NUCLEAR ENERGY

Problem 1

State the name of the element, the number of protons and neutrons, the half-life and decay reaction, and the daughter nuclide. If the daughter nuclide is radioactive, continue until the decay reactions produce a stable nuclide.

Example: $^{60}_{27}Co \rightarrow ^{60}_{28}Ni + \beta^- + \gamma_d$

Parent: cobalt-60, beta decay, $t_{1/2}$ = 5.62 y,   Daughter: nickel-60, stable

(a) $^{15}_{8}O$ (b) $^{24}_{11}Na$ (c) $^{107}_{48}Cd$ (d) $^{131}_{53}I$ (e) $^{232}_{86}Rn$

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Problem 2

Carbon dating is a technique used to age organic-derived materials. The process involves determining the amount of carbon-14 (radiocarbon) in the material. When a nitrogen-14 nucleus in the atmosphere is bombarded by `slow' neutrons emitted in cosmic radiation, a carbon-14 and proton are produced. Carbon-14 can be combined with oxygen to form $^{14}CO_2$, which constitutes 0.1 % of all CO_2 in the atmosphere. Both forms of CO_2 are absorbed by living organisms. When absorption ceases due to death of the organism, the fraction of carbon-14 begins to decrease due to radioactive decay. Estimate the age of an old manuscript if the amount of carbon-14 is measured to be 0.030 % of all carbon in the manuscript.

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Problem 3

Find the maximum kinetic energy, in MeV, of the emitted particle and that of the residual nucleus from the radioactive decay of the following radioisotopes. The energy quantities in parenthesis are the amount of gamma radiation energy (d) emitted in each decay process.

(a) uranium-234 (0.053 MeV)

(b) uranium-235 (0.186 MeV)

(c) uranium-236 (0.049 MeV)

(d) plutonium-238 ()

(e) plutonium-239 (0.052 MeV)

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Problem 4

Write the complete decay reactions, determine the activity, and calculate the energy generated, in MeV/reaction and J/reaction, of the following radioisotopes:

(a) $^{211}_{84}$Po (b) $^{95}_{40}$Zr (c) $^{30}_{15}$P (d) $^{64}_{29}$Cu (K-capture) (e) $^{87}_{36}$Kr

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Problem 5 (ME436s14m2-2 / ME405s23f-5)

When pure ordinary water passed through a reactor as a coolant-moderator, it becomes slightly radioactive. The most important of the radioactivities is due to the absorption of a neutron by an oxygen-16 nucleus. This reaction results in emission of a proton and a radioactive product nucleus that has a 7.2 second half-life.

(a) Identify the product nucleus.

(b) Calculate the percent radioactivity remaining in the water 28.8 seconds after this reaction.

Decay constant: $\lambda = \dfrac{\ln(2)}{t_{1/2}}$, Activity: $A = \lambda\, N_0\, e^{-\lambda t}$

A small part of the chart of nuclides is given below:

5	^{6}H	^{7}He	^{8}Li	^{9}Be	^{10}B	^{11}C	^{12}N	^{13}O	^{14}F	^{15}Ne	Na	Mg		
6	^{7}H	^{8}He	^{9}Li	^{10}Be	^{11}B	^{12}C	^{13}N	^{14}O	^{15}F	^{16}Ne	^{17}Na	^{18}Mg	13	14
	7	^{9}He	^{10}Li	^{11}Be	^{12}B	^{13}C	^{14}N	^{15}O	^{16}F	^{17}Ne	^{18}Na	^{19}Mg	Al	Si
	8	^{10}He	^{11}Li	^{12}Be	^{13}B	^{14}C	^{15}N	^{16}O	^{17}F	^{18}Ne	^{19}Na	^{20}Mg		^{22}Si
		9	^{12}Li	^{13}Be	^{14}B	^{15}C	^{16}N	^{17}O	^{18}F	^{19}Ne	^{20}Na	^{21}Mg	^{22}Al	^{23}Si

Answer: (a) N-16

(b) 6.25 %

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Problem 6

A sample of radioactive material is composed of two independent radioisotopes - a long-lived radioisotope and a short-lived radioisotope. A Geiger counter, from which the output is directly proportional to the total activity of the sample, gave the following results:

Time, h	0	10	20	30	40	60	90	120	180	240	360

Activity, Bq	9000	3818	1703	837	472	231	139	95	47	23	5

Determine the half-life for the two radioisotopes. (Hint: Use a semi-log plot and assume the half-life of one isotope is much longer than the second.) If the total initial activity of the mixture is 1 Ci, calculate the number of radioactive nuclei of each isotope at time 0 and 120 hours.

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Problem 7 (ME405f16f-5 / ME436s17f-5)

Tritium (very heavy hydrogen) decays by emitting low-energy (5.6 keV) β particles.

$$^{3}_{1}H \rightarrow \, ^{4}_{2}He + \, ^{0}_{-1}\beta + \nu \qquad t_{1/2} = 12.3 \text{ years}$$

This radiation acts on a phosphor producing illumination. Illumination can thus be made by adding tritium to a phosphor in the form of paint which are sealed in a plastic container that is transparent to illumination but that blocks the β particles so that no hazard is encountered. The illuminators are used for such devices as locks, timepieces, aircraft safety markers, exit signs, etc. Regulations limit the amount of original radioactivity in such devices, depending upon service.

