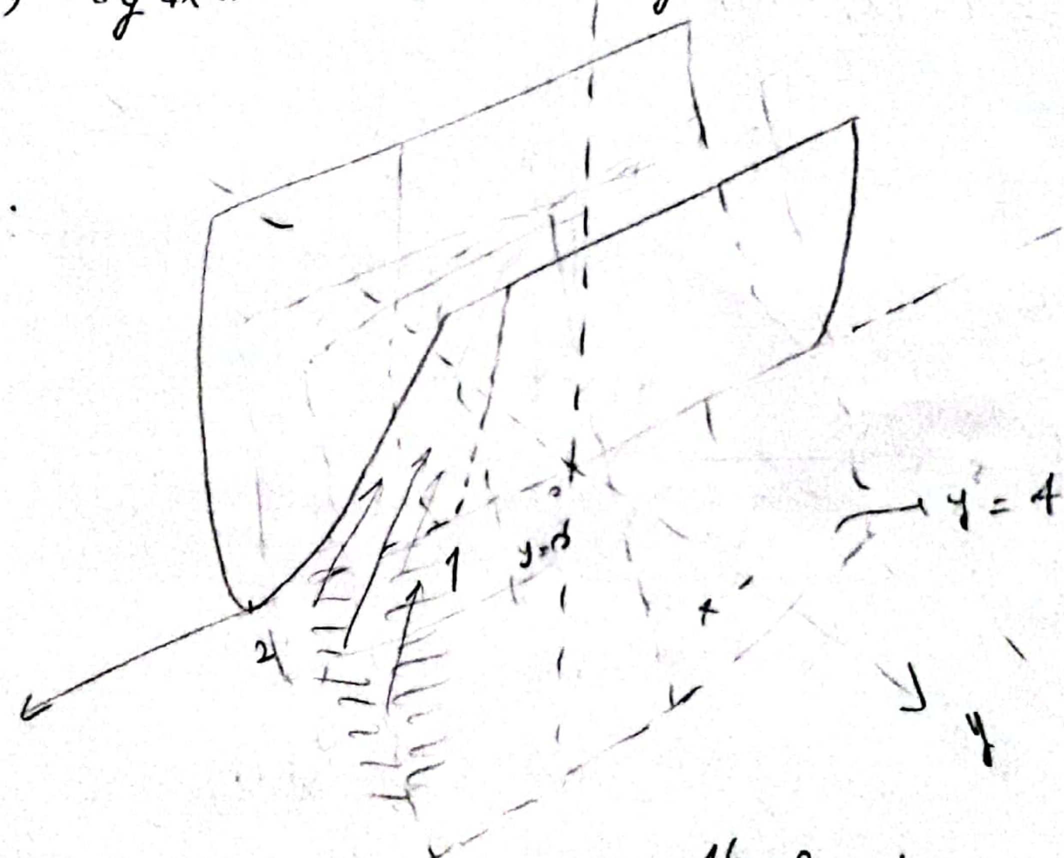
Math 120 - 2021-2 - RECITATION PROBLEMS - W12

(1) Rewrite the integral $\int_1^2 \int_0^4 \int_0^{y^2} dz\, dy\, dx$ as an equivalent iterated integral in the order

(a) $dy\, dx\, dz$ (b) $dx\, dz\, dy$

Soln :



(a) $$\int_1^2 \int_0^4 \int_0^{y^2} dz\, dy\, dx = \int_0^{16} \int_1^2 \int_{\sqrt{z}}^4 dy\, dx\, dz$$

