(1) Find the directional derivative of the function $f(x,y,z) = x^2yz + yz^2$ at the point $(1,1,1)$ in the direction normal to the surface $x^2 - xyz + z^2 - 1 = 0$.

Soln:

A normal vector to the surface $x^2 - xyz + z^2 - 1 = 0$ is $\vec{n} = \vec{\nabla} g(1,1,1) = \left\langle \left.\frac{\partial g}{\partial x}\right|_{(1,1,1)}, \left.\frac{\partial g}{\partial y}\right|_{(1,1,1)}, \left.\frac{\partial g}{\partial z}\right|_{(1,1,1)} \right\rangle$

where $g(x,y,z) = x^2 - xyz + z^2 - 1$

i.e. $\vec{n} = \left\langle 2x - yz\big|_{(1,1,1)}, -xz\big|_{(1,1,1)}, -xy + 2z\big|_{(1,1,1)} \right\rangle$

$= \langle 1, -1, 1 \rangle$.

The directional derivative of the function $f(x,y,z) = x^2yz + yz^2$ at $(1,1,1)$ in the direction $\vec{n} = \langle 1, -1, 1 \rangle$ is $D_{\vec{n}} f(1,1,1) = \vec{\nabla} f(1,1,1) \cdot \frac{\vec{n}}{\|\vec{n}\|}$

since f is differentiable function everywhere

(f_x, f_y and f_z are polynomials which are cont. everywhere)

$$D_{\vec{n}} f(1,1,1) = \vec{\nabla} f(1,1,1) \cdot \frac{\vec{n}}{\|\vec{n}\|}$$

$$= \left\langle 2xyz\Big|_{(1,1,1)},\ x^2z+z^2\Big|_{(1,1,1)},\ x^2y+2yz\Big|_{(1,1,1)} \right\rangle \cdot \frac{\langle 1,-1,1\rangle}{\sqrt{3}}$$

$$= \langle 2,2,3\rangle \cdot \left\langle \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right\rangle$$

$$= \frac{2}{\sqrt{3}} - \frac{2}{\sqrt{3}} + \frac{3}{\sqrt{3}} = \frac{3}{\sqrt{3}}.$$

!

Theorem: Suppose f_x, f_y are cont. in a nbd of the pt (x_0,y_0), f is diff'ble at (x_0,y_0).

Remark: If f_x or f_y are not cont. at (x_0,y_0) then we cannot say f is not diff'ble at (x_0,y_0).

Theorem: Let $f: D \subset \mathbb{R}^n \to \mathbb{R}$ and let P_0 be an interior pt of D. Suppose all n of the first order partial derivatives of f exists in a ball about P_0 and are cont. at P_0. Then, f is diff'ble at P_0.

② Find the directional derivative of the function

$f(x,y,z) = \dfrac{y}{x+z}$ at $(1,1,0)$ in the direction for which

the function $g(x,y,z) = \ln(x^2+y^2+z^2)$ increases most

rapidly at the same point.

Soln:

Note that if. is diff'ble, then the maximum

value of the directional derivative $D_{\vec{u}}\, g(x)$ is $|\vec{\nabla} g(x)|$.

$$\max = 1, \Rightarrow \alpha = 0$$

$$D_{\vec{u}}\, g(x) = \vec{\nabla} g(x) \cdot \vec{u} = |\vec{\nabla} g(x)| \cdot \underset{\substack{\| \\ 1}}{|u|} \cdot \cos\alpha$$

(since $\vec{u}$ is unit vector)

where α is the angle between $\vec{\nabla} g$ and $\vec{u}$

This occurs if $\vec{u}$ and $\vec{\nabla} g(x)$ have the same direction

The direction for which g increases most rapidly

at $(1,1,0)$ is $\vec{u} = \vec{\nabla} g(1,1,0)$.

i.e. $$\vec{u} = \vec{\nabla} g(1,1,0) = \left\langle \left.\frac{\partial g}{\partial x}\right|_{(1,1,0)}, \left.\frac{\partial g}{\partial y}\right|_{(1,1,0)}, \left.\frac{\partial g}{\partial z}\right|_{(1,1,0)} \right\rangle$$

$$= \left\langle \left.\frac{2x}{x^2+y^2+z^2}\right|, \left.\frac{2y}{x^2+y^2+z^2}\right|_{(1,1,0)}, \left.\frac{2z}{x^2+y^2+z^2}\right|_{(1,1,0)} \right\rangle = \langle 1, 1, 0 \rangle.$$

