

① Solve the following inequalities.

a)  $|x+3| - 2 > 3x$

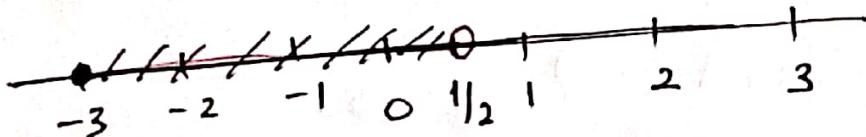
Soln:

$$|x+3| = \begin{cases} x+3 & \text{if } x \geq -3 \\ -x-3 & \text{if } x < -3 \end{cases}$$

Case 1 If  $x \geq -3$ ,  $x+3 - 2 > 3x$

$$\Rightarrow 1 > 2x \Rightarrow \frac{1}{2} > x$$

So, we have  $x \geq -3$  and  $\frac{1}{2} > x$



$$\text{So, } -3 \leq x < \frac{1}{2} \Rightarrow x \in [-3, \frac{1}{2})$$

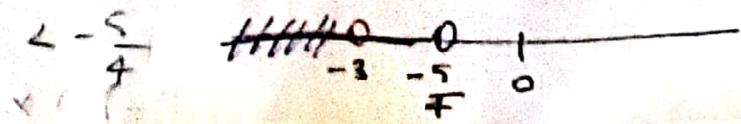
Case 2 If  $x < -3$ ,  $-x-3-2 > 3x$

$$\Rightarrow -5 > 4x$$

$$\Rightarrow \frac{-5}{4} > x \Rightarrow x \in (-\infty, -\frac{5}{4})$$

So,  $x < -3$  and  $x < -\frac{5}{4}$

So,  $x < -3$



Therefore, the solution set is the interval

$$(-\infty, -3) \cup [-3, 1/2] = (-\infty, 1/2).$$

b)  $\frac{x^3 - x^2 + 4}{x+3} \leq 1$

Soln:  $\frac{x^3 - x^2 + 4}{x+3} - 1 \leq 0$

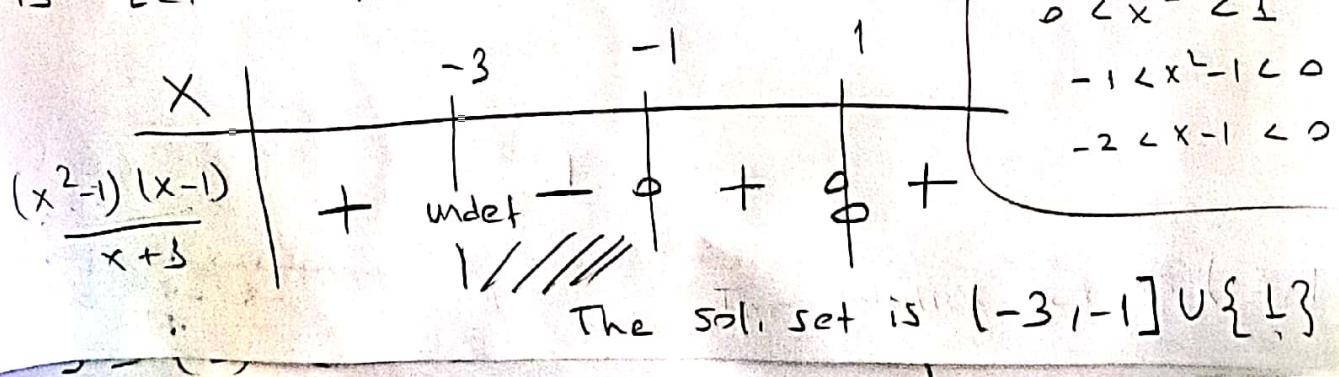
$$\frac{x^3 - x^2 + 4 - x - 3}{x+3} \leq 0$$

$$\frac{x^2(x-1) - (x-1)}{x+3} \leq 0$$

$$\frac{(x^2-1)(x-1)}{x+3} \leq 0$$

The fraction is undefined at  $x=-3$  and

it is zero if  $x=1$  and  $x=-1$ .

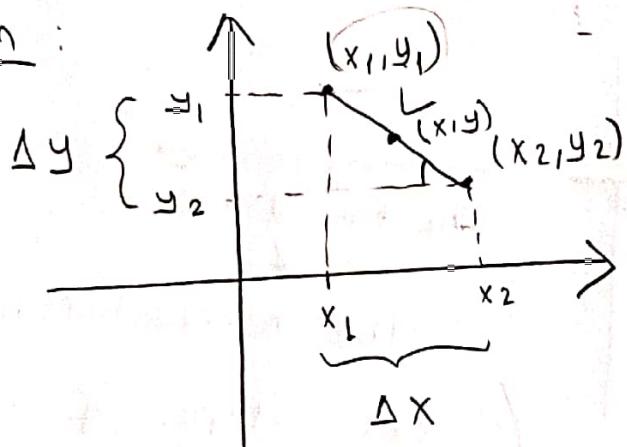


$$c) \frac{1}{2x+1} > 1-x$$

E.C.

- ② Write an equation for the line through the points  $(-1, 5)$  and  $(0, 3)$ .

Soln:



$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

↓  
the slope of L

$p(x, y)$  is a point on L

$$L: y - y_1 = m \cdot (x - x_1)$$

$$y = m \cdot (x - x_1) + y_1$$

The slope is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 5}{0 - (-1)} = \frac{-2}{1} = -2$$

Since the slope is  $-2$  and  $(-1, 5)$  is a point

on this line, the line eqn.

