Quiz-5 : Express the limit

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{2i+4n}{n^2}$$

as a definite integral. Compute this limit.

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$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{2i+4n}{n^2} = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n}\left(\frac{2i}{n}+4\right)$$

We want to interpret Riemann sum for $f(x) = 2x+4$.
The factor $\frac{1}{n}$ suggests that the interval has length 1 and is partitioned into n equal subintervals, each of length $\frac{1}{n}$

Let $c_i = \frac{i}{n}$ for $i = 1, 2, 3, \dots, n$

$c_1 = \frac{1}{n} \to 0$ as $n \to \infty$

$c_n = \frac{n}{n} \to 1$ as $n \to \infty$

Thus, the interval is $[0,1]$.
The points of the partition are $x_i = \frac{i}{n}$ (Observe that here $x_i = c_i$ for each i)

Since f is continuous on $[0,1]$, it is integrable there, and

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{2i+4n}{n^2} = \int_0^1 (2x+4)\,dx = \left(x^2+4x\right)\Big|_0^1 = 5-0 = 5 \quad //$$