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Since rational functions are continuous on their domains,

f_x, f_y, f_z are cont. on their domains. So; f is diff'ble.

$$D_{\vec{u}} f(1,1,0) = \vec{\nabla} f(1,1,0) \cdot \frac{\langle 1,1,0 \rangle}{\sqrt{2}}$$

$$= \left\langle \left.\frac{\partial f}{\partial x}\right|_{(1,1,0)}, \left.\frac{\partial f}{\partial y}\right|_{(1,1,0)}, \left.\frac{\partial f}{\partial z}\right|_{(1,1,0)} \right\rangle \cdot \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0 \right\rangle$$

$$= \left\langle \left. -\frac{y}{(x+z)^2}\right|_{(1,1,0)}, \left.\frac{1}{x+z}\right|_{(1,1,0)}, \left. -\frac{y}{(x+z)^2}\right|_{(1,1,0)} \right\rangle \cdot \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0 \right\rangle$$

$$= \langle -1, 1, 0 \rangle \cdot \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0 \right\rangle$$

$$= -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} + 0 = 0 //$$

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③ Consider the function $f(x,y) = xe^{xy}$ at the point $(\sqrt{2},0)$. Find all direction vectors u so that $D_{\vec{u}}f(\sqrt{2},0) = 2$.

soln:

The function is the composition of polynomials and exponential function, so f_x and f_y are cont. everywhere

So, f is diff'ble everywhere.

$2 = D_{\vec{u}} f(\sqrt{2},0) = \vec{\nabla} f(\sqrt{2},0) \cdot \vec{u}$ where $\vec{u} = \langle a,b\rangle$

and $|\vec{u}| = \sqrt{a^2+b^2} = 1$.

Then,

$$2 = \left\langle e^{xy} + xye^{xy}\Big|_{(\sqrt{2},0)}, \; x^2e^{xy}\Big|_{(\sqrt{2},0)} \right\rangle \cdot \langle a,b\rangle$$

$$= \langle 1, 2\rangle \cdot \langle a,b\rangle$$

$$= a+2b$$

So, we have $a+2b=2$ ① and $a^2+b^2=1$. ②

① $\Rightarrow$ $a = 2-2b$ ② $\Rightarrow$ $4(1-b)^2+b^2 = 1 \Rightarrow 4(1-2b+b^2)+b^2 = 1$

$\Rightarrow 4-8b+5b^2 = 1$

$\Rightarrow 5b^2-8b+3=0$

$\Rightarrow (5b-3)(b-1) = 0$

$\Downarrow$ $b = 3/5$ $\quad$ $\Downarrow$ $b=1$

$b = 3/5 \Rightarrow a = 2-\frac{6}{5} = \frac{4}{5} \Rightarrow \vec{u} = \langle 4/5, 3/5\rangle$

or

$b = 1 \Rightarrow a = 0 \Rightarrow \vec{u} = \langle 0,1\rangle$

(4) Compute the directional derivative of the function

$f(x,y) = \frac{1}{1+x^2+5y^2}$ at the point $(2,1)$ in the direction

$u = 5\hat{i} + 12\hat{j}$. Determine the direction at the point $(3,0)$

in which the rate of change of f is the largest

and compute this rate of change.

Soln:

Since f is a rational function, f_x and f_y are

cont. on their domains, so f is diff'ble. on Domf.

$$D_{\vec{u}} f(2,1) = \vec{\nabla} f(2,1) \cdot \frac{\langle 5,12 \rangle}{13}$$

$$= \left\langle \frac{\partial f}{\partial x}\Big|_{(2,1)}, \frac{\partial f}{\partial y}\Big|_{(2,1)} \right\rangle \cdot \left\langle \frac{5}{13}, \frac{12}{13} \right\rangle$$

$$= \left\langle \frac{-2x}{(1+x^2+5y^2)^2}\Big|_{(2,1)}, \frac{-10y}{(1+x^2+5y^2)^2}\Big|_{(2,1)} \right\rangle \cdot \left\langle \frac{5}{13}, \frac{12}{13} \right\rangle$$

$$= \left\langle \frac{-4}{100}, \frac{-10}{100} \right\rangle \cdot \left\langle \frac{5}{13}, \frac{12}{13} \right\rangle$$

$$= \frac{-20-120}{13 \cdot 100} = \frac{-14\cancel{0}}{13 \cdot 10\cancel{0}} = \frac{-14}{130}.$$

Now, we'll determine the direction v at (3,0) in which the rate of change of f is the largest.

