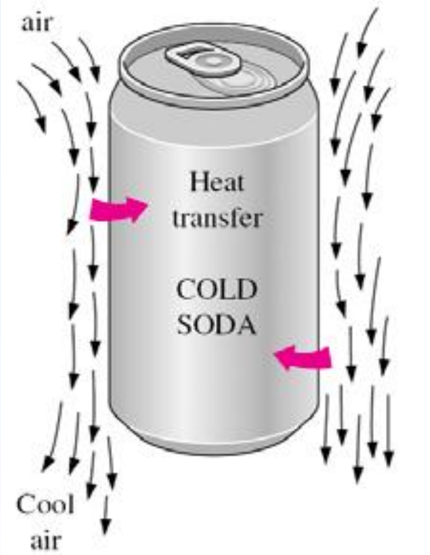
10 - Natural (Free) Convection



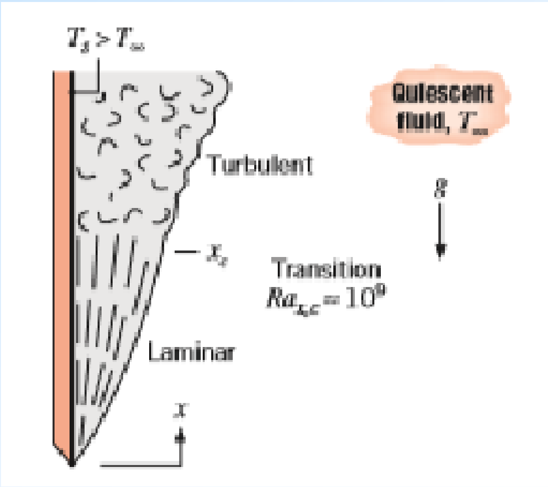
Temperature gradient —causes⟹ Density variation

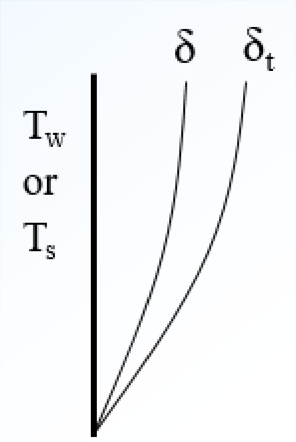
Density variation —causes⟹ Bouyancy force or body force

Bouyancy force or body force —causes⟹ Fluid motion

This is called free or natural convection. The solution is quite complicated.

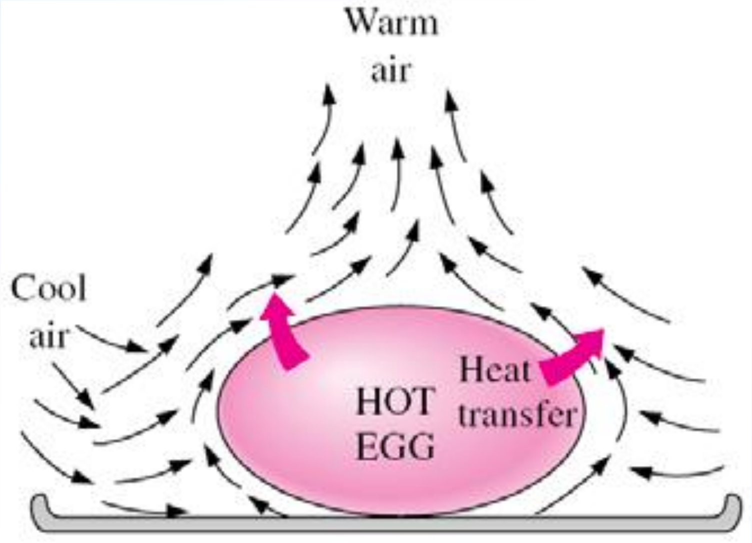
When $T_w - T_{fluid}$ is not so large, we may use equations for constant property, incompressible fluid with bouyancy forces when the density change is included in the body force term in the momentum equation.





Buoyancy forces are responsible for the fluid motion in natural convection.

Viscous forces oppose the fluid motion.



