9 - Forced Convection - Internal Flow

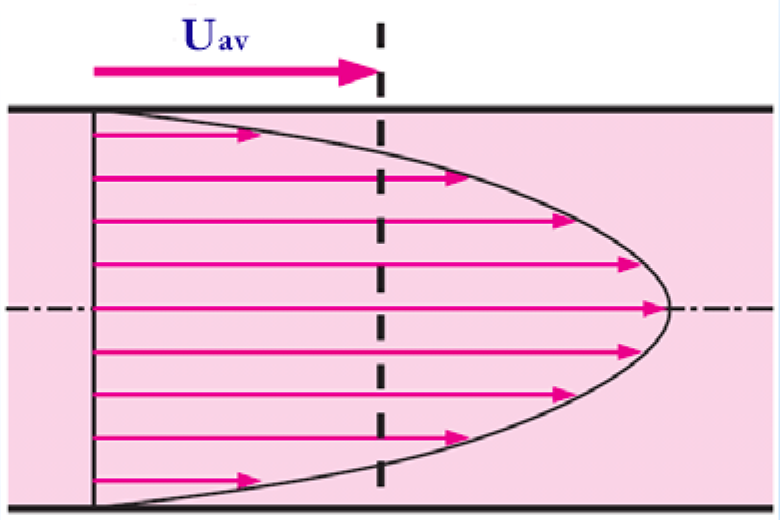
Pipe: Circular cross section

Tube: Small-diameter pipe

Duct: Non-circular cross section

The fluid velocity changes from *zero at the surface* (no-slip) to a maximum at the pipe center.

It is convenient to work with an *average velocity, which remains* constant in incompressible flow when the cross-sectional area is constant.



Average velocity: $\dot{m} = \rho \, U_{av} \, A_c$

For incompressible flow in a circular pipe of radius R:

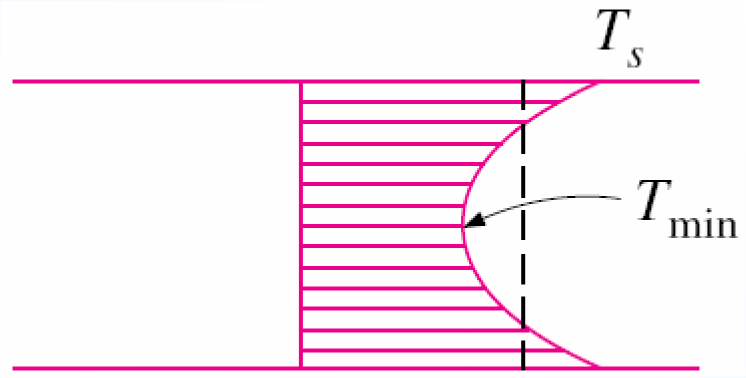
$$\dot{m} = \frac{\int_{A_c} \rho \, u(r) \, dA_c}{\rho \, A_c} = \frac{\int_0^R u(r) \, 2\pi \, r \, dr}{\pi R^2} = \frac{2}{R^2} \int_0^R u(r) \, r \, dr$$

Average temperature: Define the value of the mean temperature, T_m

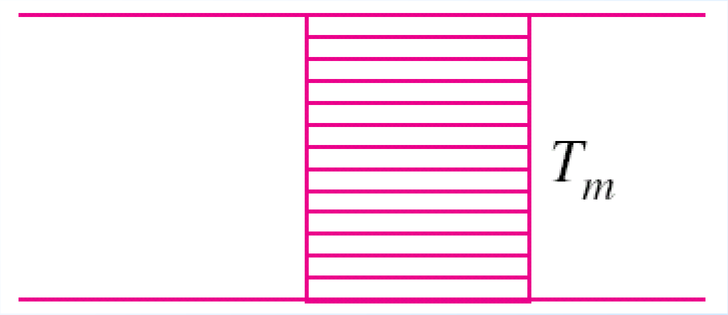
$$T_m = \frac{\int_{\dot{m}} c_p \, T(r) \, d\dot{m}}{\rho \, A_c} = \frac{\int_0^R c_p \, T(r) \, \rho \, u(r) \, 2\pi \, r \, dr}{\rho \, U_{av} \left(\pi R^2\right) c_p} = \frac{2}{U_{av} \, R^2} \int_0^R T(r) \, u(r) \, r \, dr$$

$$T_m = \frac{\int_{\dot m} c_p\, T(r)\, d\dot m}{\rho\, A_c} = \frac{\int_0^R c_p\, T(r)\, \rho\, u(r)\, 2\pi\, r\, dr}{\rho\, U_{av}\, \left(\pi R^2\right) c_p} = \frac{2}{U_{av}\, R^2} \int_0^R T(r)\, u(r)\, r\, dr$$

The **mean temperature**, T_m, (sometimes called **bulk temperature**, T_b, or **mixing-cup temperature**) of the fluid changes during heating or cooling



Actual



Idealized

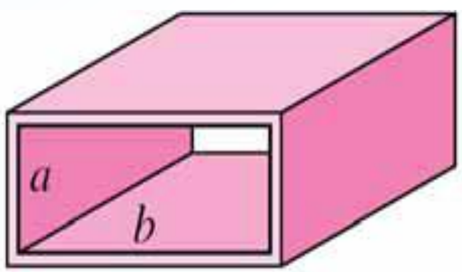
Flow in Tubes

For flow in a circular tube, the Reynolds number is defined as

$$Re = \frac{\rho\, U_{av}\, D}{\mu} = \frac{U_{av}\, D}{\upsilon}$$

For flow through noncircular tubes, D is replaced by the **hydraulic diameter**, D_h

Rectangular duct:



$$D_h = \frac{4ab}{2(a+b)} = \frac{2ab}{a+b}$$

$$D_h = \frac{4\, A_c}{P}$$

A_c: Cros sectional area

P: Wetted perimeter

Laminar flow: Re < 2300

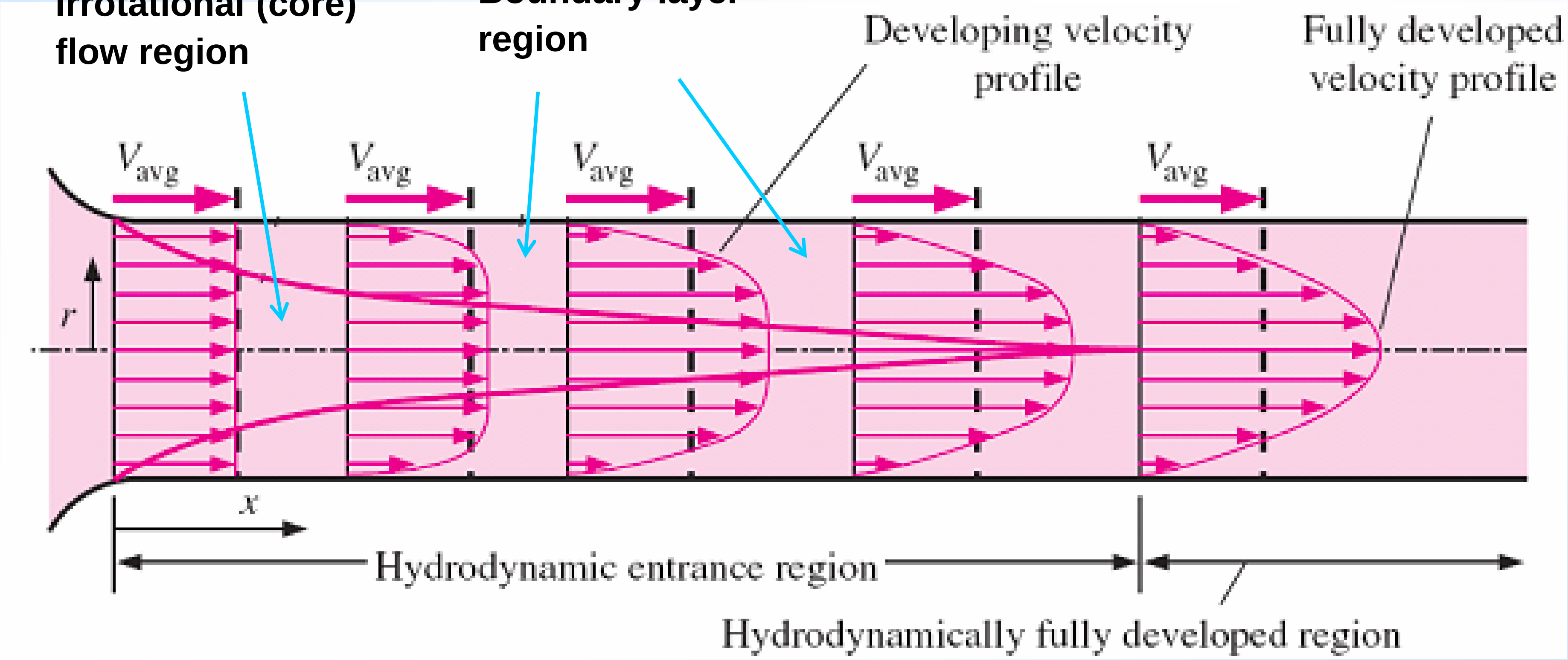
Transitional flow: 2300 ≤ Re ≤ 10,000

Fully turbulent flow : Re > 10,000

Hydrodynamic entrance region

Irrotational (core) flow region

Boundary layer region



All region is boundary layer

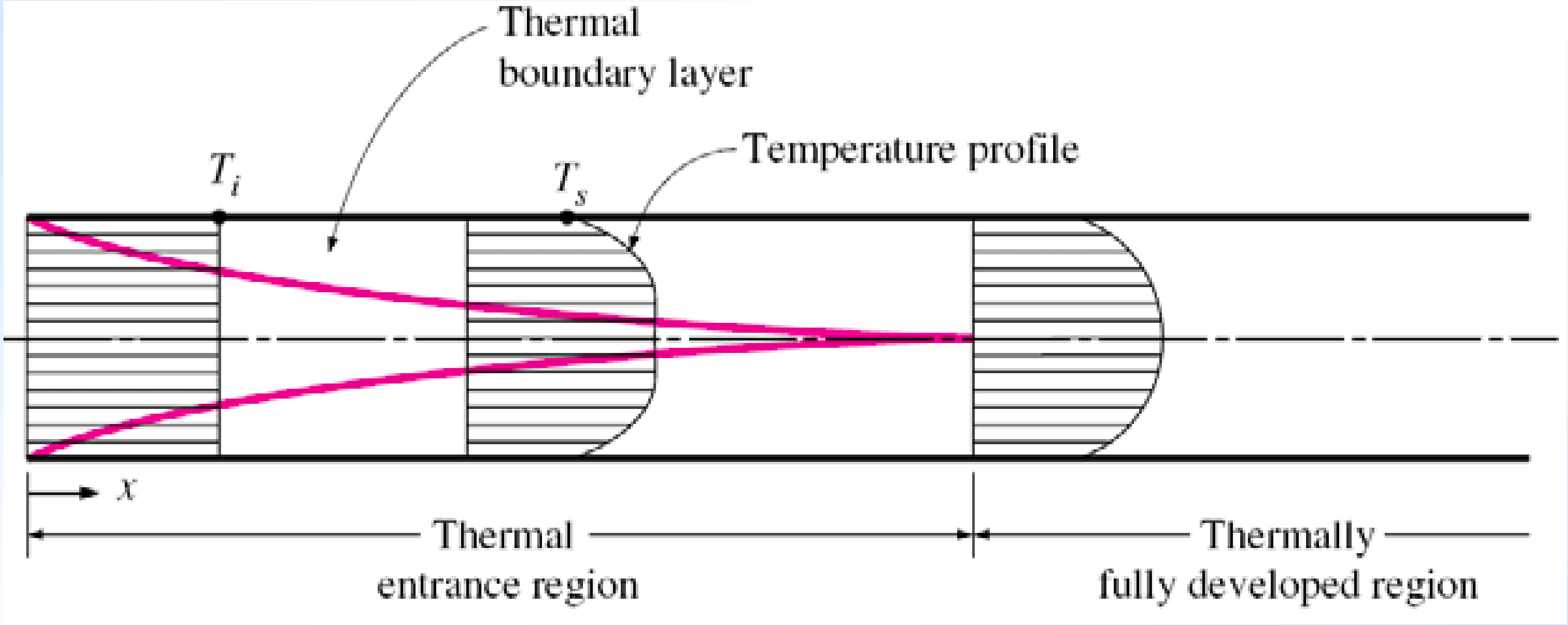
Hydrodynamic entrance region is the region from the pipe inlet to the point at which the boundary layer merges at the centerline.