Assuming that 4 mCi (milli Curie) activity are permitted for an aircraft safety device, calculate

(a) the maximum mass of tritium that can be used, in grams, and

(b) the percent decrease in luminosity (which is proportional to radioactivity) after 10 years of service.

1 Ci = 3.7 10^{10} disintegrations (or disintegrating atoms) per second

Atomic mass of tritium = 3.0160492 amu (or grams per mol)

Avagadro's number: $N_A = 6.022\ 10^{23}$ atoms per mol

Activiy: $A = \lambda N \quad , \quad A = A_0\, e^{-\lambda t}$

Decay constant: $\lambda = \dfrac{\ln(2)}{t_{1/2}}$

Answer: (a) 4.15 10^{-7} grams

(b) 43 %

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Problem 8

In fast-breeder reactors, plutonium-239 is the primary fuel. A relatively stationary ^{239}Pu nucleus is fissioned by a 1.0 MeV neutron resulting in two fission fragments: Krypton-93 and Cerium-144.

(a) Identify all decay products of these two fragments until stable isotopes are obtained.

(b) Calculate the total energy released in MeV per ^{239}Pu nucleus and kWh per gram of ^{239}Pu.

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Problem 9

For 1-g of Radium-226 calculate the percent loss of radium nuclei after 100 years. How long until the activity reaches 1 milliCurie (mCi)?

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Problem 10 (ME436s14h8)

SNAP (systems for nuclear auxiliary power) are devices that generate electric power directly from the heat generated by radioisotopic “fuels", in which case the are given odd numbers; or fission nuclear reactors, in which case they are given even numbers. Direct generation is usually accomplished by thermoelectric energy conversion. An example is the Apollo lunar surface experiment package (ALSEP), called SNAP-27, which was placed on the lunar surface by the Apollo astronauts during their lunar landings in the late 1960s and early 1970s. SNAP-27 used Plutonium-238 in the form of plutonium carbide PuC as `fuel'. If the fuel deployed has a mass of 1 kg and the thermoelectric conversion efficiency is 8 %, calculate

(a) the number of Pu-238 nuclei present,

(b) the rate of reaction,

(c) the power generated, in Watts, upon deployment, and

(d) the power generated, in Watts, 5 years after deployment

Answer: (a) $N_{Pu-238} = 2.40852\ 10^{24}$ nuclei of Pu-238

(b) $A = 6.03216\ 10^{14}$ decays/s

(c) 43.25 W

(d) 41.57 W

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Problem 11 (ME436s15h8 / ME405f15h8 / ME436s16h10)

Calculate the power generated per unit volume, in MeV/s.cm^3 and kW/m^3, of a 3.5 % enriched uranium dioxide fuel element in a thermal reactor with an effective cross section of 350 barns and a neutron flux of 10^{14} neutrons/s-cm^2. The density of UO_2 is 10.5 g/cm^3. The mean energy released per fission of Uranium-235 is approximately 200 MeV.

You may use the following data:

Mass of U-238: 238.05 amu (g/g mol)

Mass of U-235: 235.04 amu (g/g mol)

Mass of O-16: 15.994 amu (g/g mol)

3.5 % enriched uranium has 96.5 % U-238 and 3.5 % U-235 by mass (ignore the presence of U-236)

Avagadro's number = 6.022 10^{23} nuclei/mol

1 barn = 10^{-24} cm^2

Answer: 5.74 10^{15} MeV/s.cm^3 = 0.92 10^6 kW/m^3

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Problem 12 (ME436s14m2-3 / ME436s20h11)

A thermoelectric conversion device has an efficiency of 8 %. It uses Pu-238 as the fuel in the form of PuC (Plutonium carbide). If 1 kg of PuC is initially used, find the power generated, in Watts, (a) the first time it is deployed and, (b) 5 years later.

Pu-238 decay reaction: ${}^{238}_{94}Pu \rightarrow {}^{234}_{92}U + {}^{4}_{2}He$

Mass of Pu-238 atom: 238.04956 amu, Mass of U-234 atom: 234.040952 amu

Mass of He-4 atom: 4.002602 amu, Mass of C-12 atom: 12.0 amu

Mass-energy conversion: 931.5 MeV/amu

1 MeV = 1.602 10^{-13} Joules

Half life of Pu-238: 87.7 years, Decay constant: $\lambda = \frac{\ln(2)}{t_{1/2}}$, Activity: $A = \lambda N_0 e^{-\lambda t}$

Avagadro's number: N_A = 6.0225 10^{23} nuclei/mol

1 year = 31557600 seconds

Answer: (a) 43.25 W

(b) 41.57 W

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Problem 13

A PWR power plant pressurizer operates at a steady state at 2200 psia with a constant spray flow of 8 lbm/min from the cold leg at 552 F. Calculate the amount of heat added by the electric heaters, in kW, if the pressurizer heat losses to the ambient are 6144 Btu/h.