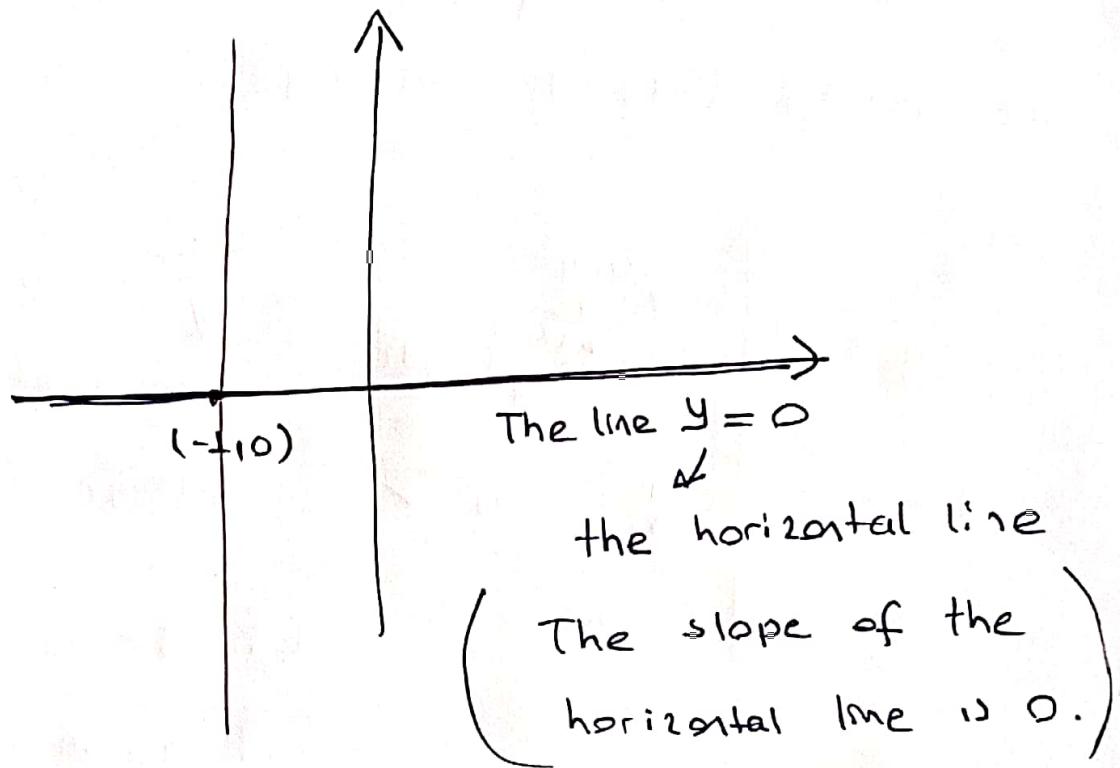
$$y - 5 = (-2) \cdot (x + 1)$$

$$y = -2x - 2 + 5 \Rightarrow \boxed{y = -2x + 3}$$

$$\boxed{| y - 3 = (-2) \cdot (x - 0) \Rightarrow y = -2x + 3}}$$

③ Find the eqn. for the (a) vertical line  
and (b) the horizontal line through the point  
 $(-1, 0)$ .

soln:



The line  $x = -1$

the vertical line

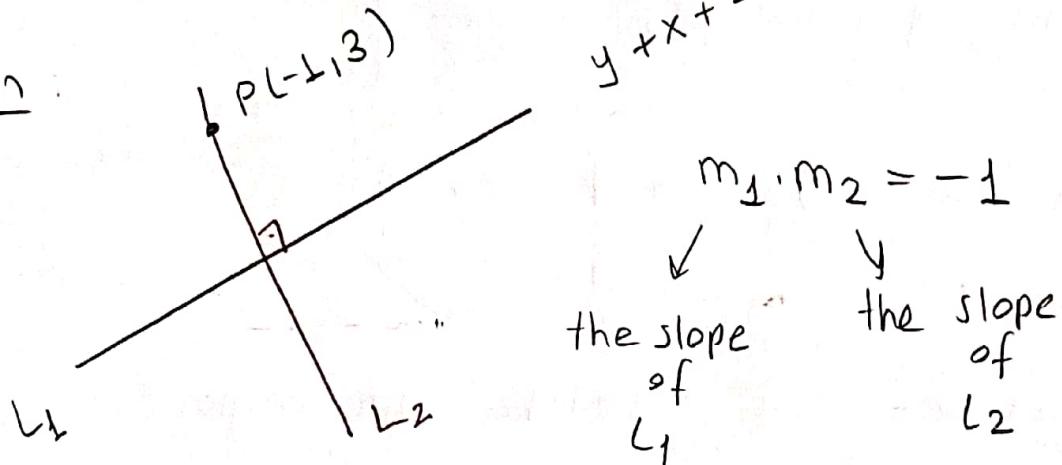
(The slope of the vertical line is )

undefined

since  $\tan \frac{\pi}{2}$  is undefined.

④ Find the equation for the line through  $P(-1, 3)$  that is perpendicular to the line  $y + x + 2 = 0$ . Find the  $x$  and  $y$ -intercepts of this line.

Soln:



$$y + x + 2 = 0$$

$$m_1 \cdot m_2 = -1$$



the slope  
of  
 $L_1$



the slope  
of  
 $L_2$

$$m_1 = ? \quad y + x + 2 = 0 \Rightarrow y = -x - 2 \\ y = (-1) \cdot (x + 2)$$

So, the slope of  $L_1$ ,  $m_L$  is  $-1$ .

