

## MATH 120 2020-2 Friday 10:40-12:30 - Quiz 1

Duration: ~ 15 min.

- Write your NAME, SURNAME, ID and SECTION.
- Write the QUESTION completely on your solution paper.
- Upload your solutions to Gradescope as a SINGLE PDF PAGE.

Question: Let  $a_n = \begin{cases} 1 & \text{if } n = 2m \\ \frac{1}{n} & \text{if } n = 2m - 1 \end{cases}$  where  $m = 1, 2, \dots$

(a) Is the sequence  $\{a_n\}$  convergent or divergent?

(b) Let  $b_n = |a_{n+1} - a_n|$ . Is the sequence  $\{b_n\}$  convergent or divergent?

a) Consider the subsequences  $a_{2n}$  and  $a_{2n+1}$  of the seqn.  $a_n$  for  $n \geq 1$ .

$$\lim_{n \rightarrow \infty} a_{2n} = \lim_{n \rightarrow \infty} 1 = 1 \quad (2)$$

$$\lim_{n \rightarrow \infty} a_{2n+1} = \lim_{n \rightarrow \infty} \frac{1}{n} = 0 \quad (2)$$

As these two subsequences do not converge to the same number,  $\{a_n\}$  is Divergent. (1)

b)  $b_n = |a_{n+1} - a_n| = |1 - \frac{1}{n}|$  as (3)

the indexes of  $a_n$  and  $a_{n+1}$  are consecutive and so one of the indexes is odd while the other is even.

Then,  $\lim_{n \rightarrow \infty} b_n = \lim_{n \rightarrow \infty} |1 - \frac{1}{n}| = 1$  (1)

∴ It is Convergent. (1)

Counter Examples for your famous mistakes

•  $a_n = \begin{cases} 1 & \text{if } n = 1, 2, 3, \dots, 10 \\ 120 & \text{if } n > 10 \end{cases}$

~~$$\lim_{n \rightarrow \infty} a_n = \begin{cases} \lim_{n \rightarrow \infty} 1 & n < 10 \\ \lim_{n \rightarrow \infty} 120 & n > 10 \end{cases} = \begin{cases} 1 & n < 10 \\ 120 & n > 10 \end{cases}$$~~

So, it is div. as  $1 \neq 120$ .

$\lim_{n \rightarrow \infty} a_n = 120$  as  $n$  takes large values.

•  $a_n = \frac{1}{n}$  is conv. seqn. as  $a_n \rightarrow 0$

However,  $\sum_{n=1}^{\infty} \frac{1}{n}$  is div. by p-test.

•  $a_n = \frac{(-1)^n}{n}$  converges to 0 although

$a_n$  is neither increasing nor decreasing.

•  $\sum_{n=1}^{\infty} 1$  is div. by n-term test but

$a_n = 1$  is conv. as  $\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} 1 = 1$