Sec 11 - Math 119 - Recitation Week 9

1) Find all pts. on the curve $y = 1 + x^{3/2}$ forthest from or closest to the point $(8,1)$

Solution

distance $= d$, $\quad d^2 = (x-a)^2 + (y-b)^2$

$y = 1 + \sqrt{x^3}$

$d^2 = (x-8)^2 + (\cancel{1} + \sqrt{x^3} - \cancel{1})^2 = x^3 + x^2 - 16x + 64$

y is defined only for $x \geq 0$ since $y = 1 + \sqrt{x^3}$

If $x = 0$ then $d^2 = 64$. Also, if $\lim_{x \to \infty} d = \infty$ (there is no forthest pt.)

Take derivative: $3x^2 + 2x - 16 = 0 \Rightarrow (3x+8)(x-2) = 0$

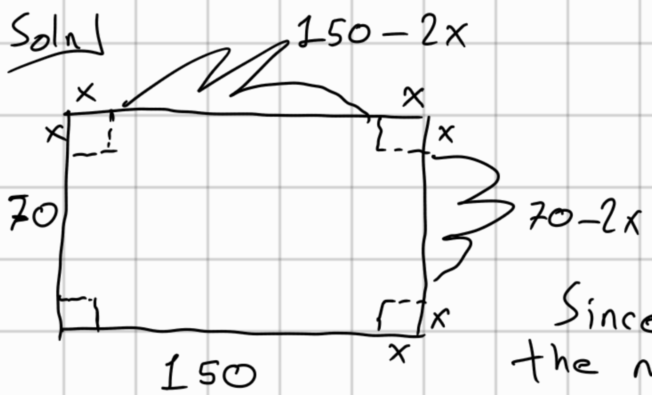
$\cancel{x = -\frac{8}{3}}$ $(x \geq 0)$ impossible, $\boxed{x = 2}$

$x = 2 \Rightarrow d^2 = 44 < 64$

$\therefore$ The minimum distance is $d = \sqrt{44} = 2\sqrt{11}$

2) A box is to be made from a rectangular sheet of cardboard 70 cm $\times$ 150 cm by cutting equal squares out of four corners. and bending up the resulting four flaps to make the sides of the box. (The box has no top) What is the largest volume of the box?

Soln



$V(x) = x(150-2x)(70-2x)$

$0 \leq x \leq 35$

Since $V(x)$ is cont. on $[0, 35]$, there exist the max. and min. values.

$V(0) = V(35) = 0$ the min. value

$V'(x) = 4(2625 - 220x + 3x^2) = 4(3x - 175)(x - 15) \Rightarrow \boxed{x = 15}$, $\cancel{x = \frac{175}{3}}$

$0 \le x \le 35$

$\therefore$ $V(15) = 72000\ cm^3$ is the largest possible volume.

3) Let $f(x)$ be a diff'ble func. where $f(-8) = -2$, $f'(-8) = \frac{1}{12}$.
Approximate the value of $f(-7.98)$.

Recall:
Linearization of f about a is the func. L defined by $L(x) = f(a) + f'(a)(x - a)$

linear approximation of f near a is

$$f(x) \approx L(x) = f(a) + f'(a)(x - a)$$

Soln

$$f(-8) = -2, \quad f'(-8) = \frac{1}{12}, \quad f(-7.98) \approx ?$$

Linearization of f about $a = -8$ is

$$L(x) = f(-8) + f'(-8)(x - 8) = -2 + \frac{1}{12}(x + 8)$$

Take $x = -7.98$, then

$$f(-7.98) \approx L(-7.98) = -2 + \frac{1}{12}(-7.98 + 8) = -2 + \frac{0.02}{12}$$

$$f(-7.98) \approx -2 + \frac{0.02}{12}$$

4) Using the linearization of a suitable func. about a suitable value, approximate $\frac{1}{\sqrt[4]{0.0083}}$.

$L(x) = f(a) + f'(a)(x - a)$; If we take $f(x) = \frac{1}{\sqrt[4]{x}}$, $a = 0.0081 = (0.3)^4$