Hydrodynamically fully developed region is the region beyond the entrance region in which the velocity profile is fully developed and remains unchanged.

The velocity profile in the fully developed region is

- ***parabolic in laminar flow***, *and*
- somewhat *flatter or fuller in turbulent flow.*

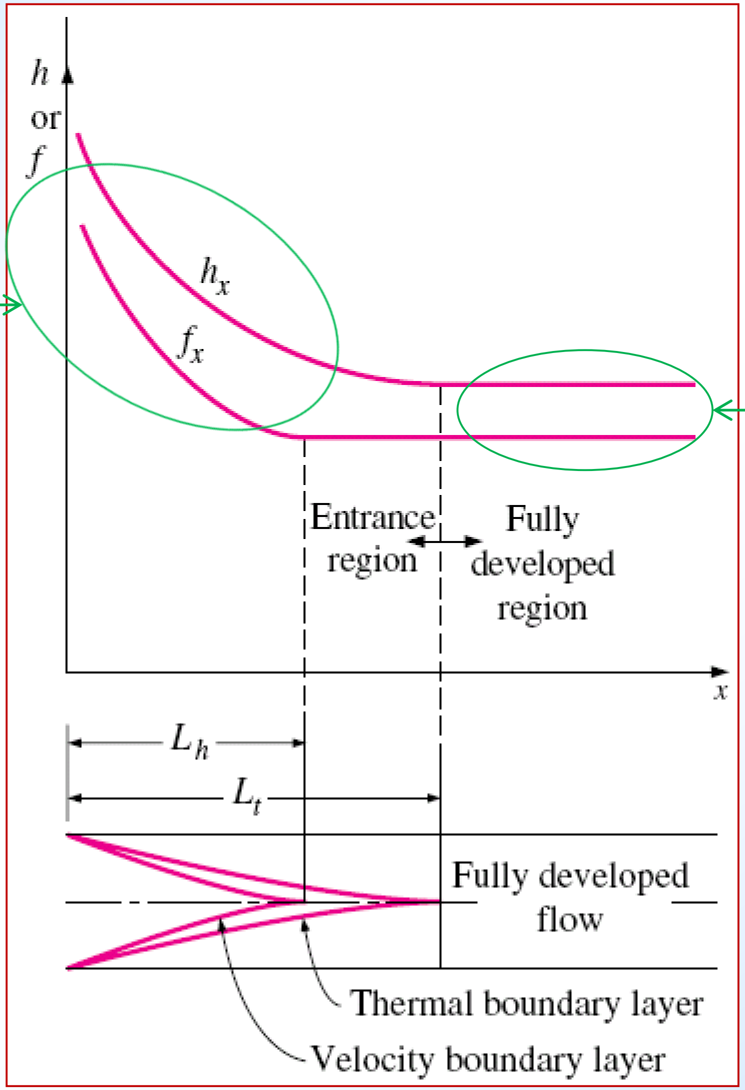
Thermal entrance region



Thermally fully developed region is the region beyond the thermal entrance region in which the dimensionless temperature profile expressed as $(T_s\text{-}T)/(T_s\text{-}T_m)$ remains unchanged.

Heat transfer coefficient and friction factor

Developing region



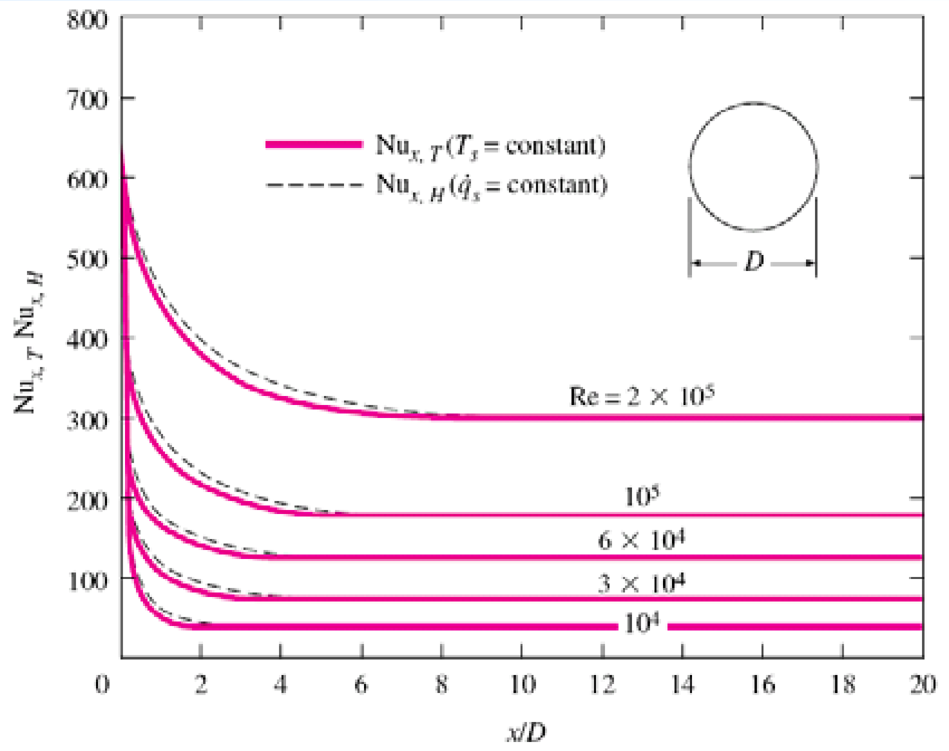
Fully developed region

Fully developed flow either laminar or turbulent depending on Re

Entry lengths

Laminar Flow	Hydrodynamic		$L_{h,laminar} = 0.065\ Re\ D$
	Thermal	Constant T_w	$L_{t,laminar} = 0.037\ Re_D\ Pr\ D$
		Constant q”	$L_{t,laminar} = 0.053\ Re_D\ Pr\ D$
Turbulent Flow	Hydrodynamic		$L_{h,turbulent} = 4.4\ Re^{1/6}\ D$
	Thermal		$L_{t,turbulent} = 10\ D$

Turbulent flow Nusselt numbers



The Nusselt numbers are much higher in the entrance region.

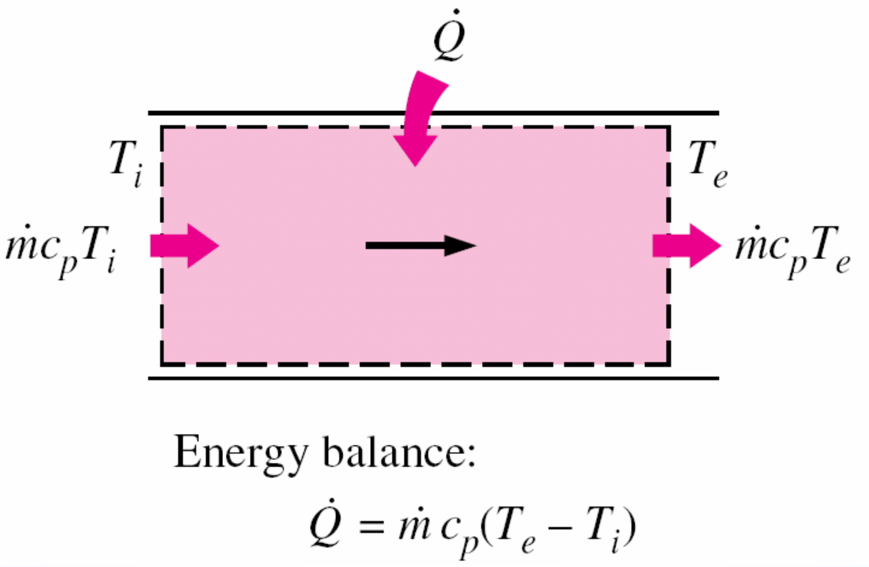
The Nusselt number reaches a constant value at a distance of less than 10 diameters.

The Nusselt numbers for the uniform surface temperature and uniform surface heat flux conditions are identical in the fully developed regions, and nearly identical in the entrance regions.

Nusselt number is insensitive to the type of thermal boundary condition.

General Thermal Analysis

For steady flow in tubes: $\dot{Q} = \dot{m}\ c_p\ (T_e - T_i)$



The thermal conditions at the surface can usually be approximated as:

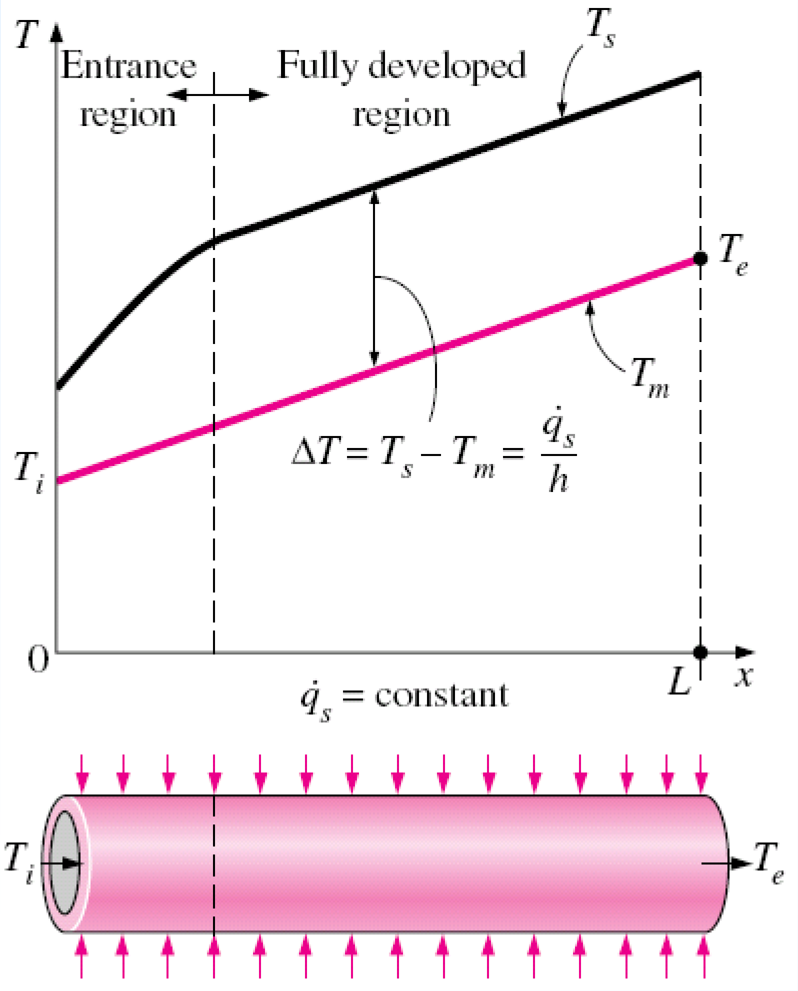
– *constant surface* ***temperature, or***

– *constant surface* ***heat flux.***

The mean fluid temperature at a cross section, T_m, *must* change during heating or cooling.