So, $\vec{v}$ and $\vec{\nabla} f(3,0)$ have the same direction.

$$\vec{\nabla} f(3,0) = \left\langle \frac{-2\cdot 3}{(1+9+0)^2}, \frac{-10\cdot 0}{(1+9+0)^2} \right\rangle$$

$$= \left\langle -\frac{6}{100}, 0 \right\rangle \text{ (not unit vector)}$$

$$\vec{v} = \frac{\vec{\nabla} f(3,0)}{\|\vec{\nabla} f(3,0)\|} = \langle -1, 0 \rangle.$$

The largest rate of change of f is

$$D_{\vec{v}} f(3,0) = \left\langle -\frac{6}{100}, 0 \right\rangle \cdot \langle -1, 0 \rangle$$

$$= \frac{6}{100}.$$

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⑤ Calculate $\frac{\partial^2 y}{\partial x \partial z}$ if x, y, z satisfy the equation

$x^3y + y^3z + z^3x = 1$.

Soln: Say $F(x,y,z) = x^3y + y^3z + z^3x - 1$.

If $\frac{\partial F}{\partial y} = x^3 + 3y^2z \neq 0$, then y can be considered as a function of x and z by the implicit func. thm.

$$\frac{\partial^2 y}{\partial x \partial z} = \frac{\partial}{\partial x}\left(\frac{\partial y}{\partial z}\right)$$

$$\frac{\partial (x^3y + y^3z + z^3x)}{\partial z} = \frac{\partial(1)}{\partial z}$$

$$\Rightarrow x^3 \cdot \frac{\partial y}{\partial z} + 3 \cdot y^2 \cdot \frac{\partial y}{\partial z} \cdot z + y^3 \cdot 1 + 3 \cdot z^2 \cdot x = 0$$

$$\Rightarrow \frac{\partial y}{\partial z}\left[x^3 + 3zy^2\right] = -3xz^2 - y^3$$

$$\Rightarrow \frac{\partial y}{\partial z} = \frac{-3xz^2 - y^3}{x^3 + 3zy^2}.$$

$$0 = 3x^2y + x^3\frac{\partial y}{\partial x} + 3y^2\frac{\partial y}{\partial x}z$$

$$\Rightarrow \frac{\partial y}{\partial x}\left[x^3 + 3y^2z\right] = -3x^2y$$

$$\Rightarrow \frac{\partial y}{\partial x} = \frac{-3x^2y}{x^3 + 3y^2z}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial y}{\partial z}\right) = \frac{\partial}{\partial x}\left(\frac{-3xz^2 - y^3}{x^3 + 3zy^2}\right)$$

Find $\frac{\partial y}{\partial x}$ and put inside!

$$\frac{\partial^2 y}{\partial x \partial z} = \frac{\left(-3z^2 - 3y^2 \cdot \frac{\partial y}{\partial x}\right) \cdot (x^3 + 3zy^2) - (-3xz^2 - y^3) \cdot \left(3x^2 + 6zy \cdot \frac{\partial y}{\partial x}\right)}{(x^3 + 3zy^2)^2}$$

⑥ If $e^{yz} - x^2 z \ln y = \pi$, calculate $\frac{\partial y}{\partial z}$ and provide the condition which guarantees the existence of solution.

Soln:

$$\frac{\partial(e^{yz} - x^2 z \ln y)}{\partial z} = \frac{\partial(\pi)}{\partial z}$$

$$\Rightarrow \quad e^{yz}\left(z\frac{\partial y}{\partial z} + y\right) - \left(x^2 \ln y + x^2 z \frac{\frac{\partial y}{\partial z}}{y}\right) = 0$$

$$\Rightarrow \quad \frac{\partial y}{\partial z}\left[z e^{yz} - \frac{x^2 z}{y}\right] = x^2 \ln y - y e^{yz}$$

$$\Rightarrow \quad \frac{\partial y}{\partial z} = \frac{x^2 \ln y - y e^{yz}}{z e^{yz} - \frac{x^2 z}{y}}$$

Say $F(x,y,z) = e^{yz} - x^2 z \ln y - \pi$.