$f(x) \approx L(x)$

So, $L(x) = f(0.0081) + f'(0.0081)(x - 0.0081)$ | $f'(x) = -\frac{1}{4}x^{-5/4}$

$$= \frac{1}{\sqrt[4]{\left(\frac{3}{10}\right)^4}} - \frac{1}{4}\left[\left(\frac{3}{10}\right)^4\right]^{-5/4}(x - (0.3)^4)$$

$$L(x) = \frac{10}{3} - \frac{1}{4}\left(\frac{10}{3}\right)^5 (x - 0.0081)$$

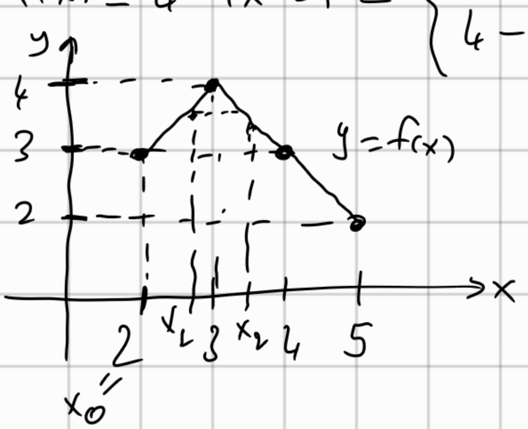
$$f(0.0083) \approx L(0.0083) = \frac{10}{3} - \frac{1}{4} \cdot \frac{10^5}{3^5} \underbrace{(0.0083 - 0.0081)}_{0.0002 = 2 \cdot 10^{-4}}$$

$$f(0.0083) \approx \frac{10}{3} - \frac{10}{2 \cdot 3^5}$$

5) Calculate lower and upper Riemann sums for the function $f(x) = 4 - |x-3|$ on the interval $[2,5]$, corresponding to the partition P of the given interval

(a) into four subintervals of equal length

$$f(x) = 4 - |x-3| = \begin{cases} 4-(3-x) & \text{if } x < 3 \\ 4-(x-3) & \text{if } x \geq 3 \end{cases} = \begin{cases} 1+x & \text{if } x \in [2,3] \\ 7-x & \text{if } x \in [3,5] \end{cases}$$



$P = \{x_0, x_1, x_2, x_3, x_4\}$ where $x_0 = 2$, $x_4 = 5$

Since the subintervals have equal length,

$$\Delta x_i = \frac{x_n - x_0}{n} = \frac{b-a}{n} = \frac{5-2}{4} = \frac{3}{4}$$

$$\underbrace{f(l_i)}_{\substack{\text{lowest value of } f \\ \text{on } [x_i, x_{i+1}]}} \leq f(x) \leq \underbrace{f(u_i)}_{\substack{\text{biggest value of } f \\ \text{on } [x_i, x_{i+1}]}} \text{ whenever } x_i \leq x \leq x_{i+1}$$

Lower Riemann Sum $L(f,P) = \sum_{i=1}^{n} f(l_i)\Delta x_i$, Upper Riemann Sum $U(f,P) = \sum_{i=1}^{n} f(u_i)\Delta x_i$

$$P = \{ \overset{x_0}{2}, \overset{x_1}{2+\frac{3}{4}}, 2+\frac{6}{4}, 2+\frac{9}{4}, \overset{x_4}{2+\frac{12}{4}} \} \qquad f = \begin{cases} 1+x & \text{on } [2,3] \\ 7-x & \text{on } [3,5] \end{cases}$$

Since f is increasing on $[2,3]$, $f(x_{i-1}) \leq f(x_i)$ where $[x_{i-1}, x_i] \subseteq [2,3]$

Since f is decreasing on $[3,5]$, $f(x_{i-1}) \geq f(x_i)$ where $[x_{i-1}, x_i] \subseteq [3,5]$

$f(x_0) = f(2) = 3$, $f(\overset{x_1}{2+\frac{3}{4}}) = 3 + \frac{3}{4} = \frac{15}{4}$, $f(\overset{x_2}{2+\frac{6}{4}}) = \frac{7}{2}$, $f(\overset{x_3}{2+\frac{9}{4}}) = \frac{11}{4}$

$f(x_4) = f(5) = 2$

On $[x_0, x_1]$, $f(x_0) \leq f(x_1)$ $\quad 3 \leq \frac{15}{4}$

on $[x_1, x_2] = [\frac{11}{4}, \frac{14}{4}]$ $\quad \frac{7}{2} \leq 4$,

On $[x_2, x_3]$ $\quad \frac{11}{4} \leq \frac{7}{2}$

On $[x_3, x_4]$ $\quad 2 \leq \frac{11}{4}$

$$\therefore L(f,P) = \sum_{i=1}^{4} f(l_i)\Delta x_i = \frac{3}{4}\left(3 + \frac{7}{2} + \frac{11}{4} + 2\right)$$

$$\therefore U(f,P) = \sum_{i=1}^{4} f(u_i)\Delta x_i = \frac{3}{4}\left(\frac{15}{4} + 4 + \frac{7}{2} + \frac{11}{4}\right)$$

exc!!

b) into six subintervals of equal length

c) into n subintervals of equal length

(i) on $[2,3]$

(ii) on $[3,5]$

(d) Evaluate the limits you obtained in part (c) as $x \to \infty$ and compare the obtained results.