) \approx L(x) = f(a) + f'(a) \cdot (x-a)$$

$$f(3.5) \approx L(3.5) = f(4) + f'(4) \cdot (3.5 - 4)$$

$$\begin{aligned} \text{choose } a = 4 \quad & f(4) = \sqrt{4} = 2 \\ & f'(4) = \frac{1}{2\sqrt{4}} = \frac{1}{4} \\ & f'(x) = \frac{1}{2\sqrt{x}} \end{aligned}$$

$$\sqrt{3.5} = f(3.5) \approx 2 - \frac{0.5}{4} = 1.875$$