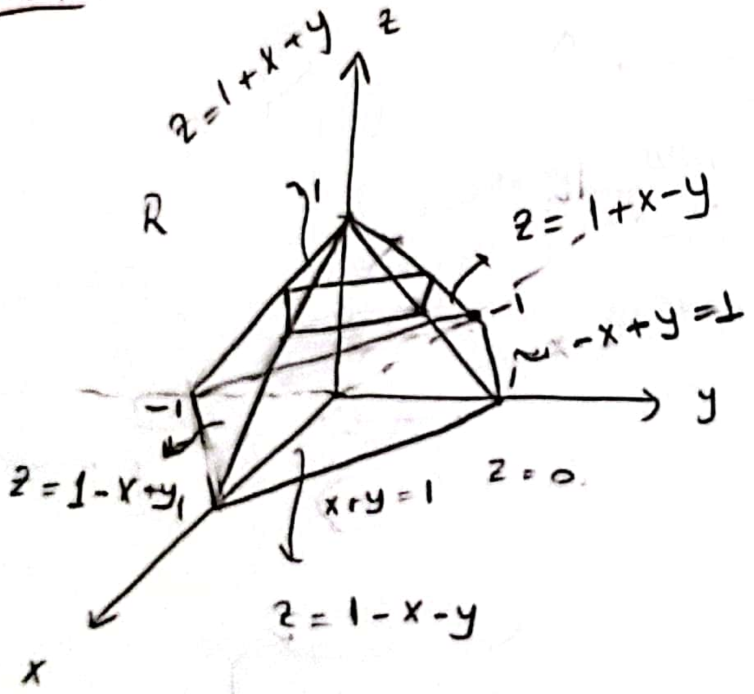
(b) $$\int_1^2 \int_0^4 \int_0^{y^2} dz\, dy\, dx = \int_0^4 \int_0^{y^2} \int_1^2 dx\, dz\, dy$$

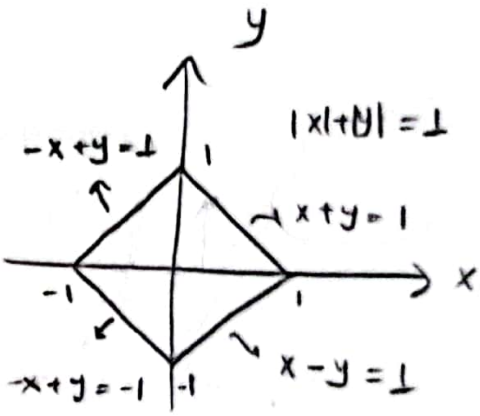
② Express the triple integral $\iiint_R f(x,y,z)\,dV$ over the region R by an iterated integral (or integrals) in the order $dz\,dy\,dx$. Do not evaluate the integral.

(a) if $f(x,y,z) = 2xy + 6z$ and R is the region given

$0 \le z \le 1 - |x| - |y|$

<u>soln</u>:



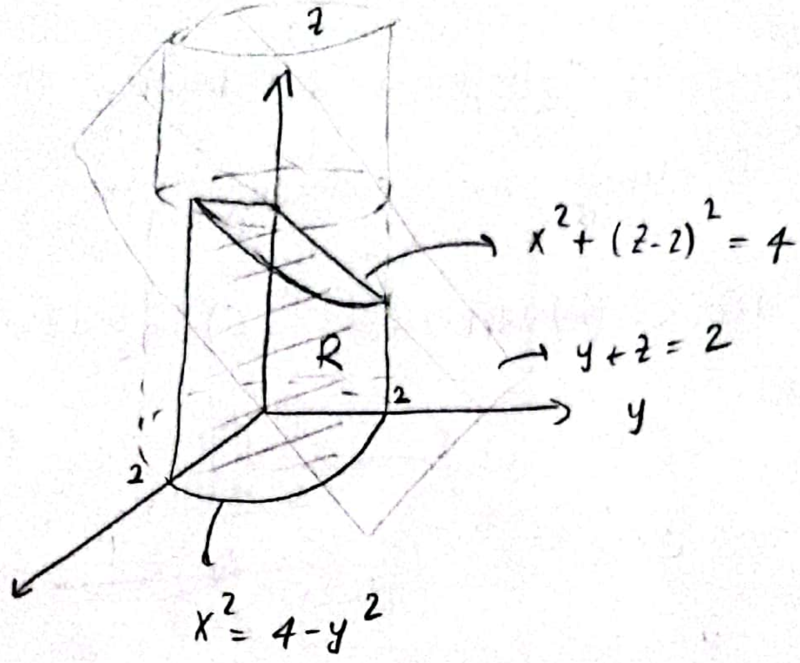


$$\iiint_R f(x,y,z)\,dV = \int_0^1\int_0^{1-x}\int_0^{1-x-y} f\,dz\,dy\,dx + \int_{-1}^{0}\int_0^{1+x}\int_0^{1+x-y} f\,dz\,dy\,dx$$

$$+ \int_{-1}^{0}\int_{-1-x}^{0}\int_0^{1+x+y} f\,dz\,dy\,dx + \int_0^1\int_{x-1}^{0}\int_0^{1-x+y} f\,dz\,dy\,dx$$