Since  $L_1$  and  $L_2$  are perpendicular,  $m_2 = 1$

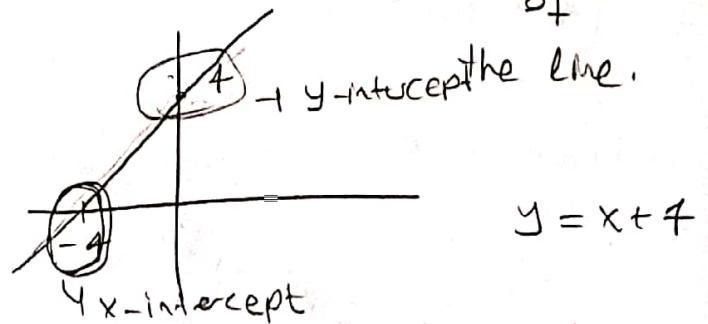
$$\left( (-1) \cdot m_L = -1 \Rightarrow m_2 = 1 \right)$$

So, the line eqn. of  $L_2$  is

$$y - 3 = 1 \cdot (x - (-1)) \Rightarrow \boxed{y = x + 4}$$

To find the  $x$ -intercept put  $y=0$  and solve for  $x$ .

$$y=0 \Rightarrow 0 = x+4 \Rightarrow x = -4 \rightarrow \text{the } x\text{-intercept}$$



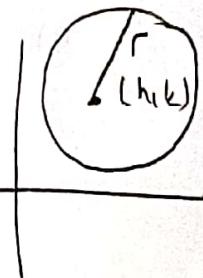
For the  $y$ -intercept

$$x=0 \Rightarrow y=0+4 \Rightarrow y=4 \rightarrow \text{the } y\text{-intercept}$$

of  
the line.

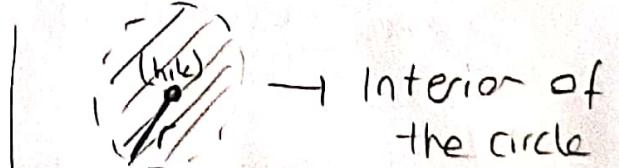
⑤ Describe and sketch the regions defined by the followings.

a)  $x^2+y^2 \leq 4$ ,  $x^2+y^2 > 2y$

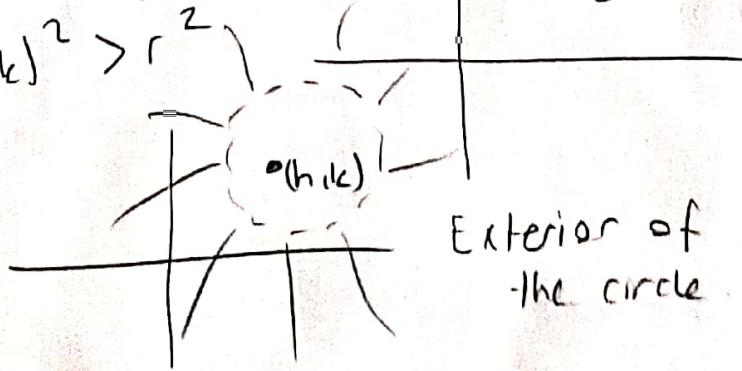


Recall:  $(x-h)^2 + (y-k)^2 = r^2$  —  
"The circle with center  $(h, k)$  and radius  $r$ "

