

# MATH 119 - CALCULUS WITH ANALYTIC GEOMETRY

## RECITATION 1

1. Solve the inequalities and express your answers in terms of intervals.

(a)  $\frac{3}{x-1} < \frac{2}{x+1}$

(b)  $|2x+5| < 1$

(c)  $x^2 - x \leq 2$

(d)  $|x-3| < 2|x|$

Solution:

(a)  $\frac{3}{x-1} < \frac{2}{x+1} \Rightarrow \frac{3}{x-1} - \frac{2}{x+1} < 0 \Rightarrow \frac{3x+3-2x+2}{(x-1)(x+1)} < 0$

$$\frac{x+5}{(x-1)(x+1)} < 0 \quad x = -5, x = 1, x = -1$$

x	-5	-1	1
$x+5$	-	+	+
$x+1$	-	-	+
$x-1$	-	-	-
$\frac{x+5}{(x-1)(x+1)}$	/\	+	/\

Thus;  $\frac{x+5}{(x-1)(x+1)} < 0$  whenever  $x \in (-\infty, -5) \cup (-1, 1)$

(b)  $|2x+5| < 1 \Rightarrow -1 < 2x+5 < 1$

$$\Rightarrow -6 < 2x < -4$$

$$\Rightarrow -3 < x < -2 \Rightarrow x \in (-3, -2)$$

(c)  $x^2 - x \leq 2 \Rightarrow x^2 - x - 2 \leq 0 \Rightarrow (x-2)(x+1) \leq 0 \quad x = 2, x = -1$

x	-1	2
$x-2$	-	-
$x+1$	-	+
$x^2 - x - 2$	+	/\

$x^2 - x - 2 \leq 0$  whenever  $x \in [-1, 2]$

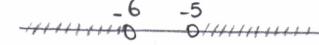
2. Write the following expressions in interval form:

(a)  $x \geq 0$  and  $x < 5$

(b)  $x > -5$  OR  $x < -6$

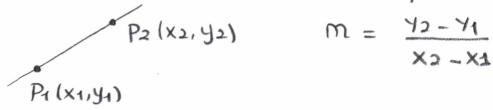
Solution:

(a)   $x \in [0, 5]$

(b)   $x \in (-\infty, -6) \cup (-5, \infty)$

### Lines

The slope of the line passing through the points  $P_1$  and  $P_2$  is



$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

An equation of a straight line  $\ell$  passing through the point  $P_1(x_1, y_1)$  with slope  $m$  is  
 $y - y_1 = m(x - x_1)$ .

3. Find an equation for the given lines:

(a) through the point  $(3, -5)$  with slope  $-2$ .

(b) through the points  $(1, 2)$  and  $(2, 5)$ .

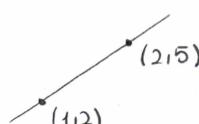
(c) through the points  $(1, 2)$  and  $(2, 2)$ .

Solution:

(a) An equation of the line is  $y - (-5) = -2(x - 3)$

$$y + 5 = -2x + 6 \Rightarrow y + 2x - 1 = 0$$

(b)

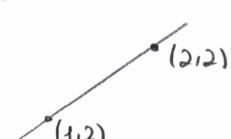


The slope of the line is  $m = \frac{5-2}{2-1} = 3$

An equation of the line is  $y - 2 = 3(x - 1)$

$$y - 2 = 3x - 3 \Rightarrow y - 3x + 1 = 0.$$

(c)

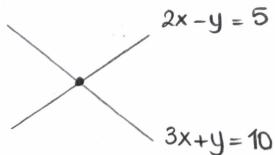


The slope of the line is  $m = \frac{2-2}{2-1} = 0$  (the line is parallel to x-axis)

An equation of the line is  $y - 2 = 0(x - 1) \Rightarrow y = 2$

4. Find the point of intersection of the lines  $2x - y = 5$  and  $3x + y = 10$ .

Solution:



To find the intersection of lines, solve the system of equations

$$\begin{cases} 2x - y = 5 \\ 3x + y = 10 \end{cases} \Rightarrow \begin{aligned} 5x &= 15 & 6 - y &= 5 \\ x &= 3 & y &= 1 \end{aligned}$$

The intersection point of these lines is  $(3, 1)$ .