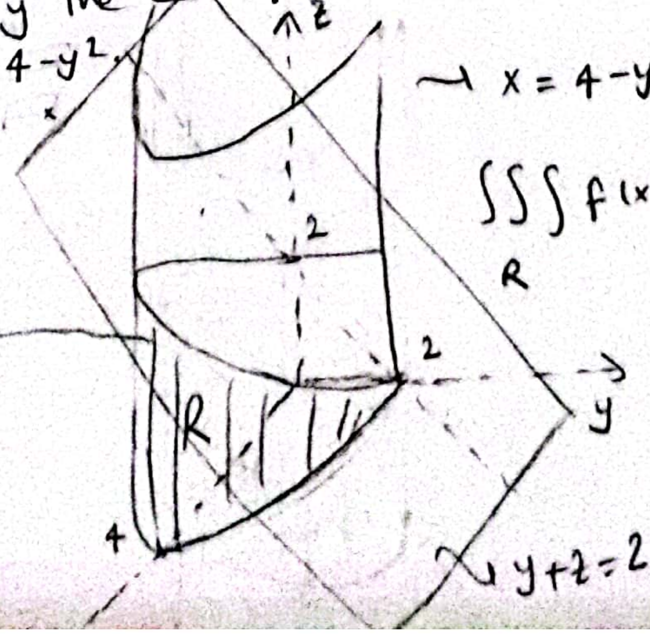
b) if $f(x,y,z) = y^2$ and R is the region in the first octant bounded by the coordinate planes, the plane $y+z=2$ and the cylinder $x^2 = 4-y^2$.

Soln:



$$\iiint_R f(x,y,z)\,dV = \int_0^2\int_0^{\sqrt{4-x^2}}\int_0^{2-y} y^2\,dz\,dy\,dx$$

c) if $f(x,y,z) = y^2$ and R is the region in the first octant bounded by the coord. planes; the plane $y+z=2$ and the parabolic cylinder $x = 4-y^2$.



$x = 4-(2-z)^2$

$x = 4-(4-4z+z^2)$

$x = 4z - z^2$

$x = z(4-z^2)$

$\rightarrow x = 4-y^2 \Rightarrow y^2 = 4-x$

$y = \mp\sqrt{4-x}$

$$\iiint_R f(x,y,z)\,dV = \int_0^4\int_0^{\sqrt{4-x}}\int_0^{2-y} y^2\,dz\,dy\,dx$$

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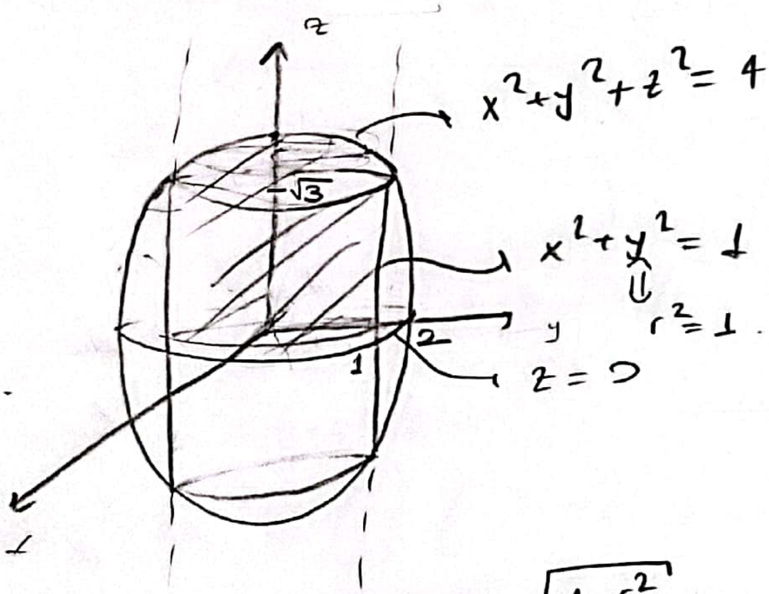
③ Let D be the region bounded below by the plane $z=0$, above by the sphere $x^2+y^2+z^2=4$ and on the sides by the cylinder $x^2+y^2=1$. Set up triple integrals in cylindrical coordinates that give the volume of D using the following orders of integration.