$$(x-h)^2 + (y-k)^2 \leq r^2$$



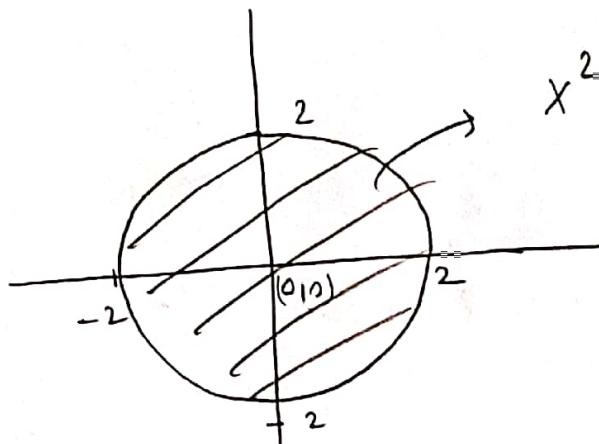
$$(x-h)^2 + (y-k)^2 > r^2$$



Exterior of  
the circle

$$x^2 + y^2 \leq 4$$

$x^2 + y^2 = 4 \rightarrow$  the circle with center  $(0, 0)$  and radius 2



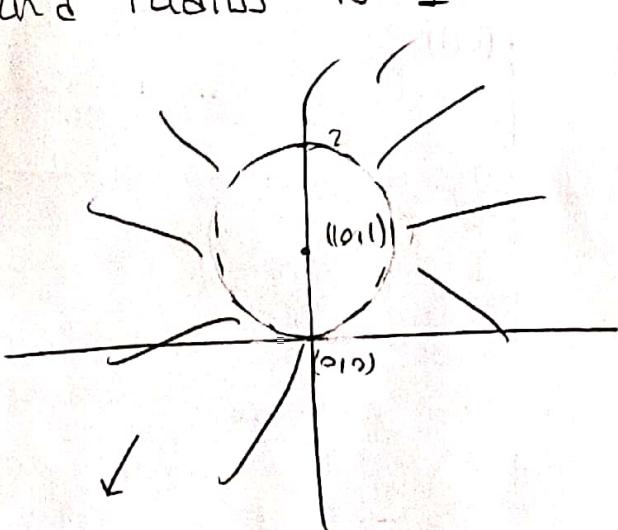
$$x^2 + y^2 \leq 4$$

$$x^2 + y^2 > 2y \Rightarrow x^2 + y^2 - 2y > 0$$

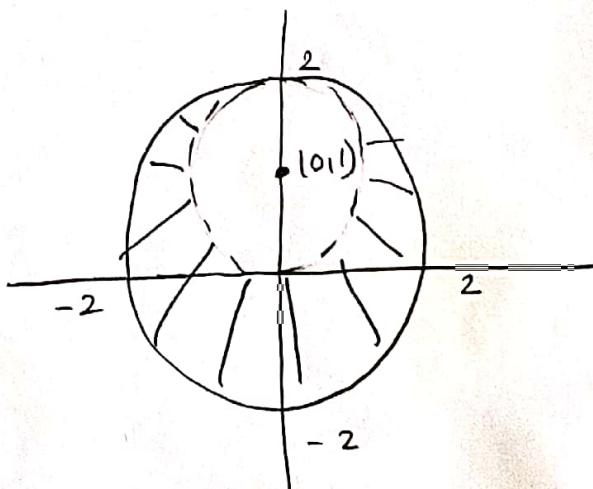
$$\Rightarrow x^2 + y^2 - 2y + 1 - 1 > 0$$

$$\Rightarrow x^2 + (y-1)^2 > 1$$

The exterior of the circle with center  $(0, 1)$  and radius is 1.



$$x^2 + y^2 > 2y$$

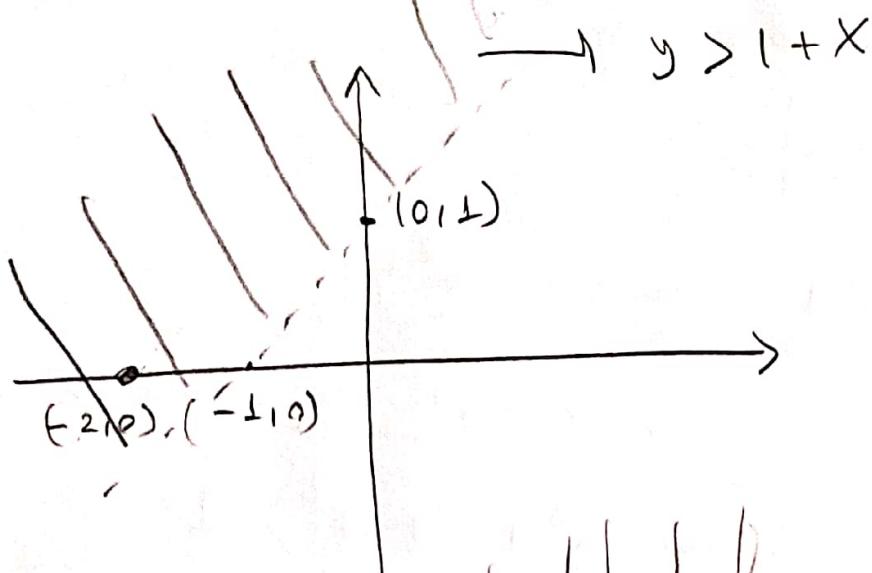


$$b) x^2 + y^2 > 2y, \quad y > 1 + x$$

We know  $x^2 + y^2 > 2y$  by part (a)