Buoyancy forces are expressed in terms of fluid temperature differences through the **volume expansion coefficient**

$$\beta = \frac{1}{V}\left.\frac{\partial V}{\partial T}\right|_P = \frac{-1}{\rho}\left.\frac{\partial \rho}{\partial T}\right|_P$$

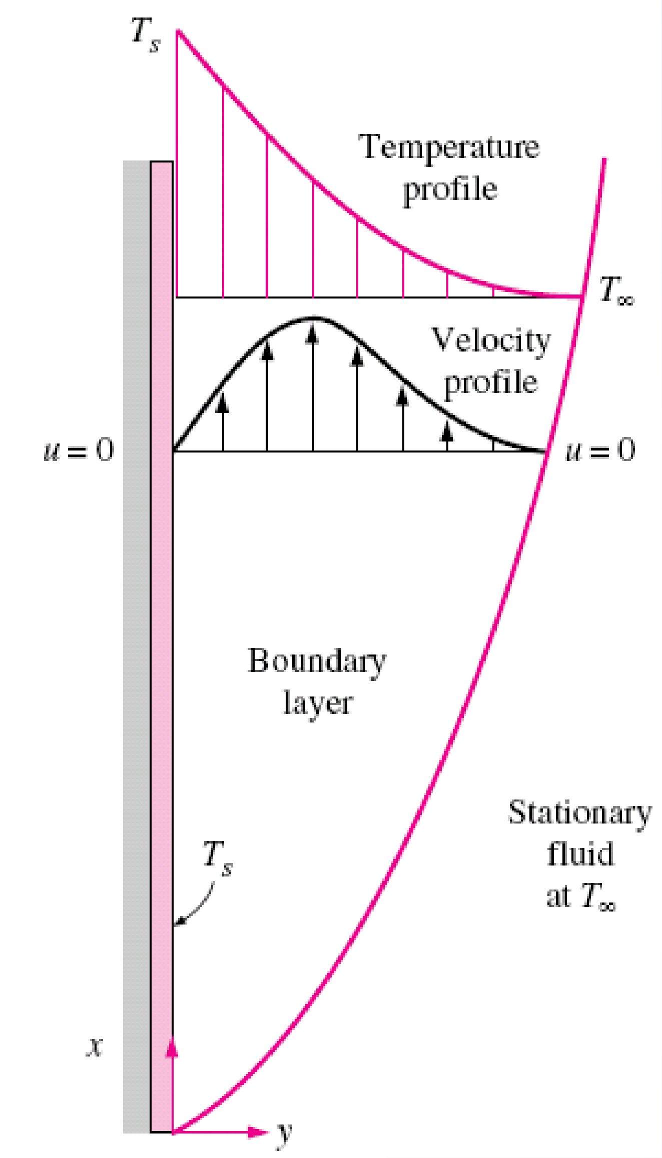
Volume Expansion Coefficient: $$\beta = \frac{1}{V}\left.\frac{\partial V}{\partial T}\right|_P = \frac{-1}{\rho}\left.\frac{\partial \rho}{\partial T}\right|_P$$

The volume expansion coefficient can be expressed approximately by replacing differential quantities by differences as

$$\beta \approx \frac{-1}{\rho}\frac{\Delta\rho}{\Delta T} = \frac{-1}{\rho}\left(\frac{\rho_\infty - \rho}{T_\infty - T}\right) \qquad \text{at constant P}$$

$$\rho_\infty - \rho = \rho\,\beta\,(T_\infty - T) \qquad \text{at constant P}$$

For an Ideal gas, (P V = m R T) $$\beta_{ideal\ gas} = \frac{1}{T} \quad \text{in } K^{-1}$$



10.1 Boundary Layer Equations

Consider a vertical hot flat plate immersed in a quiescent fluid body.

Assumptions:

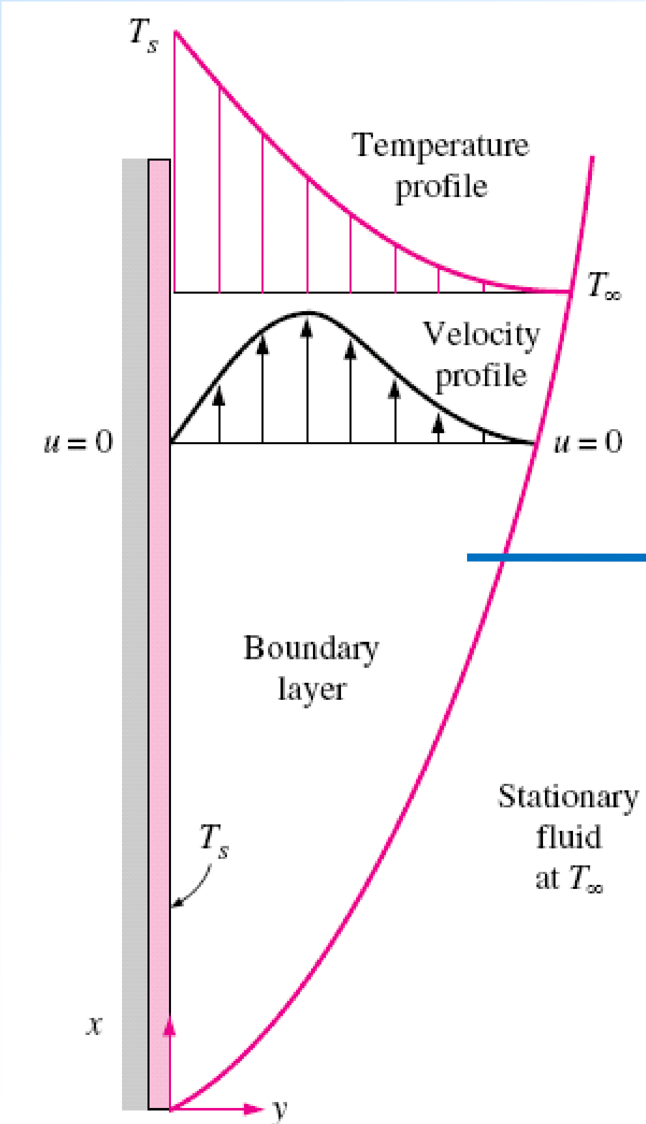
- steady,
- laminar,
- two-dimensional,
- Newtonian fluid, and
- constant properties, except the density difference (ρ - ρ_∞) (Boussinesq approximation).


M. BOUSSINESQ.

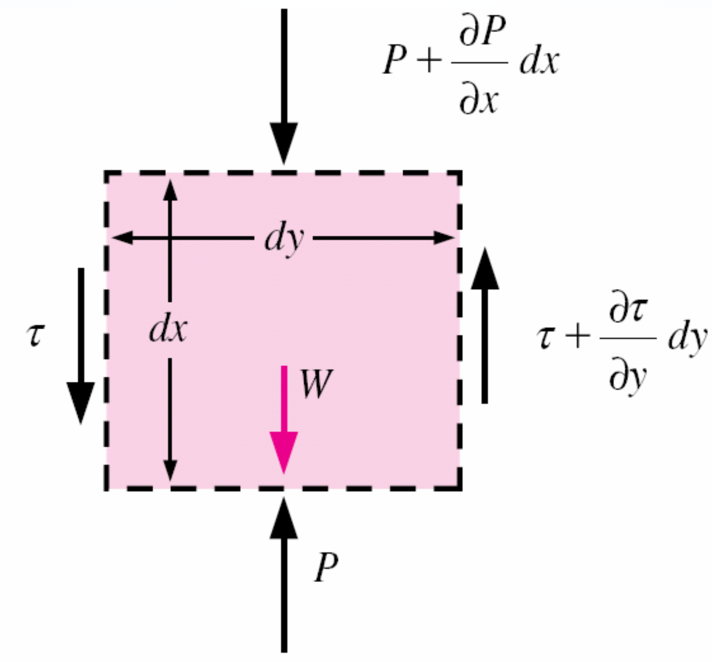
Joseph Valentin Boussinesq

French mathematician and physicist

1842 – 1929



Consider a differential volume element



Apply Newton's second law of motion to the volume element: F = mass x Acc.

Newton's second law of motion in the x direction: $F_x = \delta m \, a_x$

Net viscous force — Net pressure force — Gravitational force

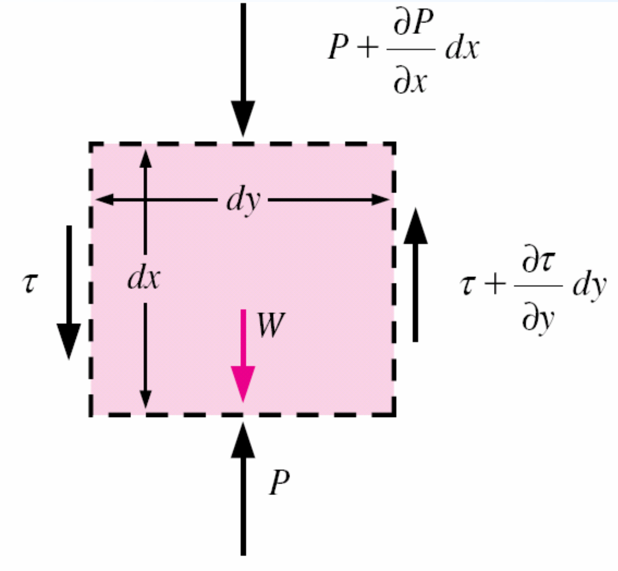
$$F_x = \left(\frac{\partial \tau}{\partial y} dy\right)(dx.1) - \left(\frac{\partial P}{\partial x} dx\right)(dy.1) - \rho\, g\,(dx.dy.1)$$