(a) $dz\,dr\,d\theta$ (b) $dr\,dz\,d\theta$ (c) $d\theta\,dz\,dr$

Soln:

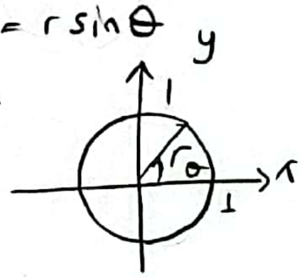


In cylindrical coord:

$x = r\cos\theta$

$y = r\sin\theta$

z



(a)

$$\text{vol}(D) = \iiint_D 1\,dV = \int_0^{2\pi}\int_0^{1}\int_0^{\sqrt{4-r^2}} 1\cdot r\,dz\,dr\,d\theta$$

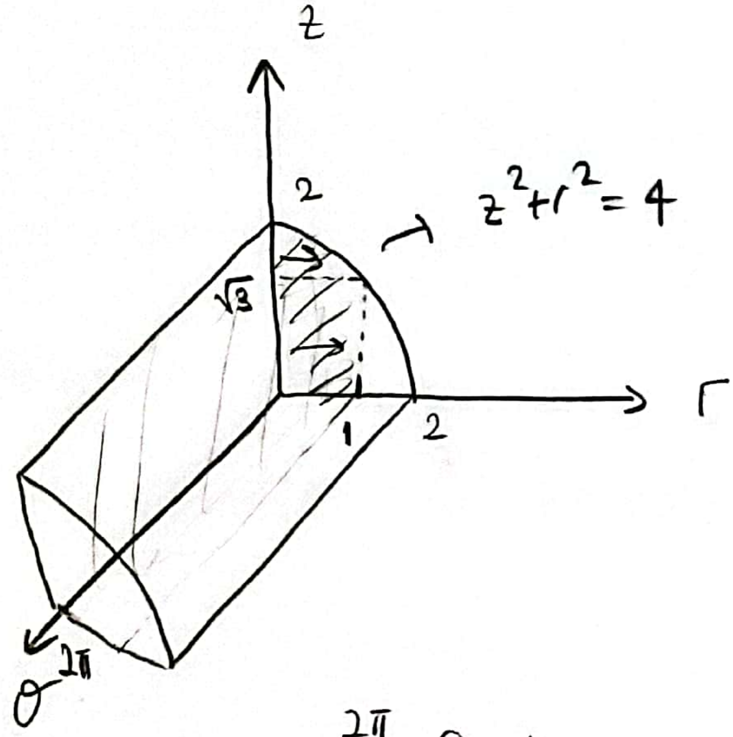
(b)

(b) $dr\,dz\,d\theta$

$0 \leq z \leq \sqrt{4-r^2}$

$0 \leq r \leq 1$

$0 \leq \theta \leq 2\pi$



$$\mathrm{Vol}(D) = \int_0^{2\pi}\int_0^{\sqrt{3}}\int_0^{1} r\,dr\,dz\,d\theta + \int_0^{2\pi}\int_{\sqrt{3}}^{2}\int_0^{\sqrt{4-z^2}} r\,dr\,dz\,d\theta$$