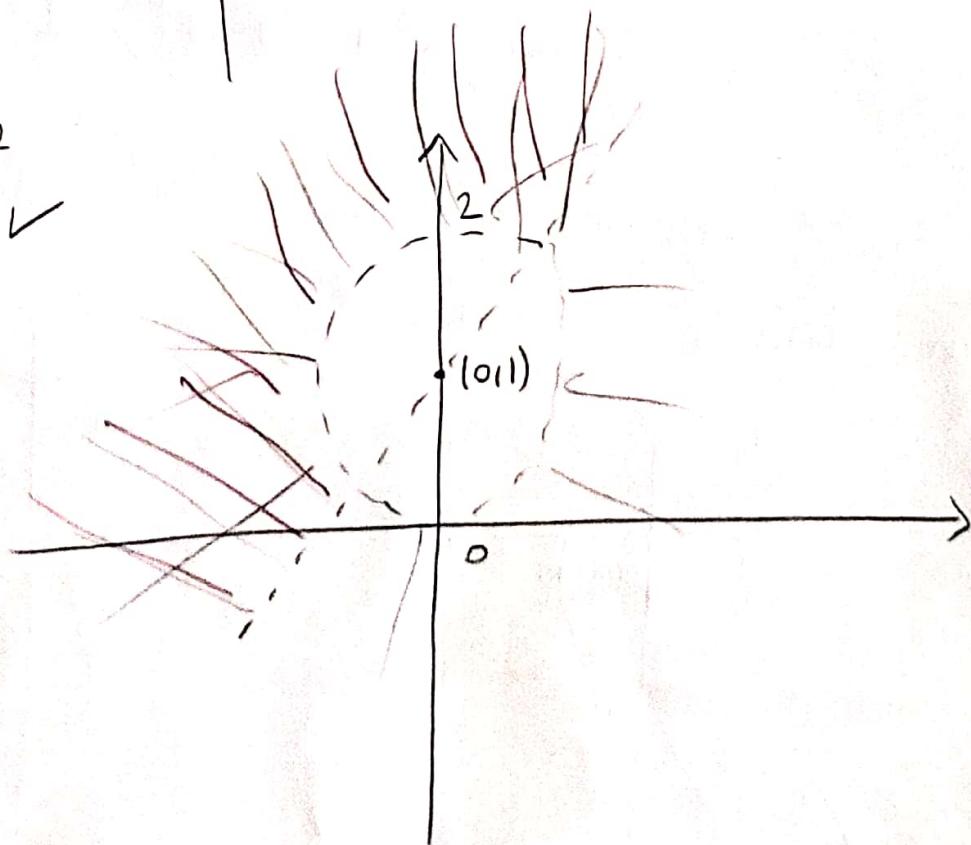
$$y > 1 + x$$

Let's consider the line  $y = 1 + x$



$$0 > 1 - 2$$

$$0 > -1 \checkmark$$



⑥ Find the points of intersections of the pairs of curves.

a)  $y = x^2 + 3$ ,  $y = 3x + 1$

Soln: To find the intersection points, we'll solve this two eqns. together.

$$x^2 + 3 = 3x + 1$$

$$x^2 - 3x + 2 = 0$$

$$(x-2)(x-1) = 0$$

$$\Rightarrow x = 2 \text{ or } x = 1$$

If  $x = 2$ ,  $y = 2^2 + 3 = 7 \rightarrow (2, 7)$

If  $x = 1$ ,  $y = 1 + 3 = 4 \rightarrow (1, 4)$

$(2, 7)$  and  $(1, 4)$  are the intersection points.

(b)  $2x^2 + 2y^2 = 5$ ,  $xy = 1$  ExC

$$c) x^2 + y^2 - 2x - 4y \leq 4 \quad \left. \begin{array}{l} \\ \end{array} \right\} \underline{\text{ExC}}$$

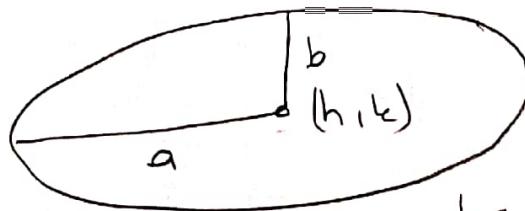
$$d) y > (x-1)^2 + 2, \quad y < 2x \quad \left. \begin{array}{l} \\ \end{array} \right\}$$

$$e) 4x^2 + (y-2)^2 \leq 4$$

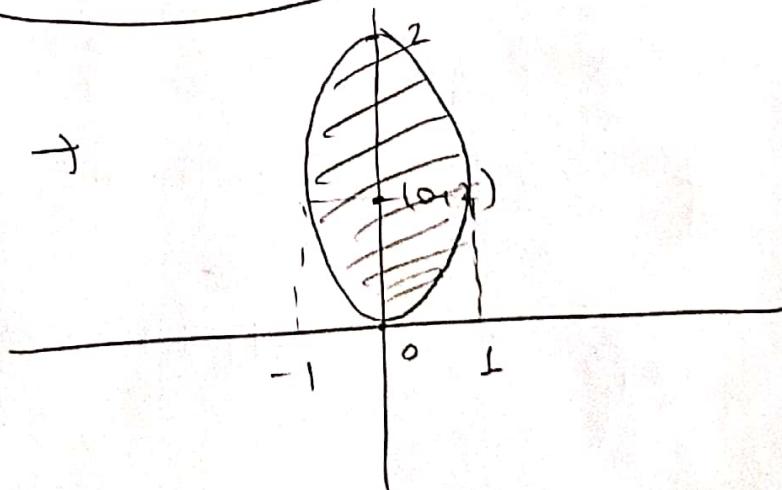
$$\underline{\text{soln}} : \frac{x^2}{1} + \frac{(y-2)^2}{4} \leq 1$$

$$\underline{\text{Recall}} \quad \frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

It is the ellipse with center  $(h, k)$



$$\frac{x^2}{1} + \frac{(y-2)^2}{4} \leq 1 \rightarrow$$



⑦ Write an eqn. of the graph obtained by shifting the graph of  $y = \sqrt{x}$

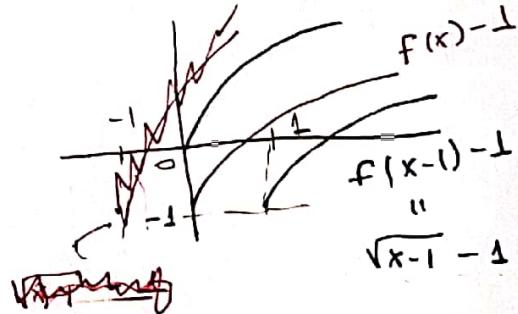
a) down 1, right 1.

Recall  $y = f(x)$ , positive real num.  $c$ .

$f(x \pm c)$  → the graph of  $f(x)$  shifted to the left by  $c$ -unit (right)

$f(x) \pm c$  → " " " shifted to the   
 upward by  $c$  unit  $y = \sqrt{x}$    
 (downward)  $f(x) - 1$

Let  $y = f(x) = \sqrt{x}$



$y = \underbrace{f(x-1)}_{y = \sqrt{x-1}} - 1$  → the graph of  $f(x)$  shifted to downward by 1-unit and to the right side 1-unit

b) down 2, left +

c) up 2, left -

d) up 1, right +

E&C

⑧ Find the domain and range of each function and sketch their graphs.