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Problem 14

PWR primary loop with 8000 ft^3 volume operates at an average temperature of 580 F. The reactor has a 1000-ft^3 vapor pressurizer that normally contains 60 % water by volume at 2200 psia. An accident occurs in which the relief valve becomes stuck in an open position and fluid is discharged into the relief tank. The system pressure steadily drops to 1600 psia, during which time the electric heaters were fully activated to help slow down the pressure drop. At 1600 psia, the primary loop average temperature is 550 F, the pressurizer is 95 % full of steam, the heaters were turned off to keep them from overheating, and the emergency core cooling system (ECCS) was activated. The ECCS replenished the primary loop with water to prevent uncovering and damaging the fuel elements. The relief tank is assumed to remain at nearly atmospheric pressure, but there is a 15.3 psig pressure drop in the line connecting the relief tank to the open pressurizer relief valve. Ignoring the effect of spray and heat losses to ambient, calculate:

1. the initial mass composition of the water-steam in the pressurizer, in lbm,
2. the condition of the fluid leaving the relief valve at the instant it opened (pressure, temperature, quality or degree of superheat),
3. the total loss of fluid, in lbm, from the primary loop (before ECCS) assuming for simplicity that the temperature remained constant, and
4. the state of the fluid exiting the relief valve (owing into the relief tank) at the instant the ECCS came on line.

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Problem 15 (ME436s14q4)

Determine the activity, in Bq, of the three uranium isotopes found in 100 kg of uranium nitrate (U_3N_4) when natural uranium is used.

The three naturally occurring uranium isotopes are:

U-234: 0.0055 %, 234.0409 amu (g/mol) , $t_{1/2}$ = 2.4 10^5 years

U-235: 0.7200 %, 235.0439 amu (g/mol) , $t_{1/2}$ = 7.1 10^8 years

U-238: 0.992745 %, 238.0508 amu (g/mol) , $t_{1/2}$ = 4.51 10^9 years

Note that the activity of 1 g Ra-226 is very small compared to the total number of nuclei (10^{10} disintegrations/second vs 10^{21} nuclei). Therefore, the activity can be treated as constant. This is true for any isotope with a long half life.

Answer: $A_{U\text{-}234}$ = 1.2517 10^9 Bq (U-234 disintegrations per s)

$A_{U\text{-}235}$ = 0.0523 10^9 Bq (U-235 disintegrations per s)

$A_{U\text{-}238}$ = 1.1342 10^9 Bq (U-238 disintegrations per s)

Total activity is the sum: A = 2.4389 10^9 Bq = 65.92 mCi (milli Curies)

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Problem 16 (ME436s14q5 / ME436s15q5 / ME405f15q8 / ME436s16q10)

Radioactive carbon-14 (6 protons, 8 neutrons in the nucleus) is used to estimate the age of materials composed of organic carbon. It is formed by a reaction between CO_2 atoms in the upper atmosphere and high energy neutrons (part of cosmic radiation) that bombards the earth. Living organic material absorbs and uses carbon dioxide from the atmosphere which contains about 1 $^{14}CO_2$. When the organism dies, the ratio of C-14 to C-12 decrease with time as the radiocarbon decays with beta emission. If the proportion of C-14 to stable carbon in an old manuscript is found to be 0.7 %, determine the age of the manuscript assuming the atmospheric bombardment of high-energy neutrons is constant. Half life of C-14 is 5730 years.

Answer: 2948.5 years

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Problem 17 (ME436s14h7 / ME436s15h7 / ME405f15h7 / ME436s16h9)

Uranium-235 undergoes α decay with emission of a 0.17 MeV gamma ray. What is the kinetic energy of the product nucleus and the α particle?

$$^{235}_{92}U \text{ --> } ^{231}_{90}Th + ^{4}_{2}\alpha + \gamma_d \text{ (0.17 MeV)}$$

$^{235}_{92}U$: 235.0439 amu (g/mol)

$^{231}_{90}Th$: 231.0347 amu (g/mol)

$^{4}_{2}He$: 4.0026 amu (g/mol)

Answer: $E_{KE,Th\text{-}231}$ = 0.102 MeV; $E_{KE,He\text{-}4}$ = 5.876 MeV

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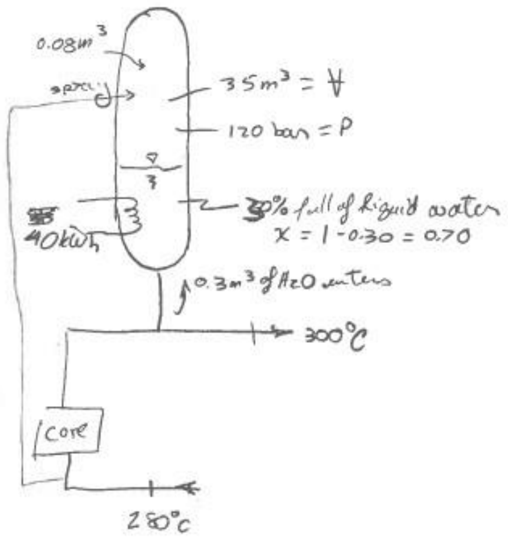
Problem 18 (ME436s14f-2)



A PWR (Pressurized Water Reactor) has inlet and exit water at 280 °C and 300 °C, respectively. It has a 35 m^3 vapor pressurizer which is normally 30 % full of water at a pressure of 120 bar.

A case of an insurge occurred during which 0.3 m^3 of water entered the pressurizer from the primary circuit hot leg, 0.08 m^3 entered through spray, and 40 kWh was added by the electric heaters. Determine the internal energy of the pressurizer contents before and after the event, in kJ. Ignore heat losses to the ambient.

