Q-2

Saturday, April 10, 2021 5:34 PM

MATH 120 2020-2 Friday 10:40- 12:30 - Quiz 2

Duration: $\sim$ 15 min.

1. Write your NAME, SURNAME, ID and SECTION.
2. Upload your solutions to Gradescope as a SINGLE JPG or PDF PAGE.

Question:

Determine whether or not the series $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n^2}\right) \sin(n)$ is convergent.

Let $a_n = \sin\left(\frac{1}{n^2}\right) \cdot \sin(n)$. It is not positive.

So, consider $\sum_{n=1}^{\infty} |a_n|$. Then,

$$0 \leq |a_n| \leq \left|\sin\left(\frac{1}{n^2}\right)\right| = \sin\left(\frac{1}{n^2}\right)$$

as $0 < \frac{1}{n^2} \leq 1 < \frac{\pi}{2}$ for all $n \in \mathbb{N}$

Consider the series $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n^2}\right)$.

Let $b_n = \frac{1}{n^2}$. Then,

$$\lim_{n\to\infty} \frac{\sin(1/n^2)}{1/n^2} = \lim_{m\to 0} \frac{\sin(m)}{m} = 1 \underset{<\infty}{\neq} 0$$

Say $m = \frac{1}{n^2}$

By L.C.T, $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n^2}\right)$ is convergent as $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is so by p-test.

$\therefore$ $\sum_{n=1}^{\infty} |a_n|$ is Conv. by Comparison test.

Hence, $\sum_{n=1}^{\infty} a_n$ is (absolutely) Convergent.

Some remarks for the common mistakes!

- $\sum_{n=1}^{\infty} \underbrace{(-1)^n \sin(n)}_{=a_n}$ is not an alternating series as $a_{n+1} \cdot a_n = (-1)^{2n+1} \cdot \sin(n) \cdot \sin(n+1)$ may not be negative for each $n \geq 1$. (though it has the factor $(-1)^n$.)

- $\sum_{n=0}^{\infty} a_n$ with $a_n \geq 0$ for all n (or ultimately)

Then, you CAN use the following test!

- Comparison / Limit Comp. Tests
- Ratio / Root Tests
- Integral Test

But, for n^{th} term test, no need to check the sign of a_n for each n.

- $-\frac{1}{n} \leq \frac{1}{n^2}$ for all $n \geq 1$.

But, $\sum_{n=1}^{\infty} -\frac{1}{n} = -\sum_{n=1}^{\infty} \frac{1}{n}$ diverges to $+\infty$, diverges to $-\infty$. although $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is convergent by p-test. $(2>1)$

Here, Comparison Test does not work!! as both $-\frac{1}{n}$ & $\frac{1}{n^2}$ are not POSITIVE (or ultimately)

Also, $\frac{1}{n}$ is convergent seq n. But, its series $\sum_{n=1}^{\infty} \frac{1}{n}$ is divergent.