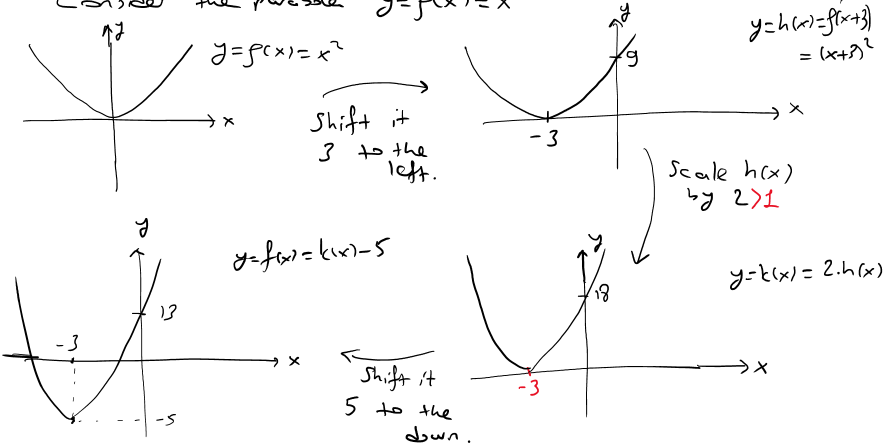
Week 3

Friday, November 5, 2021 1:31 PM

Q1] Find the domain and range of the function & sketch the graph of $f(x)=2x^2+12x+13 = 2(x^2+6x+9)-5$

$D_f=\mathbb{R}$, $R_f=[-5,\infty)$ $\qquad = \underbrace{2(x+3)^2}_{\geq 0}-5 \geq -5$

Consider the parabola $y=f(x)=x^2$



Q2] Find hos and its domain.

a) $h(x)=x-\frac{1}{x}$ and $s(x)=\frac{x+5}{x-7}$

$D_h=\mathbb{R}-\{0\}$ and $D_s=\mathbb{R}-\{7\}$. However, we cannot say $D_{hos}=\mathbb{R}-\{0,7\}$ directly

$hos(x)=h(s(x))=h\left(\frac{x+5}{x-7}\right)=\frac{x+5}{x-7}-\frac{x-7}{x+5}=\frac{24x-24}{(x+5)(x-7)}$

Check!

What is the D_{hos}?

By looking the formula of hos(x), we cannot determine D_{hos}.

i.e. $\text{Im}\, s \subset D_h$

$hos(x)=h(s(x))$

If $x\in D_{hos}$, $x\in D_s=\mathbb{R}-\{7\}$ and $s(x)\in D_h=\mathbb{R}-\{0\}$.

If $s(x)\in\mathbb{R}-\{0\}$, $s(x)\neq 0$ for any $x\in D_{hos}$. So, $x=-5\notin D_{hos}$.

"not in"

$s(x) = \frac{x+5}{x-7}$

$\therefore D_{hos}=D_s-\{-5\}=\mathbb{R}-\{-5,7\}$

b) $h(x) = \frac{1-x}{2x}$ & $s(x) = \frac{1}{1-2x}$

$$h\circ s(x) = h\left(\frac{1}{1-2x}\right) = \frac{1-\frac{1}{1-2x}}{2\cdot\frac{1}{1-2x}} = \frac{1-2x-1}{1-2x}\cdot\frac{1-2x}{2} = \underbrace{-x}_{\boxed{x \neq 1/2}}$$

But $D_{h\circ s} = \mathbb{R} - \{1/2\}$ not whole real numbers. exc!

Q3) Find the inverse funcs of the following functions and the domains of inverses if exists, or explain why it does not exit **(d.n.e.)**.

a) $f(x) = \frac{1}{2x-9}$

$D_f = \mathbb{R} - \{9/2\}$, $R_f = \mathbb{R} - \{0\}$ → exc!

Choose any $y_0 \in R_f$ and find an x_0 s.t.

$$\frac{1}{2x_0-9} = f(x_0) = y_0$$

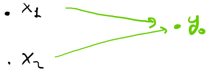
$\Rightarrow x_0 = \frac{1+9y_0}{2y_0}$ if $y_0 \neq 0$

No soln. if $y_0 = 0$

f is onto

$$f : \mathbb{R} - \{9/2\} \twoheadrightarrow \operatorname{Im} f.$$

Q: Is f 1-1?



If $f(x_1) = f(x_2)$ for any $x_1, x_2 \in D_f$,

$$\frac{1}{2x_1-9} = \frac{1}{2x_2-9} \Rightarrow 2x_1 - 9 = 2x_2 - 9$$
$$\Rightarrow x_1 = x_2$$

$\therefore f$ is 1-1. (or injective)

$\therefore f$ is invertible. Then, it has an inverse.

Say $y = f^{-1}(x)$ is its inverse.