Properties of water (from thermodynamic tables)						
	v_f, m^3/kg	v_g, m^3/kg	u_f, kJ/kg	u_g, kJ/kg	h_f, kJ/kg	h_g, kJ/kg
At 120 bar	0.001527	0.014263	1473.0	2517.3	1491.3	2684.9
At 300 °C	0.001404	0.02176	1332.0	2563.0	1344.0	2749.0
At 280 °C	0.001332	0.03017	1227.46	2586.1	1235.99	2779.6

Answer: U_1 = 14.45 10^6 kJ; U_2 = 15 10^6 kJ

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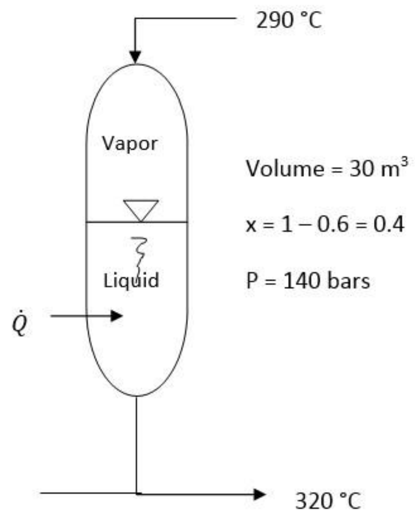
Problem 19 (ME436s14fm-2 / ME436s15q6)

A PWR (pressurized water reactor) has inlet and exit water at 290 °C and 320 °C, respectively. It has a 30 m^3 vapor pressurizer which is normally 60 % full of water at a pressure of 140 bar. A case of an insurge occurred during which 0.25 m^3 of water entered the pressurizer from the primary circuit hot leg, 0.05 m^3 entered through spray, and 50 kWh was added by the electric heaters.



Determine the internal energy of the pressurizer contents before and after the event, in kJ. Ignore heat losses to the ambient.

Properties of water (from thermodynamic tables)						
	v_f, m³/kg	v_g, m³/kg	u_f, kJ/kg	u_g, kJ/kg	h_f, kJ/kg	h_g, kJ/kg
At 140 bar	0.001611	0.011485	1548.60	2476.8	1571.10	2637.6
At 320 °C	0.001472	0.015488	1444.60	2525.5	1461.50	2700.1
At 290 °C	0.001366	0.02557	1278.92	2576.0	1289.07	2766.2

Answer: $U_1 = 19.89\ 10^6$ kJ; $U_2 = 20.366\ 10^6$ kJ

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Problem 20

Calculate the Binding Energy per nucleon of the following isotopes:

(a) H-2 or D-2 (Z = 1, A = 2, atomic mass = 2.0141 amu

(b) Fe-56 (Z = 26, A = 56, atomic mass = 55.934934 amu

(c) Ni-59 (Z = 28, A = 59), atomic mass = 58.9342 amu

(d) U-238 (Z = 92, A = 238, atomic mass = 238.0289 amu

Given data:

Mass of H-1 = 1.007825 amu

Mass of a neutron = 1.0086625 amu

1 amu = 931.5 MeV

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Problem 21 (ME436s19h9 / ME405s19h9)

An exothermic reaction occurs when common aluminum (Al-27, mass = 26.99153 amu) is bombarded with high-energy α-particles (helium nuclei, mass = 4.00260 amu) resulting in Si-30 (a heavy isotope if silicon; mass = 29.9736 amu; the most common isotope of silicon is Si-28. During the reaction, a small particle is emitted. Write the reaction and calculate the change in mass. Find out what the emitted particle is.

$$^{27}_{13}\text{Al} + ^{4}_{2}\alpha = ^{30}_{14}\text{Si} + ^{A}_{Z}\text{X}$$

Note the following: In nuclear reactions

- Mass numbers are conserved (not the mass)
- Sum of mass and energy is conserved

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Problem 22 (ME405f15m2-4 / Me436s16m2-4)

Radium-226 is a radioisotope that decays into Radon gas. If there is 1 gram of Ra-226, Compute:

(a) The decay constant; and

(b) The initial activity (at time = 0)

(c) Activity after 100 years.

Data:

$^{226}_{88}\text{Ra}$: 226.0254 amu (g/mol) , $t_{1/2}$ = 1600 years , α decay.

Avogadro number: $N_A = 6.02214\ 10^{23}$ atoms (nuclei) per mol.

Hint: Find the number of nuclei in 1 gram of Ra-226.

$$\lambda = \frac{\ln(2)}{t_{1/2}} \qquad A(t) = A_0\ e^{-\lambda t}$$

Answer: (a) $\lambda = 1.37372\ 10^{-11}\ s^{-1}$

(b) $A_0 = 3.66\ 10^{10}$ Bq

(c) $A(t) = 3.5\ 10^{10}$ Bq

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Problem 23

Determine the activity, in Bq, of the three uranium isotopes found in 100 kg of uranium nitrate (U_3N_4) when natural uranium is used.