(c) $d\theta\,dz\,dr$

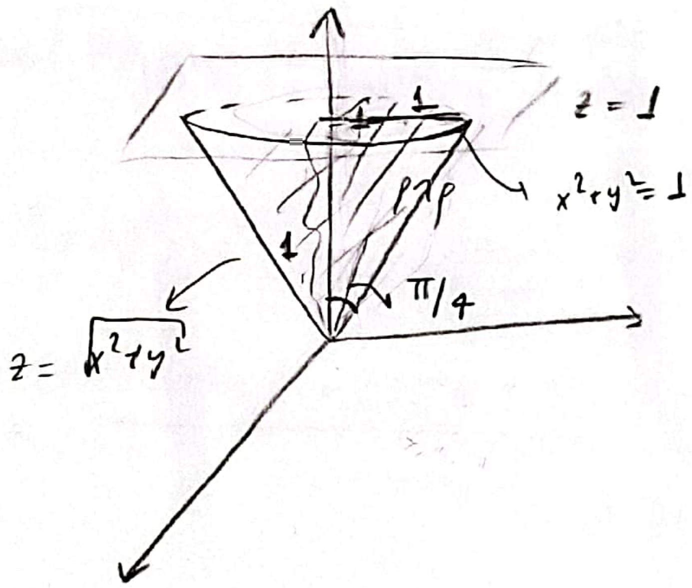
$$\mathrm{Vol}(D) = \int_0^{1}\int_0^{\sqrt{4-r^2}}\int_0^{2\pi} r\,d\theta\,dz\,dr$$

④ Let D be the region bounded below by the cone $z=\sqrt{x^2+y^2}$ and above by the plane $z=1$.

Set up triple integrals in spherical coordinates that give the volume of D using the following orders of integration.

(a) $d\overset{\rho}{\cancel{R}}\,d\phi\,d\theta$ (b) $d\phi\, d\overset{\rho}{\cancel{R}}\,d\theta$

<u>Soln</u>:



In spherical coord.

$x=\rho.\sin\phi\cos\theta$

$y=\rho\sin\phi\sin\theta$

$z=\rho\cos\phi$

$\rho^2=x^2+y^2+z^2$

$z=1=\rho.\cos\phi$

$\Downarrow$

$\rho=1/\cos\phi$

(a)

$$\text{vol}(D)=\iiint_D 1\, dV=\int_0^{2\pi}\int_0^{\pi/4}\int_0^{1/\cos\phi} 1.\rho^2\sin\phi\, d\rho\, d\phi\, d\theta$$

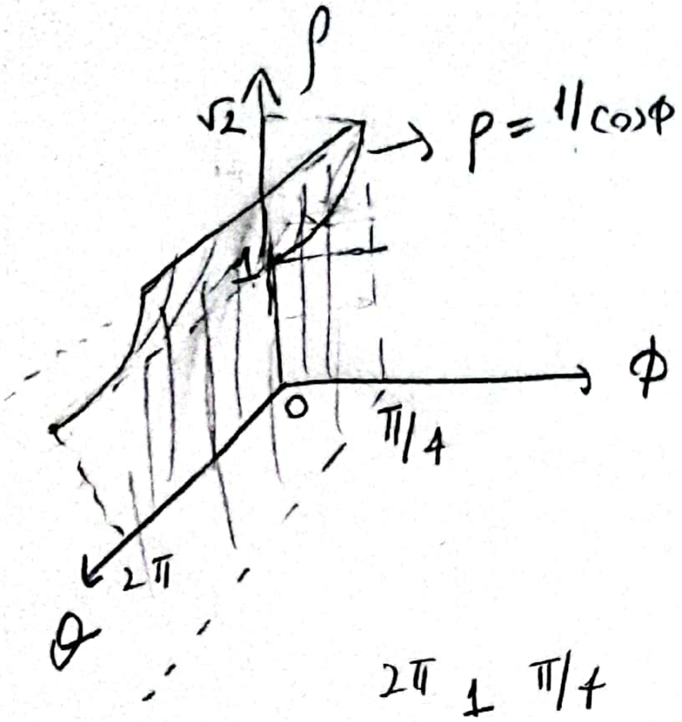
$z=\rho.\cos\phi=\underset{\overset{\|}{\sqrt{x^2+y^2}}}{\rho\sin\phi}$

$\Downarrow$

$\phi=\pi/4.$

(b)

b) $d\phi \, d\rho \, d\theta$



$0 \le \rho \le 1/\cos\phi$

$0 \le \phi \le \pi/4$

$0 \le \theta \le 2\pi$

$$\text{Vol}(D) = \int_0^{2\pi}\int_0^{1}\int_0^{\pi/4} \rho^2 \sin\phi \, d\phi \, d\rho \, d\theta \; +$$

$$\int_0^{2\pi}\int_1^{\sqrt{2}}\int_{\arccos(1/\rho)}^{\pi/4} \rho^2 \sin\phi \, d\phi \, d\rho \, d\theta$$