$$= \left(\mu \frac{\partial^2 u}{\partial y^2} - \frac{\partial P}{\partial x} - \rho\, g\right)(dx.dy.1)$$

$$\delta m = \rho\,(dx.\,dy.1)$$

$$a_x = \frac{Du}{Dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt}$$

$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$$



Conservation of momentum in the x direction in the boundary layer

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\mu\frac{\partial^2 u}{\partial y^2}\boxed{-\frac{\partial P}{\partial x}}-\rho g$$

?

x momentum equation in the quiescent fluid outside the boundary layer (u = 0)

$$\frac{\partial P}{\partial x}=-\rho_\infty g$$

In the boundary layer, v << u and $\frac{\partial v}{\partial x}\cong\frac{\partial v}{\partial y}\cong 0$

Force balance in the y direction: $\frac{\partial P}{\partial y}=0 \Rightarrow \frac{\partial P}{\partial x}=\frac{\partial P_\infty}{\partial x}=-\rho_\infty g$

Substitute: $$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\mu\frac{\partial^2 u}{\partial y^2}+\left(\rho_\infty-\rho\right)g$$

x momentum equation :
$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\mu\frac{\partial^2 u}{\partial y^2}+\left(\rho_\infty-\rho\right)g$$

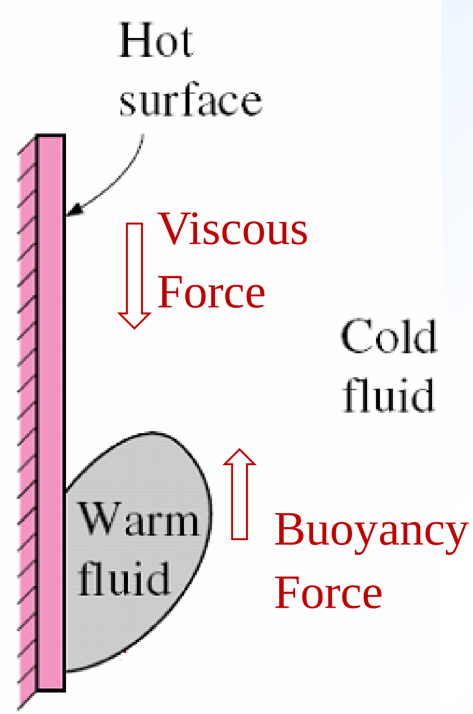
Substitute: $\rho_\infty-\rho=\rho\,\beta\left(T_\infty-T\right)$

$$\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\nu\frac{\partial^2 u}{\partial y^2}+g\,\beta\,(T-T_\infty)$$

x momentum equation in non-dimensional form, when: $T^+=\dfrac{T-T_\infty}{T_s-T_\infty}$

$$\left(u^+\frac{\partial u^+}{\partial x^+}+v^+\frac{\partial u^+}{\partial y^+}\right)=\frac{1}{Re_L}\frac{\partial^2 u^+}{\partial (y^+)^2}+\frac{g\,\beta\,(T_s-T_\infty)\,L_c^3}{\nu^2}\frac{T^+}{Re_L^2}$$

The new non-dimensional parameter represents the natural convection effects, and is called the Grashof number, Gr.