The three naturally occurring uranium isotopes are:

$^{234}_{92}U$ 0.0055 %, 234.0409 amu (g/mol) , $t_{1/2} = 2.4\ 10^5$ years

$^{235}_{92}U$ 0.7200 %, 235.0439 amu (g/mol) , $t_{1/2} = 7.1\ 10^8$ years

$^{238}_{92}U$ 0.992745 %, 238.0508 amu (g/mol) , $t_{1/2} = 4.51\ 10^9$ years

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Problem 24

Uranium undergoes α decay with emission of a 0.17 MeV gamma ray. What is the kinetic energy of the product nucleus and the α particle?

$$^{235}_{92}U \text{ --> } ^{231}_{90}Th + ^{4}_{2}\alpha + \gamma_d \text{ (0.17 MeV)}$$

$^{235}_{92}U$ 235.0439 amu (g/mol)

$^{231}_{90}Th$ 231.0347 amu (g/mol)

$^{4}_{2}He$ 4.0026 amu (g/mol)

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Problem 25 (ME405f15q9 / ME436s16q11)

Determine the reactor fuel loading in kg of U-235, in a 1200 MWe power reactor operating at a thermal efficiency of 33 %, and average neutron flux of $6\ 10^{17}$ neutrons/m^2.s in the core, and an average fission cross section of 365 barns.

Answer: $m_{U\text{-}235}$ = 2022.7 kg U-235

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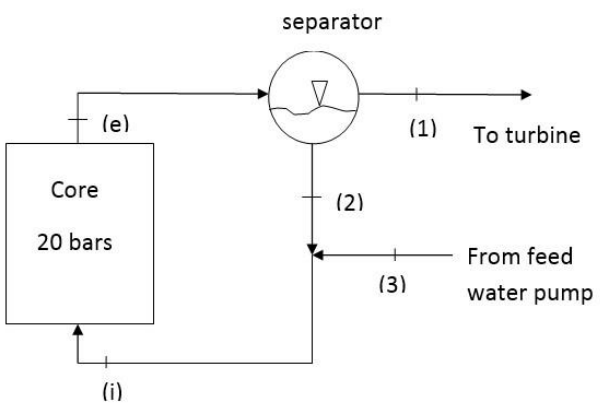
Problem 26

A 1000 MW BWR power plant with 33 % efficiency was operating at 75 % of rated load with a mass flow rate of 1150 kg/s, a reactor core pressure of 70 bar, and an average exit quality of 13.6 %. The plant uses recirculation control. Find



- The feed water temperature in °C
- The core degree of subcooling in °C
- The downcomer flow at 75 % load
- The average exit quality immediately after initiation of a load change to 80 %, and when the load has changed to 80 %
- The steam and downcomer flows, in kg/s, after load change

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Problem 27 (ME405f15q7)

Calculate the binding energy per nucleon of U-238 isotope (Z = 92, A = 238, atomic mass = 238.0289 amu)

Mass of H-1 = 1.007825 amu

Mass of a neutron = 1.0086625 amu

1 amu = 931.5 MeV

Answer: BE = 1821.758 MeV

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Problem 28 (ME405f16q9 / ME436s17q9)

A chemist determines that a sample of petrified wood has a carbon-14 decay rate of 6.00 counts per minute per gram. What is the age of the piece of wood in years? The decay rate of carbon-14 in fresh wood today is 13.6 counts per minute per gram, and the half life of carbon-14 is 5730 years.

Answer: 6765 years

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Problem 29 (ME405f15f-3 / ME436s16f-3)

A small nuclear reactor delivers 100 MW_e. It is based on a Rankine cycle where the steam is expanded in a turbine from 12.5 MPa and 350 °C to condensed water at 5 kPa and 32.9 °C. Assume that the generator is 98 % efficient, the actual cycle achieves 90 % of the efficiency of the ideal cycle, and 90 % of the thermal energy released in the nuclear reactions is transferred to the working fluid.

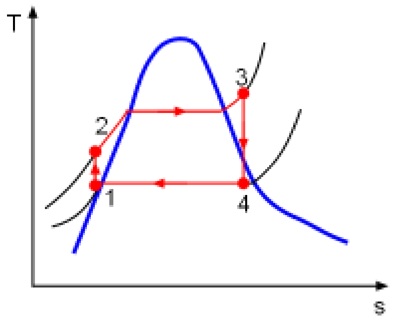
If the reactor uses uranium as a fuel which is enriched to 3 % U-235, what mass of fuel (UO_2) is consumed (burned) each year?

Assume the following:

1. The capacity factor of the plant is 100 %.
2. The fuel consists of pure natural uranium dioxide (UO_2).
3. The energy content of U-235 is 69.8 10^{12} Joules per kg U-235.

Table of thermodynamic properties (for Ideal Rankine Cycle):

State	Temperature, °C	h, kJ/kg	s, kJ/kg.K
1	32.9	137.8	0.4762
2		150.4	
3	350	2826.6	5.713
4		1740.4	5.713



Specific volume of water: 1.005 cm^3/g

Uranium has an average mass of 238 kg/kg-mol.

Answer: 5364.8 kg of UO_2

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Problem 30 (ME405f16h8 / ME436s18h8 / ME405s18h7)

Radioactive carbon-14 (6 protons, 8 neutrons in the nucleus) is used to estimate the age of materials composed of organic carbon. It is formed by a reaction between CO_2 atoms in the upper atmosphere and high energy neutrons (part of cosmic radiation) that bombards the earth. Living organic material absorbs and uses carbon dioxide from the atmosphere which contains about 1 % $^{14}CO_2$. When the organism dies, the ratio of C-14 to C-12 decrease with time as the radiocarbon decays with beta emission.