$\frac{\partial F}{\partial y} = z e^{yz} - \frac{x^2 z}{y} \neq 0$, is the condition which guarantees the existence of solution by the Implicit function theorem.

This part is at the begining.

(f) Show that the equation $xy + z^3x - 2yz = 0$ defines z as a function of x and y, (i.e, $z = z(x,y)$) near the point $(0,1)$ with $z(0,1) = 0$. Calculate $\frac{\partial z}{\partial x}$ at $(0,1)$.

Soln:

Say $F(x,y,z) = xy + z^3x - 2yz$.

$\left.\frac{\partial F}{\partial z}\right|_{(0,1)} = 3z^2x - 2y\Big|_{(0,1)} = -2 \neq 0$. By the implicit function theorem, z can be considered as a function of x and y near $(0,1)$. Also, Put $x=0$, $y=1$. $0 + z^3 \cdot 0 - 2 \cdot 1 \cdot z = 0$

$\Rightarrow z = 0$

$\Rightarrow z(0,1) = 0$.

$$\frac{\partial (xy + z^3x - 2yz)}{\partial x} = 0$$

$$y + 3 \cdot z^2 \cdot x \frac{\partial z}{\partial x} + z^3 - 2y\frac{\partial z}{\partial x} = 0$$

$$\frac{\partial z}{\partial x}\left[3z^2x - 2y\right] = -z^3 - y$$

$$\Rightarrow \frac{\partial z}{\partial x} = \frac{-z^3 - y}{3z^2x - 2y}. \quad \left.\frac{\partial z}{\partial x}\right|_{(0,1)} = \frac{-(z(0,1))^3 - 1}{3(z(0,1))^2 \cdot 0 - 2 \cdot 1} = \frac{-1}{-2} = \frac{1}{2}$$

⑧ Find and classify the critical points of the function $f(x,y) = x^4 + y^4 + 4xy$.

Soln:

Defn: (Critical pts)

A point (a,b) is called a critical pt of f if

$$f_x(a,b) = f_y(a,b) = 0$$

$\text{Dom} f = \mathbb{R}^2$

$f_x = 4x^3 + 4y$

$f_y = 4y^3 + 4x$

If (a,b) is a critical pt, then

$f_x(a,b) = 4a^3 + 4b = 0$ & $f_y(a,b) = 4b^3 + 4a = 0$

$\Downarrow$

$b = -a^3 \Rightarrow 4(-a^3)^3 + 4a = 0$

$\Rightarrow -a^9 + a = 0$

$\Rightarrow a(-a^8 + 1) = 0$

$\Rightarrow a = 0$ OR $a = \mp 1$

$\Rightarrow b = 0$ OR $b = \pm 1$. So, $(0,0)$, $(1,-1)$, and $(-1,1)$ are critical pts of f.

Recall The second derivative test

Let $f_{xx}, f_{xy}, f_{yx}, f_{yy}$ be continuous on a disk with center (a,b). Let (a,b) be a critical pt of f and

$$D = \begin{vmatrix} f_{xx}(a,b) & f_{xy}(a,b) \\ f_{yx}(a,b) & f_{yy}(a,b) \end{vmatrix} = f_{xx}(a,b)f_{yy}(a,b) - [f_{xy}(a,b)]^2$$

(by mixed partial derivative thm, $f_{xy}(a,b) = f_{yx}(a,b)$).

a) If $D > 0$ and $f_{xx}(a,b) > 0$, then $f(a,b)$ is local minimum.

b) If $D > 0$ and $f_{xx}(a,b) < 0$, then $f(a,b)$ is local maximum

c) If $D < 0$, then f has a saddle pt at (a,b)

d) If $D = 0$ then test gives no information.

(i.e. $f(a,b)$ can be local min, local max or (a,b) is a saddle pt.)

$f_{xx} = 12x^2$, $f_{xy} = 4$, $f_{yy} = 12y^2$

For the critical pt $(0,0)$,

$f_{xx}(0,0) = 0$, $f_{xy}(0,0) = 4$, $f_{yy}(0,0) = 0$

$$D = \begin{vmatrix} 0 & 4 \\ 4 & 0 \end{vmatrix} = 0.0 - (4)^2 = -16 < 0$$

So, f has a <u>saddle pt at $(0,0)$</u>.