Either T_s = *constant or* q_s = *constant at the surface of a* tube, but not both.

Constant Surface Heat Flux, q_s



$$\dot{Q} = \dot{m}\ c_p\ (T_e - T_i)$$

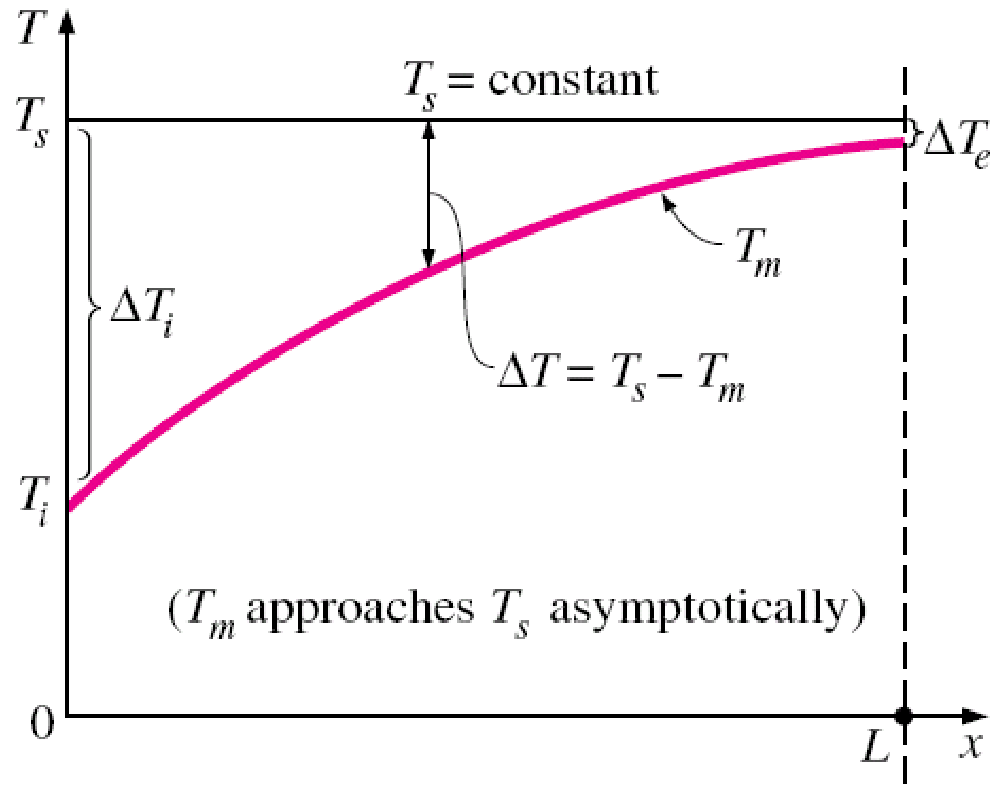
$$\dot{m} = A_c\ \rho\ U_{av} = \pi\ R^2\ \rho\ U_{av}$$

$$\dot{Q} = q_s\ A_s = q_s\ 2\ \pi\ R\ L$$

$$\dot{Q} = \int_0^L \dot{m}\ c_p\ \frac{\partial T}{\partial x}\ dx$$

$$\frac{\partial T}{\partial x} = \frac{dT_s}{dx} = \frac{dT_m}{dx} = \text{constant}$$

Constant Surface Temperature, T_s



$$\dot{Q} = \dot{m}\ c_p\ (T_e - T_i)$$

$$\dot{m} = A_c\ \rho\ U_{av} = \pi\ R^2\ \rho\ U_{av}$$

$$\dot{Q} = \int_0^L \dot{m}\ c_p\ \frac{\partial T}{\partial x}\ dx$$

$$\dot{Q} = h\ A_s\ \text{LMTD}$$

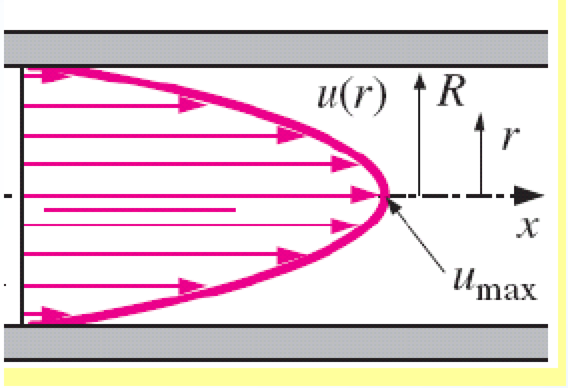
$$\text{LMTD} = \frac{\Delta T_e - \Delta T_i}{\ln\left(\frac{\Delta T_e}{\Delta T_i}\right)}$$

LMTD: Log Mean Temperature Difference

Laminar Flow in Tubes

Assumptions:
- Steady laminar flow;
- Incompressible fluid;
- Constant properties;
- Fully developed region; and
- Straight circular tube.

- The velocity profile *u(r)* remains unchanged in the flow direction.
- No motion in the radial direction.
- No acceleration.



$$u(r) = U_{max}\left(1 - \frac{r^2}{R^2}\right)$$

$$U_{max} = 2\,U_{av}$$

Pressure Drop

$$\Delta P = P_1 - P_2 = \frac{8\,\mu\,L\,U_{av}}{R^2} = \frac{32\,\mu\,L\,U_{av}}{D^2}$$

$$\Delta P = P_1 - P_2 = f\,\frac{L}{D}\,\frac{\rho\,U_{av}^2}{2}$$

Friction factor

Dynamic pressure

$$f = \frac{64\,\mu}{\rho\,D\,U_{av}} = \frac{64}{Re}$$

Temperature Profile and Nusselt Number

Laminar, Fully-developed Flow

Constant surface heat flux, q_s:

$$T(r) = T - \frac{q_s R}{k}\left(\frac{3}{4} - \frac{r^2}{R^2} + \frac{r^4}{4R^4}\right)$$

$$Nu = \frac{hD}{k} = 4.36$$

Laminar, Fully-developed Flow

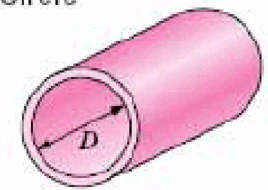
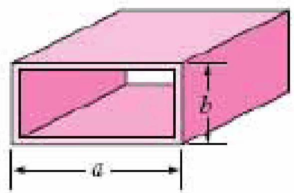
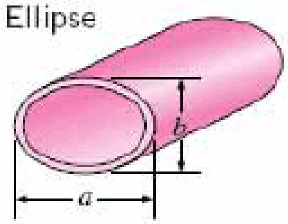
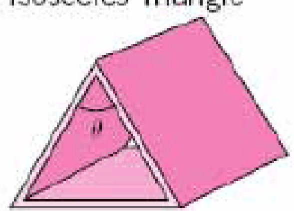
Constant surface temperature, T_s:

$$Nu = \frac{hD}{k} = 3.66$$

Laminar,

Fully-developed Flow

in Non-circular Tubes

Tube Geometry	a/b or $\theta°$	Nusselt Number T_s = Const.	Nusselt Number $\dot{q}_s$ = Const.	Friction Factor f
Circle 	—	3.66	4.36	64.00/Re
Rectangle 	a/b			
	1	2.98	3.61	56.92/Re
	2	3.39	4.12	62.20/Re
	3	3.96	4.79	68.36/Re
	4	4.44	5.33	72.92/Re
	6	5.14	6.05	78.80/Re
	8	5.60	6.49	82.32/Re
	∞	7.54	8.24	96.00/Re
Ellipse 	a/b			
	1	3.66	4.36	64.00/Re
	2	3.74	4.56	67.28/Re
	4	3.79	4.88	72.96/Re
	8	3.72	5.09	76.60/Re
	16	3.65	5.18	78.16/Re
Isosceles Triangle 	θ			
	10°	1.61	2.45	50.80/Re
	30°	2.26	2.91	52.28/Re
	60°	2.47	3.11	53.32/Re
	90°	2.34	2.98	52.60/Re
	120°	2.00	2.68	50.96/Re

Developing Laminar Flow in the Entrance Region

For a circular tube of length *L subjected to constant* surface temperature, the average Nusselt number for the *thermal entrance region (hydrodynamically* developed flow), Sieder and Tate equation is:

$$\mathrm{Nu} = 3.66 + \frac{0.0668\ (D/L)\ \mathrm{Re\ Pr}}{1+0.04\left[(D/L)\ \mathrm{Re\ Pr}\right]^{2/3}} = 3.66 + \frac{0.0668\ \mathrm{Gz}}{1+0.04\ \mathrm{Gz}^{2/3}}$$

For Flow between Isothermal Plates:

$$\mathrm{Nu} = 7.54 + \frac{0.03\ (D_h/L)\ \mathrm{Re\ Pr}}{1+0.016\left[(D_h/L)\ \mathrm{Re\ Pr}\right]^{2/3}}$$

Turbulent Flow in Tubes

Friction Factor for Smooth Tubes: (Petukhov Equation)

$$f = (0.79 \ln(Re) - 1.64)^{-2} \qquad 3000 < Re < 5\ 10^6$$

For Fully-developed Flow: (Dittus- Boelter Equation)

$$Nu = 0.023\, Re^{0.8}\, Pr^{n}$$

$Re > 10\,000$ $n = 0.4$ heating

$0.7 \le Pr \le 160$ $n = 0.3$ cooling

These are approximately valid for developing turbulent flow in the entrance region

Modified correlations exists for other conditions such as liquid metals, surface roughness, property variations due to large temperature changes, etc.

The Dittus-Boelter correlation may be used for small to moderate temperature differences, $T_{wall} - T_{avg}$, with all properties evaluated at an averaged temperature, T_{avg}.