The skin, bones and clothing of an adult female mummy discovered in Chimney Cave, Lake Winnemucca, Nevada, were dated by radiocarbon analysis. How old is this mummy if the sample retains 73.9 % of the activity of living tissue?
The half-life of C-14 is 5730 years.

Answer: 2500 years

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Problem 31 (ME436s18q11 / ME405s18q10 / ME436s19q12 / ME405s19q12 / ME436s20q12 / ME405s22q11 / ME405s23q11)

The average energy released in a fission of U-235 nucleus is about 200 MeV. Suppose we had 1.0 g of U-235, and assume fission of all of the nuclei. How much energy would be released? Estimate how much coal it would take to produce the same amount of energy?

1 eV = $1.602\ 10^{-19}$ J

Heating value of coal: 30 000 kJ/kg

Atomic Mass of U-235: 235.044 amu (g/mol)

Avagadro's number: $6.022\ 10^{23}$ atoms/mol.

Answer: 2736.3 kg

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Problem 32 (ME436s18f-5)

A small asteroid crashes to Earth. After chemical analysis, it is found to contain 1 g of technetium-99 (Tc-99) to every 3 g of ruthenium-99 (Ru-99), its stable daughter isotope. If the half-life of Tc-99 is 211 000 years, how old is the asteroid, approximately?

Relations:

Radioactivity reaction: $^{43}_{99}\text{Tc} \rightarrow {}^{44}_{99}\text{Ru} + {}^{-1}_{0}\beta$

Half life: $t_{1/2} = \dfrac{\ln(2)}{\lambda}$ where λ is the decay constant

Decay rate $\dfrac{dN}{dt} = -\lambda N \quad \Rightarrow \quad N(t) = N_0\ e^{-\lambda t}$

Assume that the asteroid was all Tc-99 at t = 0.

The mass numbers of Tc-99 and Ru-99 are about 100 amu (g/mol), approximately.

Answer: 422 000 years

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Problem 33 (ME436s19q11 / ME405s19q11 / ME436s20q11 / ME405f23q10)

A relic is found to give an activity count of 12 cpm (counts per minute) for each gram of carbon. If living trees give a count of 16 cpm, find the approximate age of the relic.

Activity $A = \lambda\, N = \frac{\ln(2)}{t_{1/2}}\, N$

The half life of C-14 is 5730 years.

Answer: t = 2378 years

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Problem 34 (ME436s19f-5 / ME405s19f-5 / ME405f23f-6)

By injecting a known volume of a specific activity serum into the blood of a subject, it is possible to determine the volume of the subject's blood. After allowing a short time for the serum to mix completely (10 to 30 minutes), a blood sample is taken. The specific activity of the sample is measured. The specific activity is equal to the total activity divided by the total blood volume of the subject. From this relation the blood volume can be determined.

Four cubic centimeters of serum with 100-μCi/cm^3 (microCurie per cm^3) specific activity is injected into a subject. Twenty minutes later, a blood sample is taken and its specific activity is found to be 0.038 μCi/cm^3.

(a) Find the blood volume of the subject assuming that the activity at the time of sampling is the same as when the serum was injected.

(b) What conditions are necessary for this approximation to be acceptable? Iodine-131 is frequently used for these measurements. Its half-life is 8 days. Is the approximation a good one for this case? Justify your answer.

$$A(t) = A_0\, e^{\left(-\frac{\ln(2)}{t_{1/2}}\, t\right)}$$

Answers: (a) $V = 10526.32\ cm^3$ (b) $\frac{A(t)}{A_0} = 0.9988$ => negligible

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Problem 35 (ME436s20h10)

The first element to be prepared that does not occur naturally on earth, technetium, was created by bombardment of molybdenum by deuterons (heavy hydrogen) in the Berkeley cyclotron by **Emilio** Segre and Carlo Perrier in 1937:

$$^{97}_{42}\text{Mo} + {}^{2}_{1}\text{H} = {}^{97}_{43}\text{Tc} + 2\,{}^{A}_{Z}\text{X}$$

Calculate the change in mass in the reaction and find out what the emitted particle, X, is.

Comment on the result.

In nuclear reactions:

- Mass numbers are conserved (not the mass)
- Atomic numbers are conserved
- Sum of mass and energy is conserved
- Momentum is conserved

Note: Emilio Gino Segre (an Italian-American physicist) who worked at Los Alamos National Labs during the Manhattan Project, received Nobel Prize in physics in 1959.

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Problem 36 (ME436s20f-5)

The nuclide Th-232 has a half life of 1.41 10^{10} years. Its radioactive decay ultimately leads to the stable isotope Pb-208.

A piece of rock contains 3.65 g of Th-232 and 0.75 g of Pb-208.

(a) Find the number of atoms of Th-232 in the rock, N_{Th}.

(b) Find the number of atoms of Pb-232 in the rock, N_{Pb}.

(c) Find the age of the rock as determined form the N_{Th} / N_{Pb} ratio?