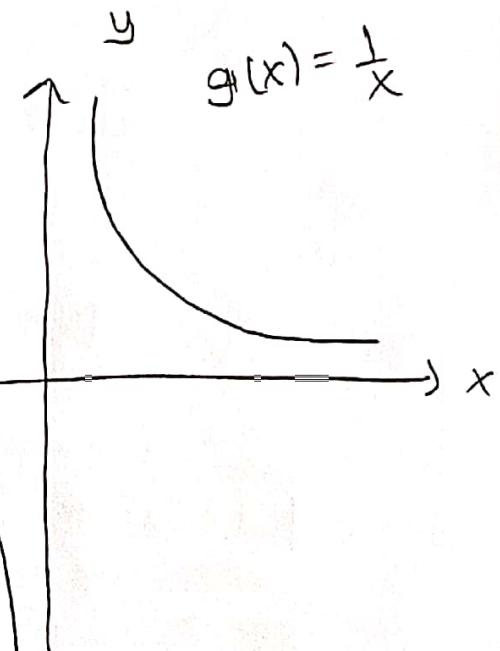
$$a) f(x) = \frac{1}{|x-2|}$$

Soln: The function  $f$  is undefined if  $x=2$  since  $|x-2|=0$  if  $x=2$

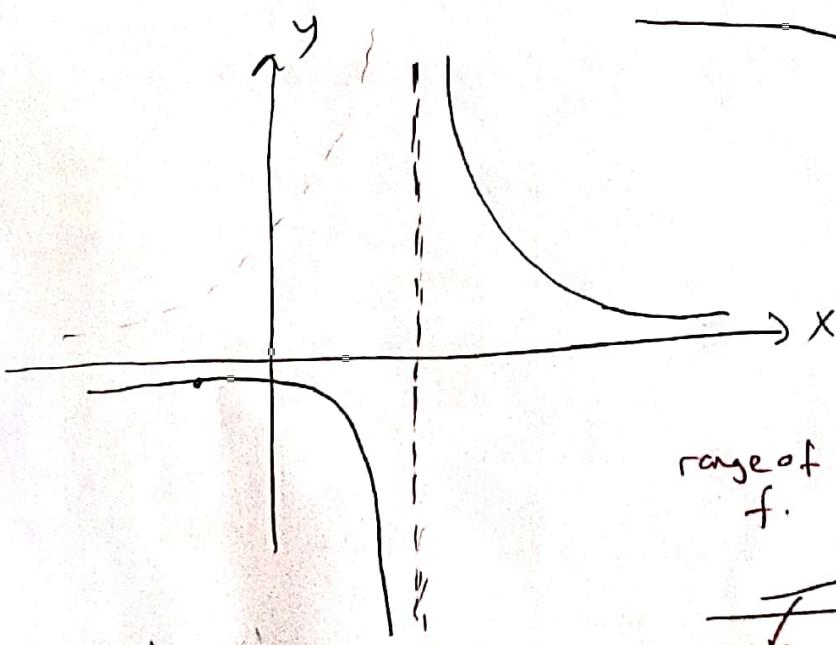
So, the domain of  $f$  is the interval

$$\text{Dom}(f) = \mathbb{R} - \{2\}$$

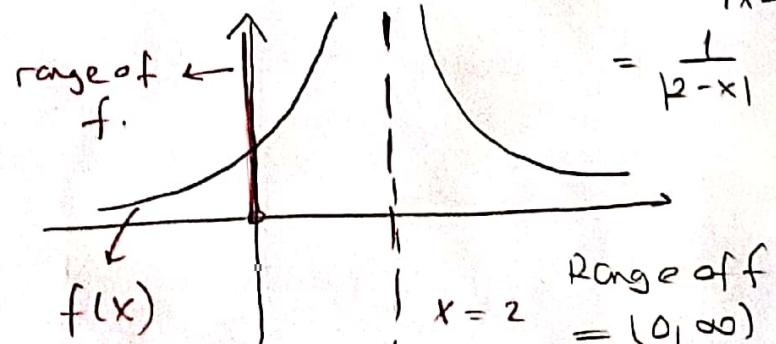
$$\text{Let } g(x) = \frac{1}{x}$$



$$g(x-2) = \frac{1}{x-2}$$



$$f(x) = |g(x-2)| = \frac{1}{|x-2|}$$



$$|g(x)| = -g(x) \quad x = -2$$

$$c) f(x) = \sqrt{x^2 - 1} \quad \underline{\underline{\Sigma x c}}$$

⑨ a) How is the graph of  $y = f(|x|)$  related to the graph of  $f$ .