5. Find the slope, y-intercept and sketch the line  $\sqrt{2}x - y = 2$ .

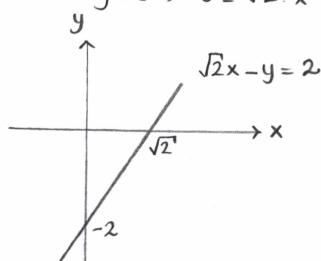
Solution:

To find the slope of the line, solve the equation for  $y$ :

$$\sqrt{2}x - y = 2 \Rightarrow y = \sqrt{2}x - 2 \quad \text{the coefficient of } x \text{ gives the slope } m = \sqrt{2}$$

When  $x=0$ ,  $y = \sqrt{2} \cdot 0 - 2 \Rightarrow y = -2$  is y-intercept

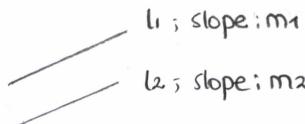
When  $y=0$ ,  $0 = \sqrt{2} \cdot x - 2 \Rightarrow x = 2/\sqrt{2} = \sqrt{2}$  is x-intercept.



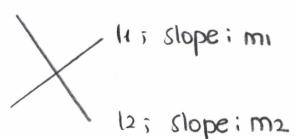
6. For what value of  $k$  is the line  $2x + ky = 3$  perpendicular to the line  $4x + y = 1$ ?

For what value of  $k$  are the lines parallel?

Solution:



$l_1$  and  $l_2$  are parallel if  $m_1 = m_2$ .  
 $(l_1 \parallel l_2)$



$l_1$  and  $l_2$  are perpendicular if  $m_1 \cdot m_2 = -1$   
 $(l_1 \perp l_2)$

$$l_1: 2x + ky = 3 \Rightarrow y = -\frac{2}{k} \cdot x + \frac{3}{k} \quad m_1 = -\frac{2}{k}$$

$$l_2: 4x + y = 1 \Rightarrow y = -4x + 1 \quad m_2 = -4$$

$$l_1 \perp l_2 \text{ if } m_1 \cdot m_2 = -1 \Rightarrow \left(-\frac{2}{k}\right) \cdot (-4) = -1 \Rightarrow k = -8$$

$$l_1 \parallel l_2 \text{ if } m_1 = m_2 \Rightarrow -4 = -\frac{2}{k} \Rightarrow k = \frac{1}{2} \quad \begin{aligned} 2x + \frac{1}{2}y &= 3 \\ 4x + y &= 1 \end{aligned}$$

7. Sketch a rough graph:

(a)  $x^2 + y^2 - 2x - y + 1 = 0$

(b)  $x^2 + y^2 - 4x + 2y > 4$

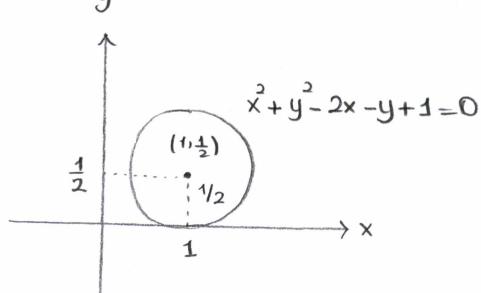
$$x+y > 1$$

(c)  $y = 6x^2 + 13x + 5$

Solution:

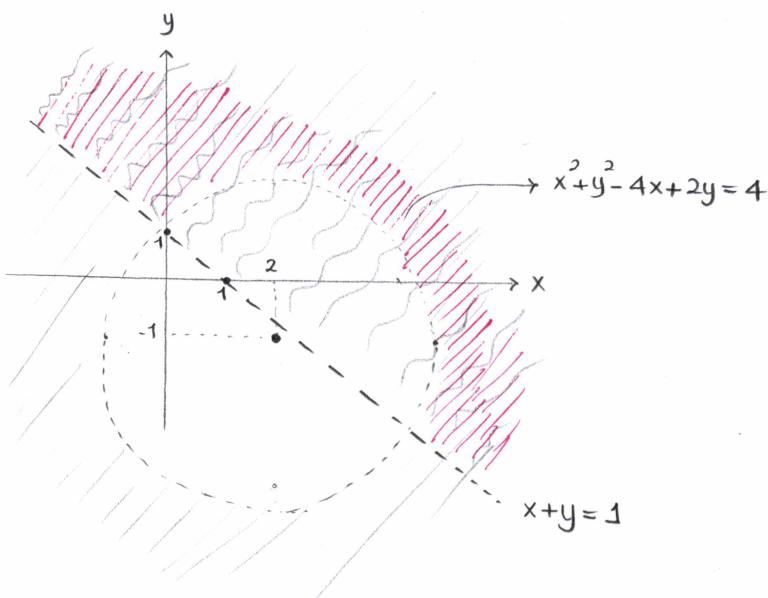
(a)  $x^2 + y^2 - 2x - y + 1 = 0 \Rightarrow x^2 - 2x + 1 + y^2 - y + \frac{1}{4} - 1 - \frac{1}{4} + 1 = 0$

$$(x-1)^2 + \left(y - \frac{1}{2}\right)^2 = \left(\frac{1}{2}\right)^2 \quad \text{Circle with center } (1, \frac{1}{2}) \text{ radius } \frac{1}{2}$$



(b)  $x^2 + y^2 - 4x + 2y > 4 \Rightarrow x^2 - 4x + 4 + y^2 + 2y + 1 - 5 > 4$   
 $(x-2)^2 + (y+1)^2 > 9$

$$x+y > 1$$



8. Find the domain of the function  $f(x) = \frac{1}{1 - \sqrt{x-2}}$ .

Solution:

When  $1 - \sqrt{x-2} = 0 \Rightarrow \sqrt{x-2} = 1 \Rightarrow x-2=1 \Rightarrow x=3$  the expression is undefined so  $x=3$  is not in the domain of  $f$ .

Moreover;  $x-2 \geq 0 \Rightarrow x \geq 2$

Thus, the domain of  $f$  is  $D(f) = [2, \infty) - \{3\}$

9. Sketch a rough graph of the given function and determine whether it is an even or odd function.

(a)  $f(x) = (x+2)^3$

(b)  $f(x) = |x^2 - 1|$

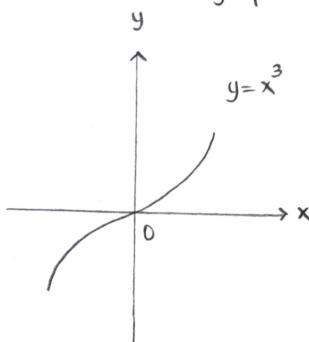
(c)  $f(x) = \sqrt{x+1}$

Solution:

•  $f$  is an even function if  $f(-x) = f(x)$ . (Graph is symmetric about  $y$ -axis)

•  $f$  is an odd function if  $f(-x) = -f(x)$ . (Graph is symmetric about origin).

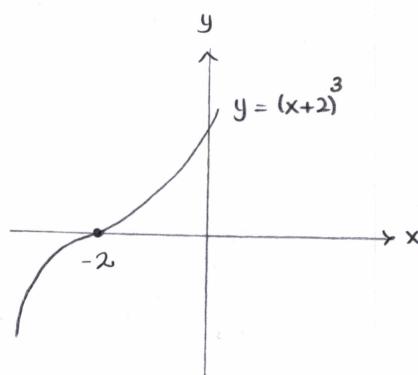
(a) Start the graph of  $y = x^3$ .



Replace  $x$  by  $x+2$

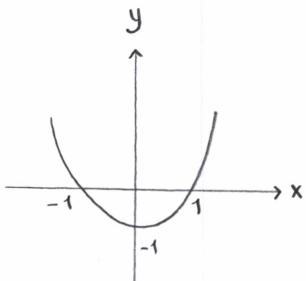
Shift the graph

2 unit left

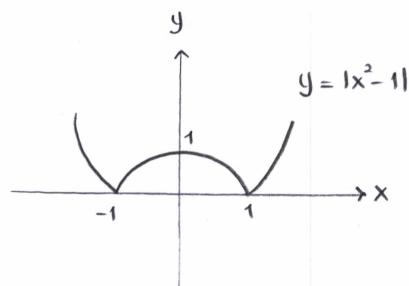


This function is not even or odd function.