Atomic mass of Th-232: 232.03805 amu (or g / mol)

Atomic mass of Pb-208: 207.97665 amu (or g / mol)

Avagadro's number: N_A = 6.022 10^{23} atoms / mol

Radioactive decay reaction: $N(t) = N_0\ e^{-\lambda t}$

Hint:

Assume that all of Pb-208 comes from the decay of Th-232. Then, the original number of atoms of Th-232 is equal to the sum of the current numbers of Th-232 and Pb-208 atoms. i.e.:

$$N_{Th}(0) = N_{Th}(t) + N_{Pb}(t)$$

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Problem 37 (ME436s21q11 / ME405s21q12 / ME436s22q12 / ME405s22q10)

Plutonium-238 is an α-emitter and a compact heat source. Coupled with a PbTe thermoelectric device, it was once used as a very reliable electrical energy source for cardiac pacemakers. What is the product of the radioactive decay of plutonium-238?

$$^{238}_{94}Pu \rightarrow {}^{4}_{2}He + ?$$

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Problem 38 (ME436s21h11 / ME405s21h12 / ME436s22h11 / ME405s22h10)

A very nasty person gives you a 10.0 g gold coin for your birthday. Why nasty? It turns out it is made entirely out of Au-198. Why is this bad?

a) What will this isotope of gold decay into? Show the entire decay equation.
b) How much time will it take for the gold coin to contain only 0.625 grams of gold-198?
c) How many grams of your decay product will there be after this amount of time?
d) If the coin were placed in an electric field, between a + charged and a - charged plate, to which plate will the decay particle be attracted? Explain

Answers: Hg-198; 10.8 days; 9.374357 g

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Problem 39 (ME436s21f-5 / ME405s21f-5)

A nuclear reactor in a submarine delivers 18 MW of shaft power at a cruising speed of 20 knots (1 knot = 1.852 km/h). If the power plant has a thermal efficiency of 25 %, how much (in kg) of the U-235 fuel is consumed on a 60 000 km trip around the world?

U-235 data: 200 MeV/fission, absorption cross section = 678 barns, fission cross section = 577 barns.

1 barn = 10^{-24} cm^2 (per neutron, per target nucleus

Atomic mass of U-235 is approximately 235 amu (or g/mol)

Avagadro's number = 6.023 10^{23} atoms/mol

Note that not all neutron captures give fission. The cross section data shows that out of 678 captures only 577 end with fission.

Conversion factor: 1.6 10^{-13} J/Mev

Answer: 6 kg

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Problem 40 (ME405f22q12)

Radioactive iodine, $^{131}_{53}I$, is used in medical diagnosis. Iodine-131 has a half-life of approximately eight days. A sample of this isotope has an activity of 20 microCuries when administered to a patient.

a. Find the activity of this material four days later.

b. How long will it take for the activity to drop to 1 percent of its initial value?

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Problem 41 (ME405f22h12)

Write a short essay (no more than one page) on the four fundamental forces of nature: gravitational force, weak nuclear force, electromagnetic force, and strong nuclear force.

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Problem 42 (ME405f22f-5)

By injecting a known volume of a specific activity serum into the blood of a subject, it is possible to determine the volume of the subject's blood. After allowing a short time for the serum to mix completely (10 to 30 minutes) a blood sample is taken. The specific activity of the sample is measured. The specific activity is equal to the total activity divided by the total blood volume of the subject. From this relation, the blood volume can be determined.

Four cubic centimeters of serum with 100-microCurie/cm^3 specific activity is injected into a subject. Twenty minutes later, a blood sample is taken and its specific activity is found to be 0.038 microCuries/cm^3.

(a) Find the blood volume of the subject assuming that the activity at the time of sampling is the same as when the serum was injected.

(b) What conditions are necessary for this approximation to be acceptable? Iodine-131 is frequently used for these measurements. Its half-life is 8 days. Is the approximation a good one for this case? Justify your answer.

Hint: $A(t) = A_0 \; e^{\left(-\frac{\ln(2)}{t_{1/2}} t\right)}$

Answer: Volume of the blood is: $V = 10526.32 \text{ cm}^3$

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Problem 43 (ME405s23q10)

Approximately 20 % of the human body by mass is carbon. Calculate the activity due to C-14 in 1 kg of carbon found in a living organism. Express the activity in Bq.

The abundance of C-14 in a living organism $= 1.3\ 10^{-12}\ \dfrac{\text{C-14 atom}}{\text{C atom}}$

Atomic mass of carbon is approximately 12 amu (g/mol)

Avagadro's number = $6.02\ 10^{23}$ Atoms/mol

Activity $A = \lambda N = \dfrac{\ln(2)}{t_{1/2}} N$

Half life of C-14 = 5730 years

Answer: A = 250 Bq

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Problem 44 (ME405s23h11)

A sealed capsule containing the radiopharmaceutical phosphorus-32 ($^{32}_{15}P$), an electron (β) emitter, is implanted into a patient's tumor. The average kinetic energy of the beta particles is 700 keV. The initial activity is 5.22 MBq. Determine the energy absorbed during a 10.0-day period in Joules.

Assume that the beta particles are completely absorbed within the tumor.

The half-life of P-32 is 14.26 days

Conversion factor: $1.6\ 10^{-16}$ J/keV

Answer: 0.4 J

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Problem 45 (ME405f23q11)

(a) Calculate the energy released in the neutron-induced fission reaction

$$^{235}U + n \rightarrow {}^{92}Kr + {}^{142}Ba + 2\ n$$

Mass of ^{92}Kr is 91.926269 amu and mass of ^{142}Ba is 141.916361 amu

(b) Confirm that the total number of nucleons and total charge are conserved in this reaction.