For the critical pt $(1,-1)$,

$f_{xx}(1,-1) = 12$, $f_{xy} = 4$, $f_{yy} = 12$

$$D = \begin{vmatrix} 12 & 4 \\ 4 & 12 \end{vmatrix} = 144 - 16 = 128 > 0.$$

Since $D = 128 > 0$ and $f_{xx}(1,-1) = 12 > 0$, f has <u>a local min. at $(1,-1)$</u>. $f(1,-1) = 1 + 1 - 4 = -2$ is the local min. value.

For the critical pt $(-1,1)$,

$f_{xx}(-1,1) = 12$, $f_{xy}(-1,1) = 4$, $f_{yy}(-1,1) = 12$

$D = 12 \cdot 12 - 4^2 = 128 > 0$

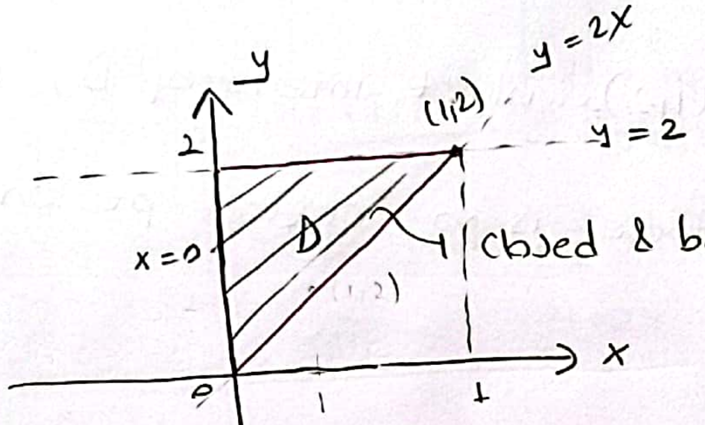
Since $D = 128 > 0$ and $f_{xx}(-1,1) = 12 > 0$, f has a local min. at $(-1,1)$. $f(-1,1) = -2$ is the local min. value.

(9) Find the absolute maximum and minimum values of the function

$$f(x,y) = 2x^2 - 4x + y^2 - 4y + 1$$

on the triangular region bounded by $x=0$, $y=2$, $y=2x$.

Soln:



closed & bounded.

f is cont. on the closed, bounded domain D, so by EVT

f has abs. max and abs. min values on D.

* Theorem (The extreme value theorem)

If f is continuous on a closed, bounded set D in $\mathbb{R}^2$, then f takes its absolute maximum and absolute minimum values on D.

* Necessary conditions for extreme values

A function $f(x,y)$ can have a local or abs. extreme value at (a,b) in its domain only if (a,b) is one of the following:

(i) critical pt. $\nabla f(a,b)=0$ (ii) singular pt $\nabla f(a,b)$ d.n.e. (iii) boundary pt of the domain of f.

To find critical pts,

$f_x(x,y) = 4x - 4 = 0 \Rightarrow x = 1$

$f_y(x,y) = 2y - 4 = 0 \Rightarrow y = 2$

So, $(1,2)$ is a critical points of f.

f has no singular pts.

The point $(1,2)$ is not interior of D, it is on the boundary. So, there is no critical pt on the interior of D.

Now, on the boundary:

(i) For $x = 0$, we have $\underbrace{f(x,y)}_{} = \underbrace{y^2 - 4y + 1}_{= g(y)}$, $y \in [0,2]$

$g'(y) = 2y - 4 = 0 \Rightarrow y = 2 \in [0,2]$

So, g has a critical pt at $y = 2$. , $\boxed{g(2) = -3 = f(0,2)}$

$[0,2]$ is closed and bounded interval, so g takes its abs. min and max. values $[0,2]$.

At the end pts: $\boxed{g(0) = 1 = f(0,0)}$, $\boxed{g(2) = -3 = f(0,2)}$

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(ii) For $y=2$, we have $f(x,y) = 2x^2 - 4x - 4 + 1 = h(x)$

where $x \in [0,1]$.

$h'(x) = 4x - 4 = 0 \Rightarrow x = 1 \in [0,1]$

So, h has a critical pt at $x=1$, $\boxed{h(1) = -5}$

At the end pts: $\boxed{\begin{matrix} h(0) = -3 \\ \| \\ f(0,2) \end{matrix}}$, $\boxed{\begin{matrix} h(1) = -5 \\ \| \\ f(1,2) \end{matrix}}$

(iii) For $y = 2x$, we have $f(x,y) \underset{\substack{\uparrow \\ y=2x}}{=} 2x^2 - 4x + 4x^2 - 8x + 1$

$= 6x^2 - 12x + 1 = k(x)$

where $x \in [0,1]$.