(b) Start with the graph of  $y = x^2 - 1$ .



Take the symmetry of  
negative values of  $y = x^2 - 1$   
with respect to  $x$ -axis



Since the graph is symmetric about  $y$ -axis  $y = |x^2 - 1|$  is even function.

10. Let  $f(x) = \frac{1}{1-x}$  and  $g(x) = \sqrt{x-1}$ . Find the functions  $f \circ f$ ,  $f \circ g$ ,  $g \circ f$ ,  $g \circ g$  and their domains.

Solution:

If  $f$  and  $g$  are two functions, the composite function is defined by  $f \circ g(x) = f(g(x))$ .  
The domain of  $f \circ g$  consist of  $x \in D(g)$  for which  $g(x) \in D(f)$ .

Domain of  $f$ :  $1-x=0 \Rightarrow x \neq 1 \quad D(f) = \mathbb{R} - \{1\}$

$$f \circ f(x) = f(f(x)) = f\left(\frac{1}{1-x}\right) = \frac{1}{1-\frac{1}{1-x}} = \frac{1}{\frac{1-x-1}{1-x}} = \frac{1-x}{-x} = \frac{x-1}{x} \quad x \neq 0$$

$$D(f \circ f) = \mathbb{R} - \{0, 1\}$$

Domain of  $g$ :  $x-1 \geq 0 \Rightarrow x \geq 1 \quad D(g) = [1, \infty)$

$$f \circ g(x) = f(g(x)) = f(\sqrt{x-1}) = \frac{1}{1-\sqrt{x-1}} \quad 1-\sqrt{x-1}=0 \Rightarrow \sqrt{x-1}=1 \Rightarrow x-1=1 \Rightarrow x=2 \\ x \neq 2$$

$$D(f \circ g) = [1, 2] \cup (2, \infty]$$

$$g \circ f(x) = g(f(x)) = g\left(\frac{1}{1-x}\right) = \sqrt{\frac{1}{1-x}-1} = \sqrt{\frac{1-1+x}{1-x}} = \sqrt{\frac{x}{1-x}}$$

$$\frac{x}{1-x} \geq 0 \quad \begin{array}{c} 0 \\[-1ex] - \end{array} \begin{array}{c} 1 \\[-1ex] + \end{array} \quad D(f) = \mathbb{R} - \{1\} \Rightarrow D(g \circ f) = [0, 1)$$

11. Let  $f$  be an even function and  $g$  be an odd function defined on  $\mathbb{R}$ . Are the following functions even, odd, neither:  $f+g$ ,  $f \cdot g$ ,  $f \cdot f$ ,  $f \circ f$ ,  $f \circ g$ ,  $g \circ f$ ,  $g \circ g$ ?

Solution:

Assume that  $f$  and  $g$  are non-zero.

$f$  is even function:  $f(-x) = f(x)$

$g$  is odd function:  $g(-x) = -g(x)$

$f+g$ :  $(f+g)(-x) = f(-x) + g(-x) = f(x) - g(x) \neq -(f+g)(x)$  neither

$f \cdot g$ :  $(f \cdot g)(-x) = f(-x) \cdot g(-x) = f(x) \cdot (-g(x)) = -(f \cdot g)(x)$  odd

$f \cdot f$ :  $(f \cdot f)(-x) = f(-x) \cdot f(-x) = f(x) \cdot f(x) = (f \cdot f)(x)$  even.

$f \circ g$ :  $(f \circ g)(-x) = f(g(-x)) = f(-g(x)) = f(g(x)) = (f \circ g)(x)$  even

$g \circ f$ :  $(g \circ f)(-x) = g(f(-x)) = g(f(x)) = (g \circ f)(x)$  even.