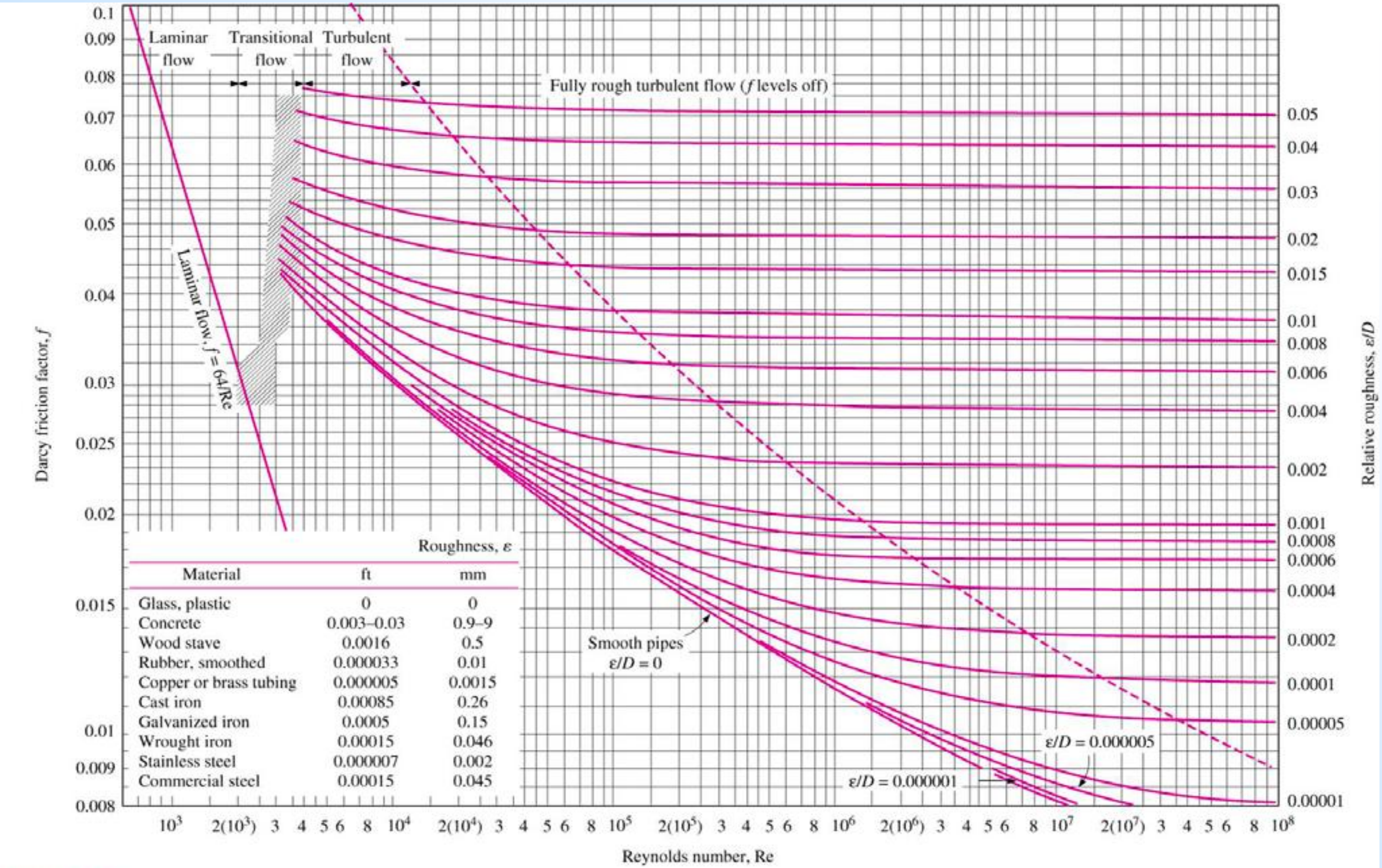
When the difference between the surface and the fluid temperatures is large, it may be necessary to account for the variation of viscosity with temperature. Therefore a modified form of Dittus-Boelter equation was proposed by **Sieder and Tate** (1936).

$$Nu = 0.027\, Re^{0.8}\, Pr^{n} \left(\frac{\mu}{\mu_s}\right)^{0.14}$$

$Re > 10\,000$ $\qquad$ $n = 0.4$ heating

$0.7 \le Pr \le 16700$ $\qquad$ $n = 0.3$ cooling

$\frac{L}{D} > 10$



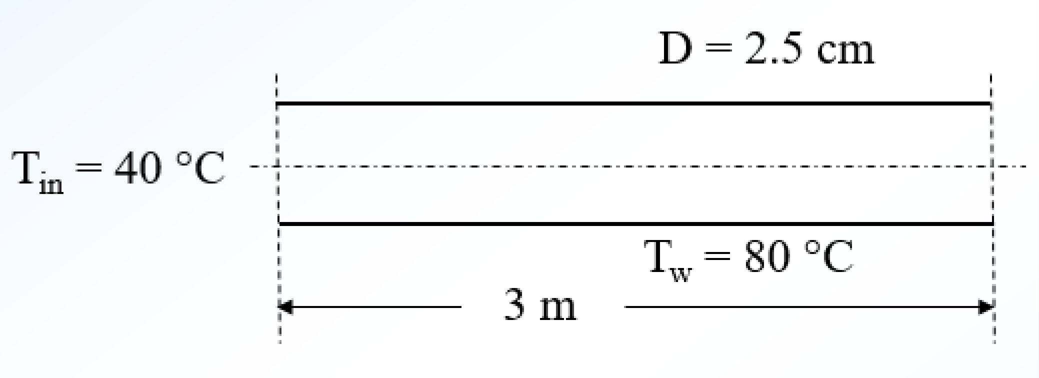
Material	Roughness, ε ft	Roughness, ε mm
Glass, plastic	0	0
Concrete	0.003–0.03	0.9–9
Wood stave	0.0016	0.5
Rubber, smoothed	0.000033	0.01
Copper or brass tubing	0.000005	0.0015
Cast iron	0.00085	0.26
Galvanized iron	0.0005	0.15
Wrought iron	0.00015	0.046
Stainless steel	0.000007	0.002
Commercial steel	0.00015	0.045

FIGURE A–20

The Moody chart for the friction factor for fully developed flow in circular pipes for use in the head loss relation $\Delta P_L = f\frac{L}{D}\frac{\rho V^3}{2}$. Friction factors in the turbulent flow are evaluated from the Colebrook equation $\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right)$.

Example 1

Water at a temperature of 40 °C flows through a 2.5 cm OD smooth-walled tube at a rate of 0.5 kg/s. The tube wall is kept at 80 °C. Find h and Q if the length of the tube is 3 m. Assume that the velocity and temperature profiles are fully developed.



m = 0.5 kg/s

T_w = 80 °C

T_{in} = 40 °C

Solution

For heating the fluid

$$Nu = 0.023\, Re^{0.8}\, Pr^{0.4}$$

Evaluate properties at the bulk temperature $T_b = \dfrac{T_{in} + T_{out}}{2}$ $T_{out} = ?$

Assume a value for T_{out} and then iterate.

Assume $T_{out} = 60\ ^\circ C$ $T_b = \dfrac{T_{in} + T_{out}}{2} = \dfrac{40 + 60}{2} = 50\ ^\circ C$

Properties of water at 50 °C

- $\rho = 987\ kg/m^3$
- $k = 0.646\ W/m.K$
- $\mu = 5.5\ 10^{-4}\ kg/m.s$
- $Pr = 3.5$
- $c_p = 4.176\ kJ/kg.K$

$$Nu = 0.023\ Re^{0.8}\ Pr^{0.4} = 0.023 \left(\frac{U_{mean}\ D}{\mu / \rho} \right)^{0.8} (3.5)^{0.4} \quad \text{if turbulent}$$

$$U_{mean} = \frac{\dot{m}}{\rho\, A} = \frac{0.5}{(987)\ \dfrac{\pi\ (0.025)^2}{4}} = 1.032 \text{ m/s}$$

$$Re = \frac{U_{mean}\ D}{\mu / \rho} = \frac{(1.032)\ (0.025)}{5.5\ 10^{-4} / 987} = 4.6\ 10^4 \quad > 2300 \quad \text{Turbulent}$$

$$Nu_D = 0.023\ (4.6\ 10^4)^{0.8}\ (3.5)^{0.4} = 205$$

$$h = Nu\ \frac{k}{D} = 205\ \frac{0.646}{0.025} = 5298 \text{ W/m}^2.\text{K}$$

Now check T_{out}

$$Q = h\ A\ \Delta T = (5928)\ (\pi)\ (0.025)\ (3)\ (80 - 50) = 37449 \text{ W}$$

$$Q = \dot{m}\ c_p\ (T_{out} - T_{in})$$

$$Q = \dot{m}\, c_p\, (T_{out} - T_{in}) \quad \Rightarrow \quad T_{out} = T_{in} - \frac{Q}{\dot{m}\, c_p} = 40 - \frac{37449}{(0.5)\,(4179)} = 57.9\ ^{\circ}C$$

This is close enough to the first asumption 60 °C.

If not, repeat the calculation using the new T_{out}.

Dimensional Analysis

This is for forced convection heat transfer in tube (pipe) flow.

Write down all the parameters that h depends on: $h = f(D, \rho, \mu, c_p, k, U_m)$

Assume that there is a power relation: $h = C\,(D)^a\,(\rho)^b\,(\mu)^c\,(c_p)^d\,(k)^e\,(U_m)^f$

Try to find all these powers, a, b, c, d, e, and f, by expresing every term in basic units first.

Basic units are: Legth : L (meters)

Mass : m (kilograms)

Time : T (seconds)

Temperature : θ (°C)

The proper units of the terms in terms of these basic units:

Write down all the parameters that h depends on: $h = f(D, \rho, \mu, c_p, k, U_m)$

h (W/m².K) : $(M L^2 T^{-3}) (L^{-2}) (\theta^{-1}) = M T^{-3} \theta^{-1}$

W : kg.m²/s³ (for $M L^2 T^{-3}$)

D diameter or characteristic length : L

ρ (kg/m³) : $M L^{-3}$

μ (kg/m.s) : $M L^{-1} T^{-1}$

c_p (J/kg.K) : $M L^2 T^{-2} M^{-1} \theta^{-1} = L^2 T^{-2} \theta^{-1}$

k (W/m.K) : $M L^2 T^{-3} L^{-1} \theta^{-1} = M L T^{-3} \theta^{-1}$

U_∞ (m/s) : $L T^{-1}$

Substitute in $h = C\ D^a\ \rho^b\ \mu^c\ c_p^{\,d}\ k^e\ (U_m)^f$

$$M\,T^{-3}\,\Theta^{-1} = C\ (L)^a\ (M\,L^{-3})^b\ (M\,L^{-1}\,T^{-1})^c\ (L^2\,T^{-2}\,\theta^{-1})^d\ (M\,L\,T^{-3}\,\theta^{-1})^e\ (L\,T^{-1})^f$$

The powers of the same units on both sides of the equation must be equal.

Mass (M) : $1 = b + c + e$

Length (L) : $0 = a - 3b - c + 2d + e + f$

Time (T) : $-3 = -c - 2d - 3e - f$

Temperature (θ) : $-1 = -d - e$

There are 6 unknowns but 4 equations.

Chose any two and express the other unknown powers in terms of these two.