$$Gr_L = \frac{g\,\beta\,(T_s - T_\infty)\,L_c^3}{\nu^2}$$

$$Gr_L = \frac{\textit{Buoyancy Force}}{\textit{Viscous Force}}$$

The flow regime in natural convection is governed by the *Grashof number*

$Gr_L > 10^9$ Flow is turbulent

Franz Grashof

German Engineer

1826 – 1893

$\frac{Gr_L}{Re^2}$ Bouyancy force / Inertia force

If $\frac{Gr_L}{Re^2} << 1$ No free convection, no buoyancy

If $\frac{Gr_L}{Re^2} >> 1$ Buoyancy dominates

Natural Convection over Surfaces

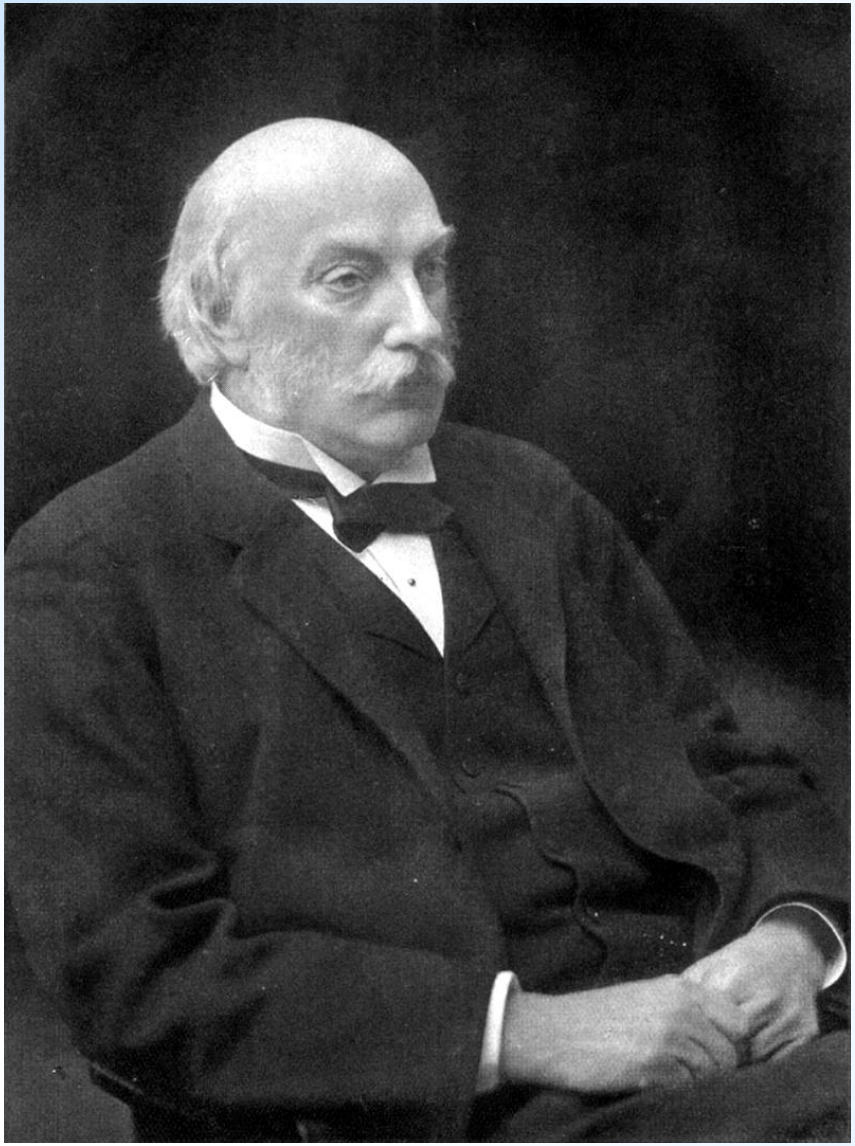
In free (or natural) convection, buoyancy forces are the only driving force that generates the flow field because there is no external field. Therefore, Re cannot be an independent parameter. Instead, Gr is.

$$Nu = f(Gr, Pr)$$

Define **Rayleigh number**: $Ra = Gr\,Pr = \dfrac{g\,\beta\,(T_s - T_\infty)\,L_c^3}{\nu\,\alpha}$

Sometimes, Ra is used in the relations: $Nu = f(Ra)$

See Table 12-3 for the correlations over various surfaces.



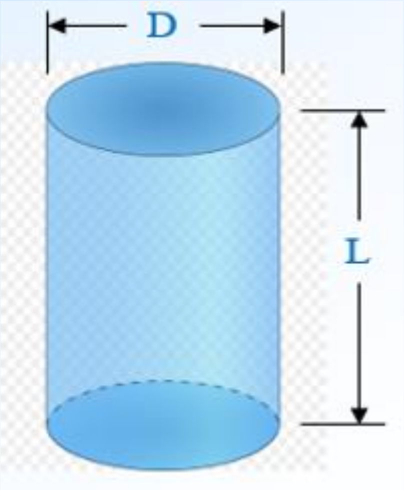
John William Strutt

3^{rd} Baron Rayleigh

British Mathematician and Physicist

1842 – 1919

Example 1



Cylinder in water

$D = 2.5$ cm

$L = 0.5$ cm

$T_\infty = 20$ °C

$T_w = 55$ °C

h = ?