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Problem 46 (ME405f23h10)

After the sudden release of radioactivity from the Chernobyl nuclear reactor accident in 1986, the radioactivity of milk in Poland rose to 2000 Bq/L due to iodine-131 present in the grass eaten by dairy cattle. Radioactive iodine, with half-life 8.04 days, is particularly hazardous because the thyroid gland concentrates iodine. The Chernobyl accident caused a measurable increase in thyroid cancers among children in Belarus.

(a) For comparison, find the activity of milk due to potassium. Assume that one liter of milk contains 2.00 g of potassium, of which 0.0117 % is the isotope K-40 (39.1 g/mol) with half-life 1.28 10^9 years.

(b) After what time interval would the activity due to iodine fall below that due to potassium?

There are 3.156 10^7 seconds in a year

Answers: (a) A = 61.8 Bq (b) t = 40.3 days

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Problem 47

The electrical power output of a large nuclear reactor facility is 900 MW. It has a 35 percent efficiency in converting nuclear power to electrical power.

(a) What is the thermal nuclear power output in megawatts?

(b) Assuming the average fission produces 200 MeV, how many U-235 nuclei fission each second?

(c) What mass of U-235 is fissioned in 1 year of full-power operation?

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Problem 48

What are the properties of a good moderator in a nuclear reactor?

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Problem 49

In fast-breeder reactors, plutonium-239 is the primary fuel. A relatively stationary ^{239}Pu nucleus is fissioned by a 1.0 MeV neutron resulting in two fission fragments, Krypton-93 and Cerium-144.

(a) Identify all decay products of these two fragments until stable isotopes are obtained.

(b) Calculate the total energy released in MeV per ^{239}Pu nucleus and kWh per gram of ^{239}Pu.

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Problem 50 (ME405f24q7)

The atomic mass of ^{14}C is 14.003242 au. Show that the β^- decay of ^{14}C is energetically possible, and calculate the energy released in the decay.

Production: $^{14}_{7}\text{N} + \text{n} \rightarrow {}^{14}_{6}\text{C} + \text{p}$

Decay: $^{14}_{6}\text{C} \rightarrow {}^{14}_{7}\text{N} + \beta + \text{neutrino}$

Mass of an electron = 0.0005491 au

Mass of ^{14}N = 14.003074 au

Answer: E = 156 keV

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Problem 51 (ME405f24q8)

A monoenergetic beam of neutrons having an intensity of 4 10^{10} neutrons/cm^2.s impinges on a target 1 cm^2 in area and 0.1 cm thick. There are 0.048 10^{24} atoms per cm^3 in the target and the total cross section at the energy of the beam is 4.5 barns.

(a) What is the macroscopic total cross section?

(b) How many neutron interactions per second occur in the target?

Answers: (a) $\bar{\Sigma}$ = 0.0216 cm^2 (b) Reaction rate = 10^8 reactions /s

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Problem 52 (ME405f24h7)

Radioactive Tracers: Radioactive isotopes are often introduced into the body through the bloodstream. Their spread through the body can then be monitored by detecting the appearance of radiation in different organs. ^{131}I, a β (electron) emitter, with a half-life of 8 days, is one such tracer. Suppose a scientist introduces a sample with an activity of 375 Bq and watches it spread to the organs.

(a) Assuming that the sample all went to the thyroid gland, what will be the decay rate in that gland 24 days (about 3.5 weeks) later?

(b) If the decay rate in the thyroid 24 days later is actually measured to be 17.0 Bq, what percentage of the tracer went to that gland?

(c) What isotope remains after the ^{131}I decays?

Answers: (a) 46.9 Bq; (b) 36.2 %; (c) $^{131}_{54}Xe$

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Problem 53 (ME405f24h8)

The β-emitter Al-28 (half-life 2.30 minutes) can be produced by the radiative capture of neutrons by Al-27. The 0.0253-eV cross section for this reaction is 0.23 barns. Suppose that a small, 0.01-g aluminum target is placed in a beam of 0.0253-eV neutrons having an intensity of 3 10^8 neutrons/cm^2.s, which strikes the entire target. Calculate

(a) The rate at which Al-28 is produced;

(b) The maximum activity which can be produced in this experiment.

Answers: (a) 1.54 10^4 reactions /s; (b) Same as (a).

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Problem 54 (ME405f24f-2)

The H-3 isotope of hydrogen, which is called tritium (because it contains three nucleons), has a half-life of 12.33 years, emitting an electron and a neutrino. It can be used to measure the age of objects up to about 100 years. It is produced in the upper layers of the atmosphere by cosmic rays and brought to Earth by rain.

(a) Complete the decay formula: $^{3}_{1}H \rightarrow \ldots$.

(b) As an application, determine approximately the age of a bottle of wine whose radiation activity is about 1/10 of that present in new wine.

Because the tritium in water is being replenished, we assume that the amount is constant until the wine is made, and then it decays.

$$t_{1/2} = \frac{\ln 2}{\lambda} \quad ; \quad A = \lambda N$$

Answer: 41 years

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