$D_{f^{-1}} \underset{\text{by def'n}}{:=} R_f = \mathbb{R} - \{0\}$, $R_{f^{-1}} := D_f = \mathbb{R} - \{9/2\}$

One-to-one (1-1) ≡ injective

onto ≡ surjective

$$y = f^{-1}(x) \underset{\text{as } f \text{ is invertible}}{\Longrightarrow} f(y) = x \Rightarrow \frac{1}{2y-9} = x$$

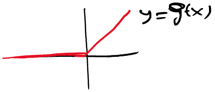
$$\Rightarrow 1 = 2xy - 9x \Rightarrow 2xy = 1 + 9x \Rightarrow \boxed{y = \frac{1+9x}{2x} = f^{-1}(x)}$$

b) $f(x) = x + |x|$

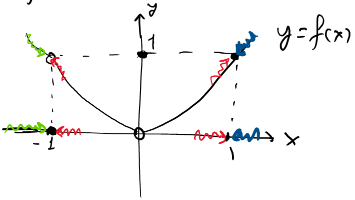
Chose $x_1 = -153$ & $x_2 = -3$

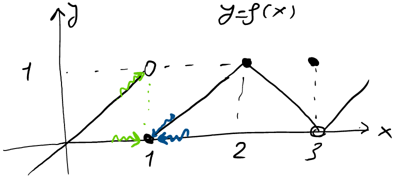
$f(x_1) = 0$ & $f(x_2) = 0$ but $x_1 \neq x_2 \implies f$ is not even injective

∴ It is not invertible.



Q4| Find the limits for the following funcs. whose graphs given below!





a) $\lim_{x \to -1} f(x) =$ $f(-1) = 0$

$\lim_{x \to -1^+} f(x) = 1$ $=$ $\lim_{x \to -1^-} f(x) = 1$ $\implies$ $\lim_{x \to -1} f(x) = 1$

"Right Limit" ↳ $x > -1$ but close to -1

"Left limit" ↳ $x < -1$ but close to -1

but $f(-1) = 0$

b) $\lim_{x \to 0} f(x)$ exc!

c) $\lim_{x \to 1} f(x) =$

$$\left.\begin{array}{l}\lim_{x \to 1^+} f(x) = 1 \\ \lim_{x \to 1^-} f(x) = 1\end{array}\right\} = \implies \lim_{x \to 1} f(x) = 1$$

$f(1) = 1$

d) $\lim_{x \to 1} g(x)$

$$\left.\begin{array}{l}\lim_{x \to 1^+} g(x) = 0 \\ \lim_{x \to 1^-} g(x) = 1\end{array}\right\} \neq$$

So, the limit does not exist. (d.n.e)

$\lim_{x \to 1} g(x) =$ d.n.e

e) $\lim_{x\to 2} f(x) =$ exc!

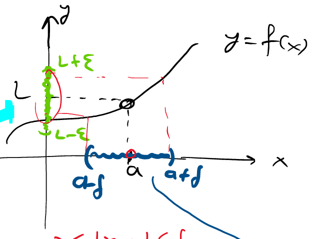
Q5) Find suitable $\delta > 0$ s.t. if $0 < |x-a| < \delta$, then $|f(x)-L| < \varepsilon$
where $\lim_{x\to a} f(x) = L$

ε, δ

a) $\lim_{x\to 2} x+8 = 10$ & $\varepsilon = 2$

$-\varepsilon < f(x)-L < \varepsilon$

$\iff L-\varepsilon < f(x) < L+\varepsilon$



$0 < |x-a| < \delta$

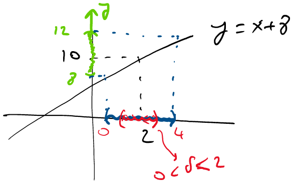
$\iff -\delta < x-a < \delta$ $x \neq a$
if and only if

$\iff a-\delta < x < a+\delta$
$x \neq a$

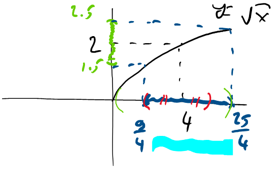
$|f(x) - L| = |x+8-10| < 2$

$\iff |x-2| < 2$
if and only if
(i.e. iff)

Choose $\delta = 2$ (or any positive number ≤ 2)



b) $\lim_{x\to 4} \sqrt{x} = 2$ & $\varepsilon = 0.5$



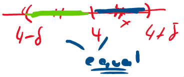
$|\sqrt{x} - 2| < 0.5$

$\iff -0.5 < \sqrt{x}-2 < 0.5$

$\iff \frac{3}{2} < \sqrt{x} < \frac{5}{2}$

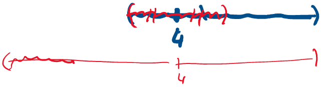
$\iff \frac{9}{4} < x < \frac{25}{4} \iff -\frac{5}{4} < x-4 < \frac{9}{4}$

Aim: Find $\delta > 0$ s.t. if $0 < |x-4| < \delta \implies |\sqrt{x} - 2| < 0.5$



Choose $\delta = \frac{5}{4}$ (or any number less than $\frac{5}{4}$)

$\Downarrow$

$0 < |x-4| < \frac{5}{4}$