Q = ?

Evaluate properties of water

at the mean temperature

$$\bar{T} = \frac{T_w + T_\infty}{2} = \frac{55 + 20}{2} = 37.5\ ^\circ C$$

Properties of water

at 37.5 °C

$c_p = 4.174$ kJ/kg.K

$\rho = 993$ kg/m^3

$\mu = 6.82\ 10^{-4}$ kg/m.s

$k = 0.63$ W/m.K

$Pr = 4.53$

$$\frac{g\,\beta\,\rho^2\,c_p}{\mu\,k} = 3.3\ 10^{10}\ m^{-3}.K^{-1}$$

$$Gr_L \ Pr = Ra_L = \frac{g\,\beta\,\rho^2\,c_p}{\mu\,k} L^3 \left(T_w - T_\infty\right)$$

$$Ra_L = (3.3\ 10^{10})\ (0.5)^3 \left(55 - 20\right) = 1.03\ 10^{11} \quad > 10^9 \quad \Rightarrow \text{ Turbulent}$$

$$\frac{g\,\beta\,\rho^2\,c_p}{\mu\,k} = 3.3\ 10^{10}\ m^{-3}.K^{-1}$$

$$g = 9.8\ m/s$$

$$\beta = 3.5\ 10^{-4}\ K^{-1} \text{ or } {}^\circ C^{-1}$$

$$Nu_{mean} = \frac{h_{mean}\ L}{k} = C\left(Gr_L\ Pr\right)^m$$

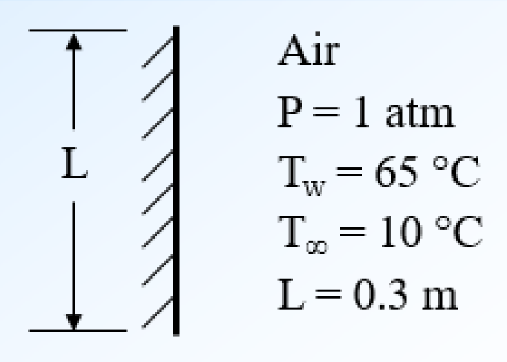
Use Table => $C = 0.1$ and $m = 1/3$

$$Nu_m = (0.1)\left(1.03\ 10^{11}\right)^{1/3} = 46.875$$

$$h_m = \frac{0.63}{0.5}(46.875) = 59.06\ W/m^2.K$$

$$Q = h_m\ A\ (T_w - T_\infty) = (59.06)\ \pi\ (0.025)\ (0.5)\ (55 - 20) = 58.51\ W$$

Example 2



h = ?

Q = ?

Evaluate properties of air

at the mean temperature

$$\bar{T} = \frac{T_w + T_\infty}{2} = \frac{65 + 10}{2}$$

$$= 37.5\ ^\circ C = 310.5\ K$$

Properties of air

at 310.5 K

c_p = 1.006 kJ/kg.K

k = 0.027 W/m.K

ρ = 1.18 kg/m^3

Pr = 0.707

μ = 2.0 10^{-5} kg/m.s

α = 0.23 10^{-4} m^2/s

ν = 15.7 10^{-6} m2/s

$$Gr_L = \frac{g\,\beta\,(T_w - T_\infty)\,L^3}{\upsilon^2} = \frac{(9.8)\,\frac{1}{283}\,(65 - 10)\,(0.3^3}{(15.7\,10^{-6})^2} = 2.086\,10^8$$

$Gr_L\ Pr = (2.086\,10^8)\,(0.707) = 1.475\,10^8 < 10^9$ Flow is Laminar

Use Table for $Ra \approx 1.5\,10^8$ } $C = 0.59$, $m = 1/4$ }

$$\bar{N}u = \frac{\bar{h}\,L}{k} = (0.59)\,(Ra)^{1/4}$$

$$= (0.59)\,(1.5\,10^8)^{1/4} = 65.02$$

$$\bar{h} = \frac{k}{L}\,\bar{N}u = \frac{0.027}{0.3}\,(65.02) = 5.85\ W/m^2.K$$

Alternatively, we may use Table 7.2 to find h.

11 – Boiling and Condensation