Chose, for example b and c, then using the above four equations find

$$d = b + c \qquad f = b$$
$$e = 1 - b - c \qquad a = b - 1$$

Substitute in the original equation $h = C\ D^{(b-1)}\ \rho^b\ \mu^c\ c_p^{(b+c)}\ k^{(1-b-c)}\ U_m^{\ b}$

Collect the terms with the same powers

$$h = C \left(\frac{U_m\ D\ \rho\ c_p}{k} \right)^b \left(\frac{\mu\ c_p}{k} \right)^c \frac{k}{D}$$

Rearrange

$$\underbrace{\frac{h\ D}{k}}_{Nu} = C \underbrace{\left(\frac{U_m\ D\ \rho\ c_p}{k} \right)^b}_{Re\ Pr} \underbrace{\left(\frac{\mu\ c_p}{k} \right)^c}_{Pr}$$

Call $m = b$, $n = b + c$

$$Nu = C\ (Re)^m\ (Pr)^n$$

Re Pr = Pe Peclet number

Jean Claude Eugène Péclet

French physicist

1793 – 1857

Reynolds Analogy

Remember laminar flow over a flat plate

$$\left.\begin{aligned} C_{f\,x} &= 0.664\, Re_x^{-1/2} \\ Nu_x &= 0.332\, Re_x^{1/2}\, Pr^{1/3} \end{aligned}\right\} \quad Nu_x = \frac{1}{2} C_{f\,x}\, Re_x\, Pr^{1/3}$$

This is a similarity between Nu_x and $C_{f\,x}$

Can we extend this result to other cases (what other cases?) so that we may use it to predict h?

$$\frac{Du^+}{Dt^+} = -\frac{\partial P^+}{\partial x^+} + \frac{1}{Re}\nabla^{+2} u^+$$

$$\frac{DT^+}{Dt^+} = \frac{1}{Pe}\nabla^{+2} T^+ + \frac{E}{Re}\Phi$$

If

$$\frac{\partial P^+}{\partial x^+} \ll 1$$

$$E \ll 1$$

Eckert number

$$\frac{Du^+}{Dt^+} = \frac{1}{Re}\nabla^{+2} u^+$$

$$\frac{DT^+}{Dt^+} = \frac{1}{Re\,Pr}\nabla^{+2} T^+$$

Equations are similar.

$$\frac{Du^+}{Dt^+} = \frac{1}{Re} \nabla^{+2} u^+ \quad \text{BC's:} \quad \begin{cases} u^+(0) = 0 \\ u^+(\infty) = 1 \end{cases}$$

$$\frac{DT^+}{Dt^+} = \frac{1}{Re\,Pr} \nabla^{+2} T^+ \quad \text{BC's:} \quad \begin{cases} T^+(0) = 1 \\ T^+(\infty) = 0 \end{cases}$$

$$T^+ = \frac{T - T_\infty}{T_w - T_\infty}$$

Same BC's if $T^+ \rightarrow 1 - T^+$

If Pr =1 then expect identical solution, i.e., $1 - T^+ = \theta^+ \rightarrow u^+$

Then, there must be a relation between C_f and Nu, or h.

Let's have a closer look at C_f and h.

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho U_\infty^2}$$

$$\tau_w = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0} \Rightarrow \tau_w = \frac{\mu U_\infty}{L} \left.\frac{\partial u^+}{\partial y^+}\right|_{y=0}$$

$$C_f = \frac{2\mu}{\rho U_\infty L} \left.\frac{\partial u^+}{\partial y^+}\right|_{y^+=0}$$

$$C_f = \frac{2}{Re} \left.\frac{\partial u^+}{\partial y^+}\right|_{y^+=0}$$

$$h\left(T_w - T_\infty\right) = -k \left.\frac{\partial T}{\partial y}\right|_{y=0}$$

$$h\left(T_w - T_\infty\right) = -k \frac{\left(T_w - T_\infty\right)}{L} \left.\frac{\partial T^+}{\partial y^+}\right|_{y^+=0} = k \frac{\left(T_w - T_\infty\right)}{L} \left.\frac{\partial \theta}{\partial y^+}\right|_{y^+=0}$$

$$Nu = \left.\frac{\partial \theta}{\partial y^+}\right|_{y^+=0}$$

If $Pr = 1$, then $u^+ \rightarrow \theta$ The profiles are identical. So

$$Nu = \frac{1}{2} C_f \, Re$$

This is called Reynods analogy (for Pr = 1)

If $Pr \neq 1$, then the profiles are similar, but not identical.

Known from the exact solution that $Pr = \left(\frac{\delta}{\delta_t}\right)^3$

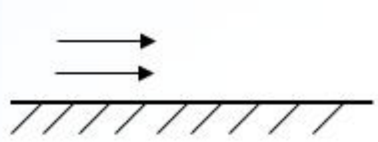
$$\frac{\left.\frac{\partial \theta}{\partial y^+}\right|_0}{\left.\frac{\partial u^+}{\partial y^+}\right|_0} = \frac{1/\delta_t/L}{1/\delta/L} = \frac{\delta}{\delta_t} = Pr^{1/3}$$

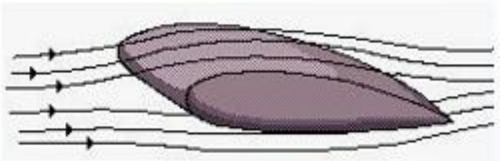
$$Nu = \frac{1}{2} C_f \, Re \, Pr^{1/3}$$

When is this Reynolds analogy used?

☆ E << 1 → Low-speed flow (E is Eckert Number)

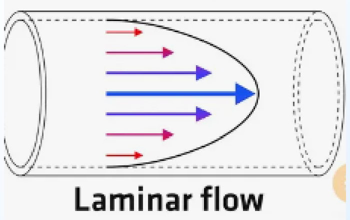
☆ $\frac{\partial P^+}{\partial x^+} << 1$ Flow over flat plates or aerodynamic shapes

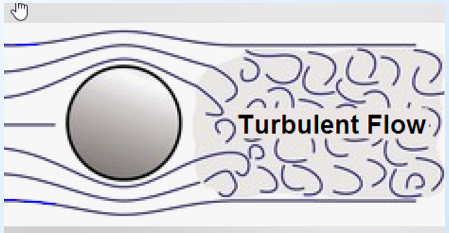


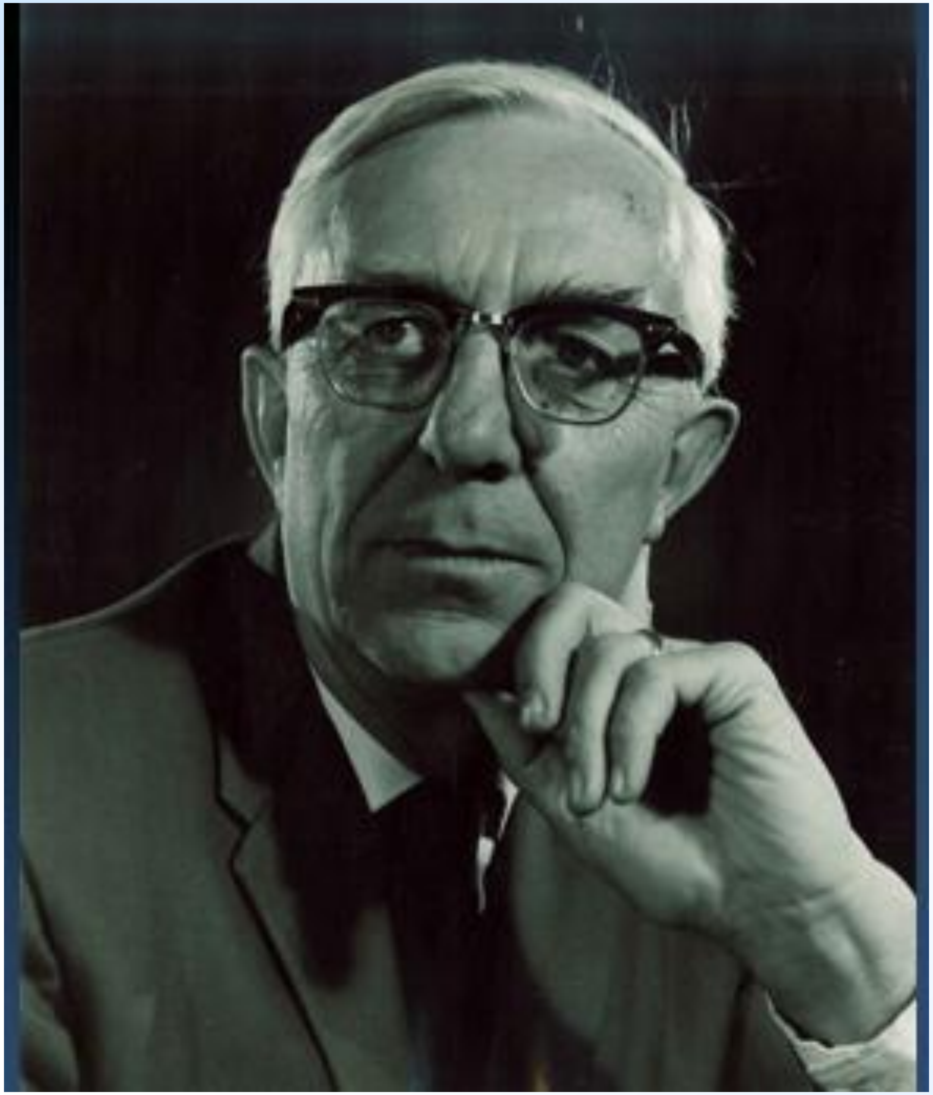


☆ Turbulent pipe flow, because mixing dominates both friction and heat transfer

Reynolds Analogy is not valid for







Ernst R. G. Eckert

Austrian American Scientist

1904 - 2004

Stanton Number: $St = \dfrac{Nu}{Re\ Pr}$

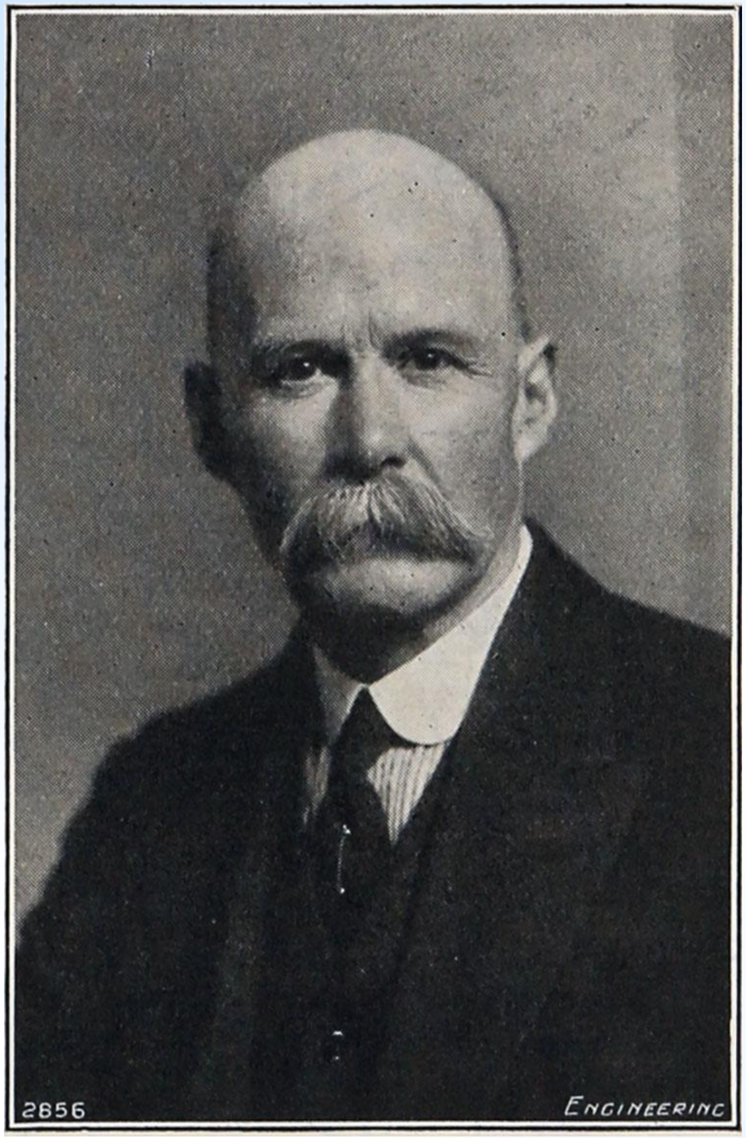
$$St\ Pr^{2/3} = \frac{1}{2} C_f \quad \text{or} \quad \frac{Nu}{Re\ Pr^{1/3}} = J$$

$$J = \frac{1}{2} C_f$$

J is Colburn factor

This is called Colburn analogy

For average values of C_f, C_D may be used as long as the drag is mainly due to skin friction.