Soln

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

$$f(|x|) = \begin{cases} f(x) & \text{if } x \geq 0 \\ f(-x) & \text{if } x < 0 \end{cases}$$

The graph of  $f(|x|)$  as follows:

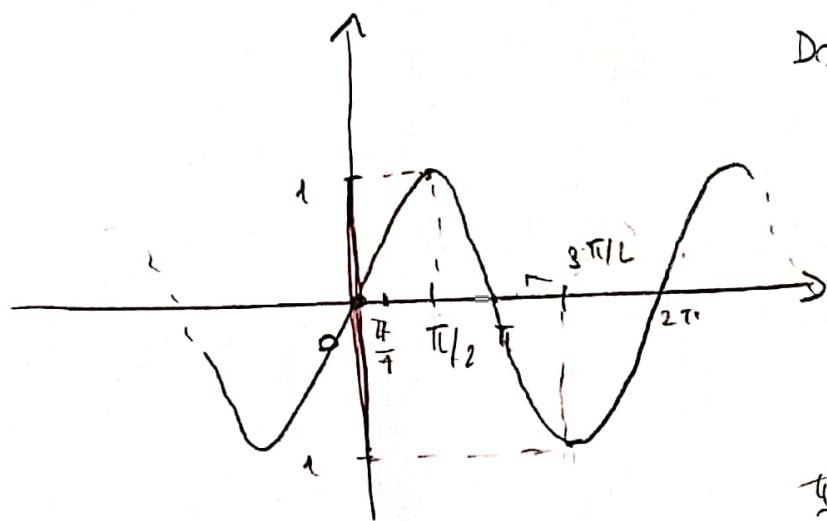
$f(|x|)$  is the same of <sup>the</sup> left side of graph of  $f(x)$  at the right hand side of the

$$x=0$$

$f(|x|)$  is the reflection of the right side of the graph of  $f(x)$  w.r.t.  $y$ -axis.

$$b) y = 1 + \sin\left(x + \frac{\pi}{4}\right) = f(x)$$

Soln Let  $y = \sin x = g(x)$   $g(x) = g(x+2\pi)$



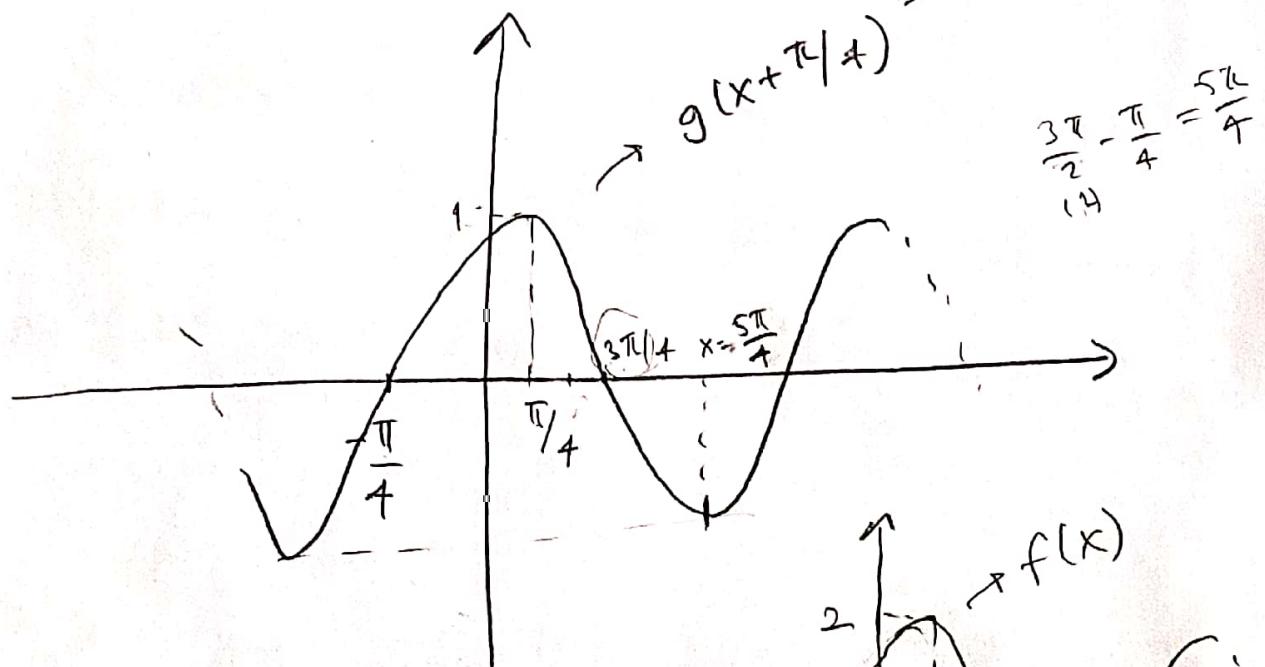
$$\text{Dom}(g(x)) = \mathbb{R}$$

$$\begin{aligned} \text{Range of } g(x) \\ = [-1, 1] \end{aligned}$$

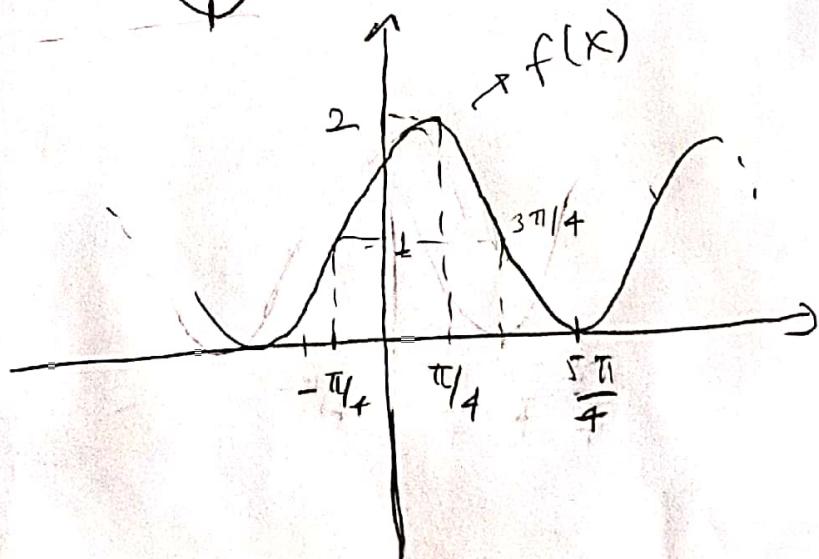
$$\frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$$