$k'(x) = 12x - 12 = 0 \Rightarrow x = 1 \in [0,1]$

So, k has a critical pt at $x = 1$, $\boxed{k(1) = -5}$

At the end pts: $\boxed{\begin{matrix} k(0) = 1 \\ \| \\ f(0,0) \end{matrix}}$, $\boxed{\begin{matrix} k(1) = -5 \\ \| \\ f(1,2) \end{matrix}}$

Comparing $f(0,0) = 1$, $f(0,2) = -3$ and $f(1,2) = -5$,

we conclude that the abs. min. of f is -5 and occurs at $(1,2)$, and the abs max of f is 1 and occurs at $(0,0)$.

(10) Find the abs. max. and min. values of the function

$$f(x,y,z) = 2x + 3y + z$$

subject to the unit sphere $x^2+y^2+z^2=1$.

Soln:

Recall: (method of Lagrange Multipliers)

To find the max. or min. values of $f(x,y,z)$ subject to the constraint $g(x,y,z)=k=0$

(i) Find all values of x,y,z satisfying

$$\vec{\nabla} f(x,y,z) = \lambda \vec{\nabla} g(x,y,z) \quad \text{and } g(x,y,z)=k=0 \text{ if } \lambda \neq 0$$

(i.e. $f_x = \lambda g_x$, $f_y = \lambda g_y$, $f_z = \lambda g_z$ and $g(x,y,z) = k$)

(ii) Evaluate f at these pts. The largest one is the maximum and the smallest one is the minimum. ⊥

Let $g(x,y,z) = x^2+y^2+z^2-1=0$

$f_x = 2$, $g_x = 2x \Rightarrow 2 = \lambda 2x$ (i) $\Rightarrow x = \frac{1}{\lambda}$, $\lambda \neq 0$

$f_y = 3$, $g_y = 2y \Rightarrow 3 = \lambda 2y$ (ii) $\Rightarrow y = \frac{3}{2\lambda}$, $\lambda \neq 0$

$f_z = 1$, $g_z = 2z \Rightarrow 1 = \lambda 2z$ (iii) $\Rightarrow z = \frac{1}{2\lambda}$, $\lambda \neq 0$

Since $g(x,y,z) = x^2+y^2+z^2 = 1$,

$$\left(\frac{1}{\lambda}\right)^2 + \left(\frac{3}{2\lambda}\right)^2 + \left(\frac{1}{2\lambda}\right)^2 = 1$$

$$\Rightarrow \quad \frac{4+9+1}{4\lambda^2} = 1 \Rightarrow 14 = 4\lambda^2$$

$$\Rightarrow \lambda = \mp \frac{\sqrt{14}}{2}$$

$$\Rightarrow x = \mp \frac{2}{\sqrt{14}}, \quad y = \mp \frac{3}{\sqrt{14}}, \quad z = \mp \frac{1}{\sqrt{14}}$$

The solutions are: $\left(\frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}, \frac{1}{\sqrt{14}}\right)$ and $\left(-\frac{2}{\sqrt{14}}, -\frac{3}{\sqrt{14}}, -\frac{1}{\sqrt{14}}\right)$

$$f\left(\frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}, \frac{1}{\sqrt{14}}\right) = \frac{4}{\sqrt{14}} + \frac{9}{\sqrt{14}} + \frac{1}{\sqrt{14}} = \frac{14}{\sqrt{14}}$$

$$f\left(-\frac{2}{\sqrt{14}}, -\frac{3}{\sqrt{14}}, -\frac{1}{\sqrt{14}}\right) = -\frac{4}{\sqrt{14}} - \frac{9}{\sqrt{14}} - \frac{1}{\sqrt{14}} = -\frac{14}{\sqrt{14}}$$

So, $f\left(\frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}, \frac{1}{\sqrt{14}}\right) = \frac{1}{\sqrt{14}}$ is the max. value of f and

$f\left(-\frac{2}{\sqrt{14}}, \frac{-3}{\sqrt{14}}, -\sqrt{14}\right) = -1/\sqrt{14}$ is the min. value of f.