Sir Thomas Ernest Stanton

British Mechanical Engineer

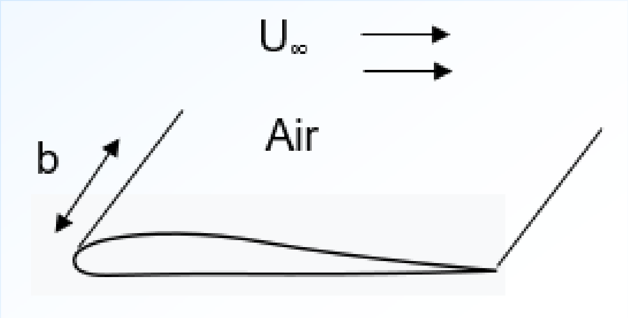
1865 - 1931

Allan Philip Colburn

American Chemical Engineer

1904 - 1955

Example 2



Air flows over a NASA 2405 profile as shown.

U_∞ = 100 m/s

T_∞ = - 40 °C

T_w = 20 °C

Find the heat flow rate, Q, in W/m.

Evaluate properties of air

at $\bar{T} = \dfrac{-40 + 20}{2} = -10\ ^\circ C$

ν = 10.7 10-6 m^2/s

k = 0.023 W/m.K

Pr = 0.72

$Re = \dfrac{(100)\ (1)}{10.7\ 10_{-6}} = 9.35\ 10^6$

Find the drag coefficient, C_D,

using Re and an experimental Figure.

$C_D \cong 7.0\ 10^{-3} = \dfrac{\text{Total drag} / (b)\ (L)}{(1/2)\ \rho\ U_\infty^2}$

$C_f \cong C_D / 2 = 3.5\ 10^{-3}$

$$Nu = \frac{1}{2}\, C_f \; Re\, Pr^{1/3} = \frac{1}{2}\, (3.5\ 10^{-3})\ (9.35\ 10^{6})\ (0.72)^{1/3} = 14665.4$$

$$h = Nu\, \frac{k}{L} = (14665.4)\ \frac{0.023}{1} = 337.3\ W/m^2.K$$

$$\dot{Q} = (335.3)\ (2)\ (1)\ (20 + 40) = 40\ 476.5\ W$$

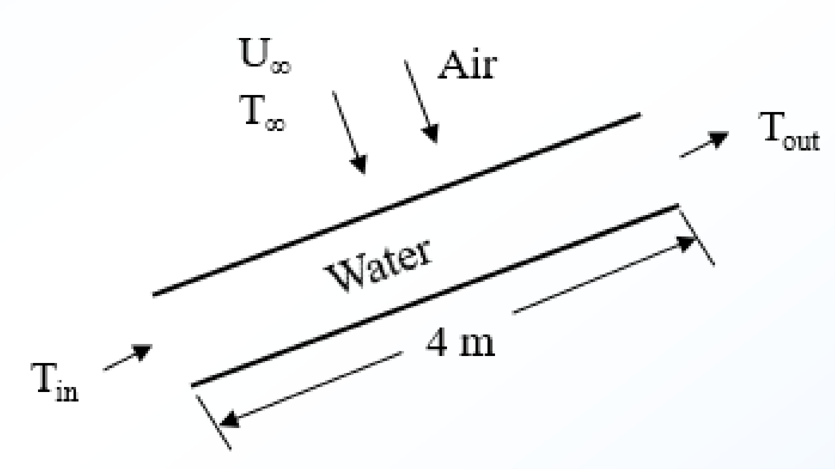
Note that purely hydrodynamic correlations are used.

Example 3

Water flows in a thin-walled tube, 40 mm OD, length 4 m. The inlet temperature of the water is 30 °C. Atmospheric air flows across the tube on the outside. The velocity and temperature of the air are 100 m/s and, 225 °C, respectively.

Estimate the exit temperature of the water.

Assume fully developed conditions for the internal flow of water.



D = 40 mm

L = 4 m

m = 0.25 kg/s

U_∞ = 100 m/s

T_∞ = 225 °C

T_{in} = 30 °C

T_{out} = ?

Assumptions: - Steady state

- Tube wall thermal resistance is neglected
- Fully-developed internal flow
- Constant properties
- Changes in KE, PE, flow-work are negligible

This is a typical trial-and-error solution. Assume a value for T_{out}; then check whether it is close enough to the correct value. If not, repeat the calculation with a new assumption for T_{out}.

Assume T_{out} = 70 °C First, evaluate properties at the average temperature

$$\frac{T_{in} + T_{out}}{2} = \frac{70 + 30}{2} = 50\ ^\circ C$$

Properties of water

at the average temperature

$$\frac{T_{in} + T_{out}}{2} = \frac{70 + 30}{2} = 50\ ^\circ C = 315\ K$$

$\rho = 1/v_f = 991.1\ kg/m^3$

$c = 4179\ J/kg.K$ $\quad$ $k = 0.634\ W/m.K$

$\mu = 631\ 10^{-6}\ N.s/m^2$ $\quad$ $Pr = 4.16$

Properties of air, 1 atm

at T_∞ = 225 °C = 315 K

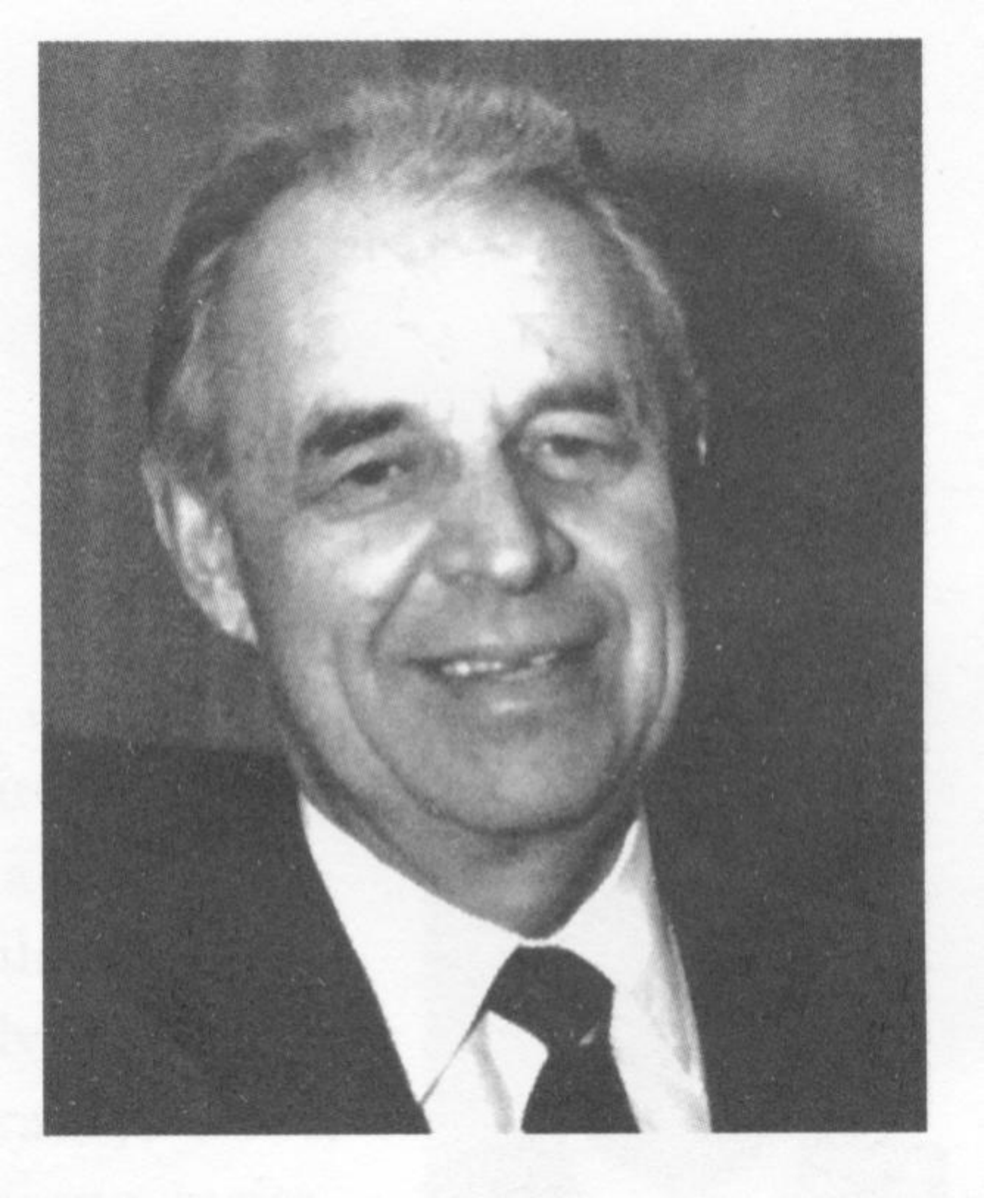
$\nu = 38.79\ 10^{-6}\ m^2/s$

$k = 40.7\ 10^{-3}\ W/m.K$

$Pr = 0.684$

Note that properties are evaluated at the free stream temperature. This is according to the relation given by Žukauskas.

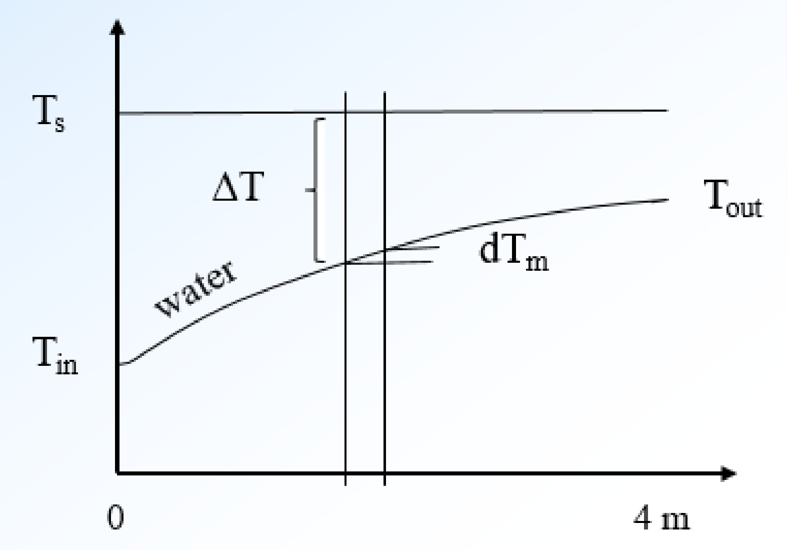
Additional assumption: Use constant surface temperature, T_s.