$$g(x + \frac{\pi}{4}) = \sin\left(x + \frac{\pi}{4}\right)$$

$$= \sin(x + \pi/4)$$

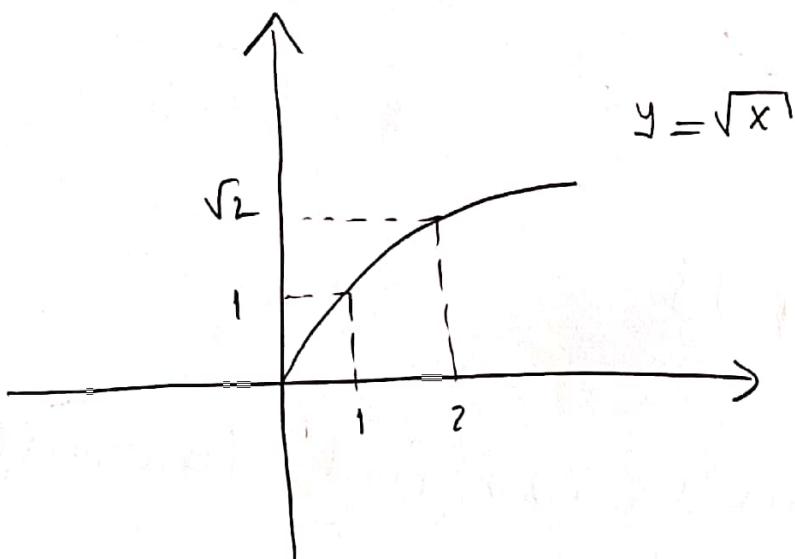


$$f(x) = 1 + g(x + \pi/4)$$

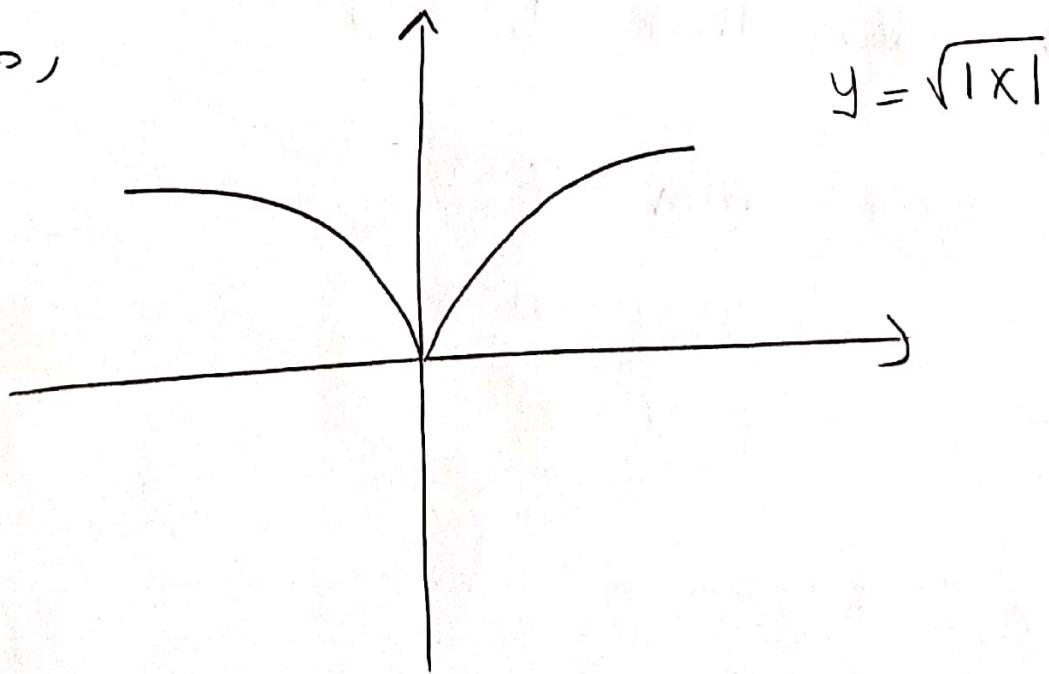


c) sketch the graph of  $y = \sqrt{|x|}$

Soln Since the domain of  $y = f(x) = \sqrt{x}$  is  $[0, \infty)$  and the graph of  $f$  is as follows



So,



⑩ a) Find  $f \circ g$  and its domain where

$$f(x) = x + \frac{1}{x} \quad \text{and} \quad g(x) = \frac{x-1}{x+3}$$

soln

$$\begin{aligned} f(g(x)) &= f\left(\frac{x-1}{x+3}\right) = \frac{x-1}{x+3} + \frac{1}{\frac{x-1}{x+3}} \\ &= \frac{(x-1)^2 + (x+3)^2}{(x-1)(x+3)} \end{aligned}$$

$$\text{Dom}(f \circ g) = \mathbb{R} - \{-1, -3\}$$

b) Given  $F(x) = \sin^2(x-5)$ , find functions  $f, g, h$  such that  $F = f \circ g \circ h$ .

soln Let  $h(x) = x-5$

$$g(x) = \sin x$$

$$f(x) = x^2$$

Check  $f \circ g \circ h = F$  !