Algirdas Žukauskas

Lithuanian Scientist

1923 - 1997



At any cross section, $dQ = \dot{m}\, c_p\, dT_m$

$$dQ = h_x\, P\, dx\, \left(T_s - T_m\right)$$

$$\frac{dT_m}{dx} = \frac{P}{\dot{m}\, c_p}\, h_x\, \underbrace{\left(T_s - T_m\right)}_{\Delta T}$$

$$\int_{\Delta T_{in}}^{\Delta T_{out}} \frac{d(\Delta T)}{\Delta T} = -\frac{P}{\dot{m}\, c_p} \int_0^L h_x\, dx \qquad \ln\left(\frac{\Delta T_{out}}{\Delta T_{in}}\right) = \frac{P\, L}{\dot{m}\, c_p}\, \bar{h} \qquad \frac{\Delta T_{out}}{\Delta T_{in}} = \exp\left(\frac{P\, L}{\dot{m}\, c_p}\, \bar{h}\right)$$

$$\frac{T_s - T_{m,out}}{T_s - T_{m,in}} = \exp\left(\frac{P\, L}{\dot{m}\, c_p}\, \bar{h}\right) \quad , \quad T_s = \text{constant}$$

$$Q = \dot{m}\, c_p \left[\left(T_s - T_{m,in}\right) - \left(T_s - T_{m,out}\right) \right] = \dot{m}\, c_p \left(\Delta T_{in} - \Delta T_{out}\right)$$

$$= \bar{h}\, A_s\, (LMTD)$$

Using these derivations: $$T_{m,out} = T_{out} = T_s - \left(T_s - T_{m,in}\right) \exp\left(\frac{P\,L}{\dot{m}\, c_p}\, \bar{h} \right)$$

Replace T_s with T_∞ and h with U where $$U = \frac{1}{\dfrac{1}{h_i} + \dfrac{1}{h_o}}$$

$$T_{m,out} = T_{out} = T_\infty - \left(T_\infty - T_{m,in}\right) \exp\left(\frac{P\,L}{\dot{m}\, c_p}\, U \right)$$

Internal Flow: $$Re = \frac{4\,\dot{m}}{\pi\, D\, \mu} = 12\ 611$$

$$\bar{h}_i = \frac{k}{D}\ 0.023\ Re^{4/5}\ Pr^{0.4} = 1230\ W/m^2.K$$

External Flow: $$Re = \frac{U_\infty\, D}{\nu} = 1031\ 10^5$$

$$Nu = \frac{\bar{h}_o\, D}{k} = C\ Re^m\ Pr^n \left(\frac{Pr_\infty}{Pr_s}\right)^{1/4}$$

Let $Pr_\infty = Pr_s$

Use Table: $C = 0.26$, $m = 0.6$

$n = 0.37$ for $Pr \le 10$

$\bar{h}_o = 234\ W/m^2.K$

$$U = \frac{1}{\frac{1}{h_i} + \frac{1}{h_o}} = \frac{1}{\frac{1}{1230} + \frac{1}{234}} = 197\ W/m^2.K$$

$$T_{m,out} = T_s - \left(T_s - T_{m,in}\right) \exp\left(\frac{P\,L}{\dot{m}\,c_p}\,\bar{h}\right)$$

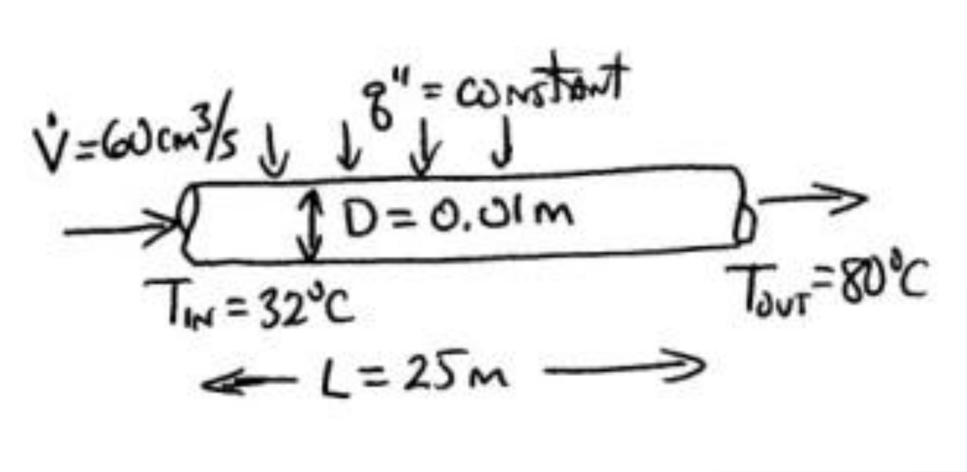
$$= 225 - \left(225 - 30\right) \exp\left(\frac{\pi\,(0.04)\,(4)}{(0.25)\,(4179)}\,(197)\right) = 47.6\ ^\circ C$$

Find $T_s = 63.2\ ^\circ C$ Find $Pr_s = 0.687$ So $Pr_\infty \approx Pr_s$

It checks.

Example 4

In a pharmaceutical application, the product is subjected to a final sterilization by heating it from 32 ºC to 80 ºC. A flow of 60 cm^3/s is passed through a 10-mm tube that is heated with a uniform heat flux produced by wrapping the tube with an electric resistance heater. If product properties can be approximated by those of ethylene glycol, and the tube is 25-m long.



Determine.

a) the required power (in W);

b) the wall temperature at the tube exit (in ºC).

The required heat transfer rate can be obtained from conservation of energy and the given information. The wall temperature at the tube outlet can be calculated using the basic rate equation ($q'' = h \, \Delta T$).

Assumptions:

1. The system is steady with no work or potential or kinetic energy effects.
2. The liquid is ideal with constant specific heat.

(a) For conservation of energy on the liquid, assume steady, no work, no potential or kinetic energy effects, and an ideal liquid with constant specific heat so that enthalpy change = $c_p \, \Delta T$

$$\dot{Q} = \dot{m} \, c_p \, \left(T_{out} - T_{in} \right)$$

The properties required to calculate the heat transfer coefficient are evaluated at $T_{avg} = \frac{32 + 80}{2} = 56\ ^\circ C \cong 330\ K$

$\rho = 1089.5\ kg/m^3$

$c = 2.549\ kJ/kg.K$ $k = 0.260\ W/m.K$

$\mu = 56.1\ 10^{-4}\ N.s/m^2$ $Pr = 55$

$$\dot{Q} = (1089.5)\ (60\ 10^{-6})\ (2.549\ 10^{3})\ (80 - 32) = 8\ 000\ W$$

(b) For wall temperature at the tube outlet:

$$q'' = h\ (T_s - T_{out}) \Rightarrow T_s = T_{out} + \frac{q''}{h}$$

$$T_s = T_{out} + \frac{\dot{Q}/A}{h} = T_{out} + \frac{\dot{Q}}{\pi\ D\ L\ h}$$

To calculate the heat transfer coefficient, we need the Reynolds number:

$$Re = \frac{\rho\ V\ D}{\mu} \Rightarrow V = \frac{\dot{m}}{\rho\ (\pi/4)\ D^2}$$

$$Re = \frac{4\ \dot{m}}{\pi\ \mu\ D} = \frac{4\ (1089.5)\ (60\ 10^{-6})}{\pi\ (56.1\ 10\text{-}4)\ (0.01)} = 1480$$

Re = 1480 is laminar flow, so we will check the entrance length

$$L_{ent} \simeq 0.053 \, Re \, Pr \, D$$
$$\simeq 0.053 \, (1480) \, (55.0) \, (0.01) = 43.2 \text{ m}$$

This is significantly longer than the tube length, so the entrance effects must be taken into account.

We do not have a correlation that gives the heat transfer coefficient for laminar, developing flow with a constant wall heat flux boundary condition. So, we will use the Seider and Tate correlation.

Assuming $T_s \sim 350$ K,

$\mu_s = 34.2 \; 10^{-4}$ N.s/m^2

$$Nu = 1.86 \, Gz^{1/3} \left(\frac{\mu}{\mu_s} \right)^{0.14}$$
$$= 186 \left[\frac{(1480)\,(55.0)\,(0.01)}{25} \right]^{1/3} \left(\frac{56.1 \; 10^{-4}}{34.2 \; 10^{-4}} \right)^{0.14}$$
$$= 6.37$$

$$h = \frac{Nu\,k}{D} = \frac{(6.37)\,(0.260)}{0.01} = 166\ W/m^2.K$$

$$T_s = 80 + \frac{8000}{\pi\,(0.01)\,(25)\,(166)} = 141.3\ ^\circ C$$

Re-evaluating μ_s at 373 K (highest temperatures for which we have the properties):

$\mu_s = 21.5\ 10^{-4}\ N.s/m^2$

Nu = 6.80

$h = 177\ W/m^2.K$

Ts = 137.5 °C

The wall temperature will be lower than this, since μ_s should be smaller than with what we have available.

10 – High Speed Flow

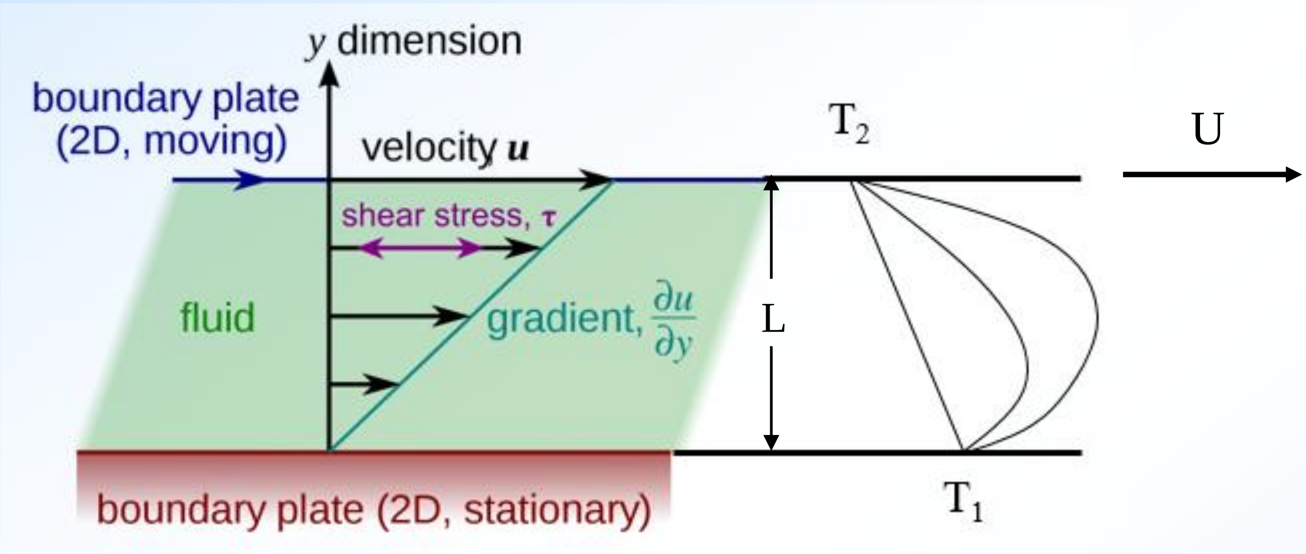
Effects of high speed:

- → Viscous dissipation (E) cannot be neglected (for incompressible fluids)
- → Viscous dissipation (E) + work of compression (M) cannot be neglected (for compressible fluids)
- → Temperature gradients in the boundary layer becomes so large that properties of fluid vary significantly

There are number of techniques developed. We will not see them all. Instead, we will define an average temperature, T_{av}, in $q = h\left(T_w - T_{av}\right)$

Let's examine two cases, incompressible fluid and compressible fluid.

Incompressible fluid – Couette Flow



$$u(y) = U \frac{y}{L}$$

$$u^{+} = \frac{u}{U} = \frac{y}{L} = y^{+}$$

$$\frac{\partial u^{+}}{\partial y^{+}} = 1$$

$$T^{+} = \frac{T - T_1}{T_2 - T_1}$$

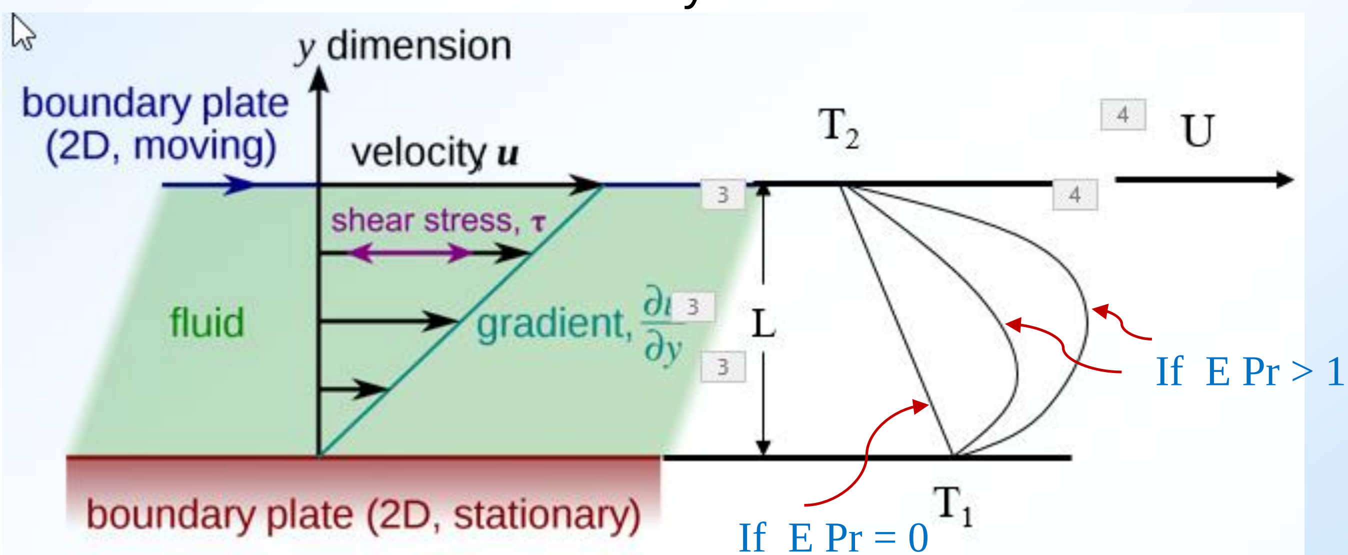
In fluid dynamics, **Couette flow** is the flow of a viscous fluid in the space between two surfaces, one of which is moving tangentially relative to the other with velocity **U**. The relative motion of the surfaces imposes a shear stress on the fluid and induces flow.

It is named after Maurice Couette, a Professor of Physics at the French University of Angers in the late 19th century.

Non-dimensional energy equation:

$$u^+ \cancelto{0}{\frac{\partial T^+}{\partial x^+}} + v^+ \cancelto{0}{\frac{\partial T^+}{\partial y^+}} = \frac{1}{\mathrm{Re}\,\mathrm{Pr}}\left[\cancelto{0}{\frac{\partial^2 T^+}{\partial x^{+2}}} + \frac{\partial^2 T^+}{\partial y^{+2}}\right] + \frac{E}{\mathrm{Re}}\cancelto{1}{\Phi}$$

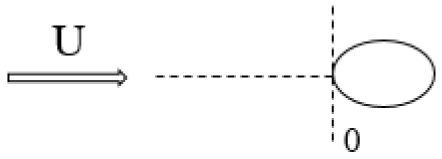
$$\frac{\partial^2 T^+}{\partial y^{+2}} = -\,E\;\mathrm{Pr} \qquad \Phi = \left(\frac{\partial u^+}{\partial y^+}\right)^2$$



Compressible fluid

Dissipation $\rightarrow$ E What about compression work?

Assume: No friction (no dissipation)
No heat transfer (adiabatic compession)
Perfect gas



Define T_0, total or stagnation temperature

$$\frac{1}{2}\, m\, U^2 = m\, c_p \left(T_0 - T\right) \quad \Rightarrow \quad c_p\, T + \frac{U^2}{2}\, c_p\, T_0 \quad \Rightarrow \quad \frac{T_0}{T} = 1 + \frac{U^2}{2\, c_p\, T}$$

But $c_p - c_v = R$ universal gas constant

$$\frac{1}{c_p} = \frac{k-1}{k}\,\frac{1}{R} \quad , \quad \frac{c_p}{c_v} = k \text{ or } \lambda \quad , \quad k = 1.4 \text{ for air}$$

$$\frac{T_0}{T} = 1 + \left(\frac{k-1}{2\,k}\right)\left(\frac{U^2}{R\,T}\right)$$

Acoustic speed: $a = \sqrt{k R T}$

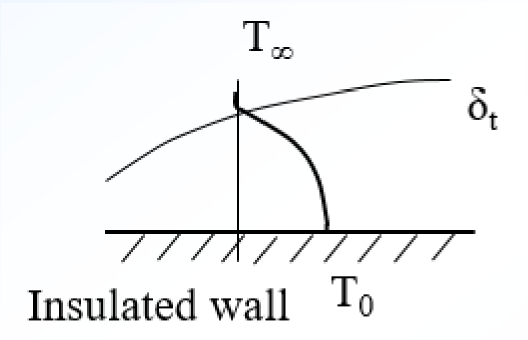
Mach number: $M = \dfrac{U}{a}$

$$\frac{T_0}{T} = 1 + \frac{k - 1}{2} M^2$$

Compression work is related to M^2

$E \rightarrow M^2$ for both dissipation and compression

For gases M^2 replaces E



T_∞: Fluid temperature away from the wall

T_0: Wall temperature due to compression only

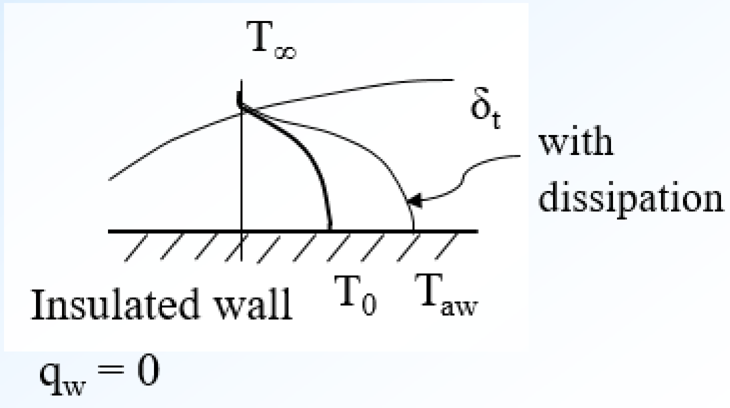
(Fluid particles coming to rest on the wall)

No fricton, no heat transfer in the fluid

No viscous dissipation

No heat transfer, $q_w = 0$

$$\frac{T_0}{T_\infty} = 1 + \frac{k - 1}{2} M^2$$



Actually:

T_{aw}: Wall temperature with viscous dissipation and with heat transfer in the fluid

Define recovery factor:

$$r = \frac{T_{aw} - T_\infty}{T_0 - T_\infty}$$

- If no friction and no heat transfer $\rightarrow$ r = 1
- If friction balances heat transfer $\rightarrow$ r = 1
- If friction dominates $\rightarrow$ r > 1
- If heat transfer dominates $\rightarrow$ r < 1

Friction $\rightarrow \nu$

Heat transfer $\rightarrow \alpha$

$r \rightarrow Pr$

$r = Pr^{1/2}$ Laminar

$r = Pr^{1/3}$ Turbulent

$$\frac{T_{aw}}{T_\infty} = 1 + \frac{k-1}{2} M^2$$

Therefore, use T_{aw} in $q = h\,(T_w - T_{aw})$

For air, $\sqrt{k\,R} \simeq 20$ and $a \simeq 20\sqrt{T\ (K)}$

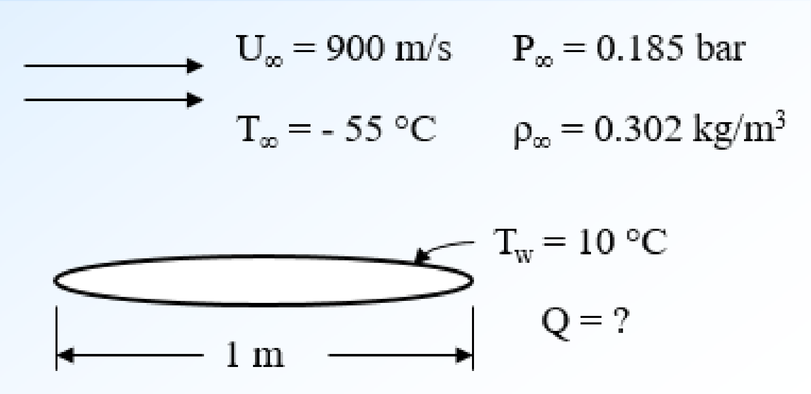
The properties are to be evaluated at T*

$$T^* = T_\infty + 0.5\,(T_w - T_\infty) + 0.22\,(T_{aw} - T_\infty)$$

For h or Nu, use the low-speed relations, evaluated at T* if $Re^* < 10^7$

For $Re^* > 10^7$, see other references.

Example 5



Altitude: 12 000 m

Assume turbulence $r = Pr^{1/3} = 0.9$ constant

$$T_{aw} = T_\infty \left[1 + r \frac{k-1}{2} M^2 \right]$$

$$a = \sqrt{k\,R\,T} \cong 20\sqrt{T\,(K)} = 20\sqrt{218}$$

$$M = \frac{900}{20\sqrt{218}} = 3.05$$

$$T_{aw} = 218 \left[1 + (0.9)\,(0.2)\,(3.05)^2 \right]$$

$$= 583 \text{ K} = 310\ ^\circ\text{C}$$

$$T_{aw} = 310\ ^\circ\text{C}$$

$$T_\infty = -55\ ^\circ\text{C}$$

Wow!

$$Q = h\,A\,(T_w - T_{aw}) = h\,(2\ m^2)\,(10 - 310) \quad => \quad h = ?$$

$$Nu = 0.036\ Re^{0.8}\ Pr^{1/3}$$

Evaluate properties at T* $\quad T^* = -55 + (0.5)(10 + 55) + (0.22)(310 + 55) = 57\ ^\circ C$

$$Re = \frac{U_\infty L}{\nu} \qquad \nu = \frac{\mu}{\rho} \qquad \rho = \frac{P}{R\,T} = \frac{(0.185\ 10^5)(29\ 10^{-3})}{(8.31)(330)} = 0.195\ kg/m^3$$

$\nu \cong 10^{-4}$ $\quad Re = 9\ 10^6$ $\quad$ turbulent but $< 10^7$

$Nu = 1.19\ 10^4$ $\quad h = 332\ W/m^2.K$ $\quad Q \cong 200\ 000\ W/m$

Note the significance