8. Forced Convection - External Flow

Drag and Heat Transfer

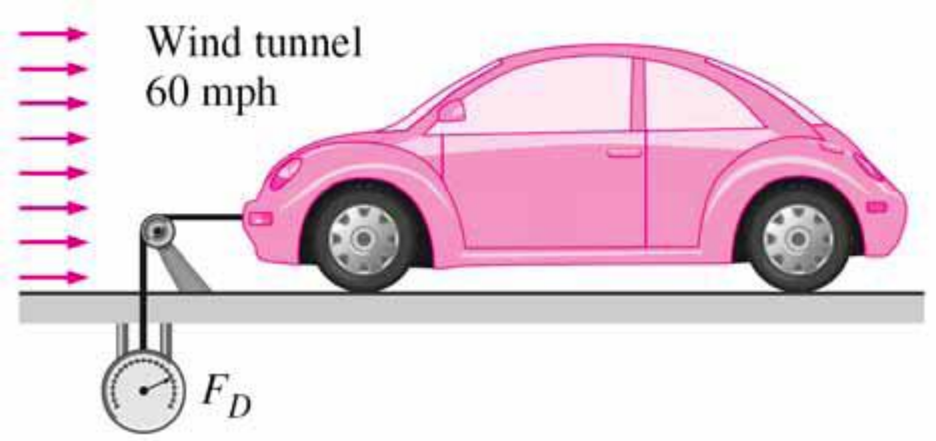
Fluid flow over solid bodies is responsible for numerous physical phenomena such as

- *drag force* (automobiles and power lines)
- *lift force* (airplane wings)
- *cooling of metal or plastic sheets.*

Free-stream velocity — the velocity of the fluid relative to an immersed solid body sufficiently far from the body.

The fluid velocity ranges from zero at the surface (the no-slip condition) to the free-stream value away from the surface.

Friction and Pressure Drag



The force a flowing fluid exerts on a body in the flow direction is called **drag.**

Drag is composed of:

- – pressure drag,
- – friction drag (skin friction drag).

The drag force F_D ***depends on the***

- – density, ρ, of the fluid,
- – the upstream velocity, U_∞, *and*
- – the size, shape, and orientation of the body.

The dimensionless **drag coefficient, *C_D*, *is defined as***

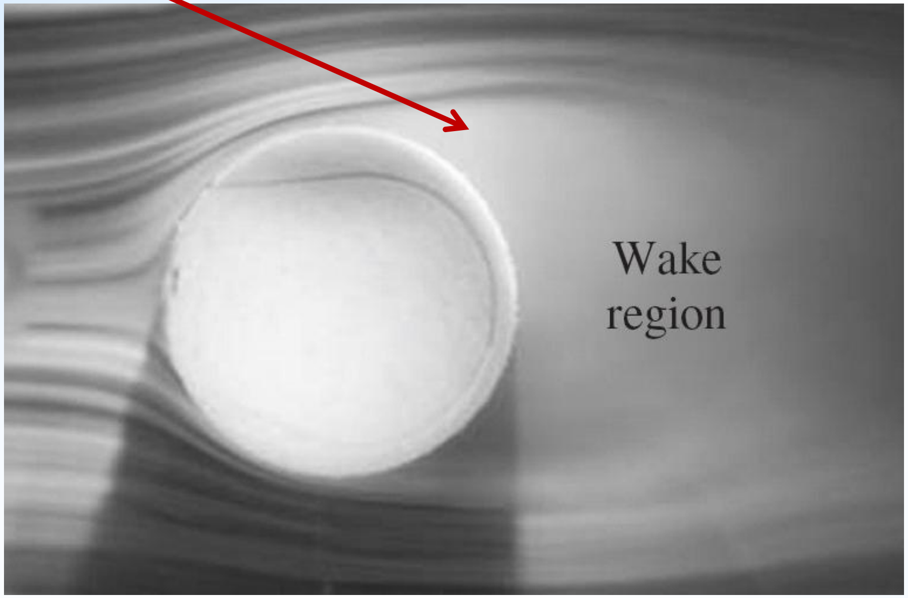
$$C_D = \frac{F_D}{\frac{1}{2}\,\rho\,U_\infty^2\,A}$$

where A is a reference area.

The friction drag is proportional to the surface area. The reference area, A, is the surface area.

At low Reynolds numbers, most drag is due to friction drag.

The pressure drag is proportional to the frontal area and to the *difference between the pressures acting on* the front and back of the immersed body. The reference area, A, is the frontal area.



The pressure drag is usually dominant for blunt bodies and negligible for streamlined bodies.

When a fluid separates from a body, it forms a separated region between the body and the fluid stream.

The larger is the separated region, the larger is the pressure drag.

Heat Transfer

The phenomena that affect friction drag also affect heat transfer.

The local friction drag and convection coefficients vary along the surface as a result of the changes in the velocity boundary layers in the flow direction.

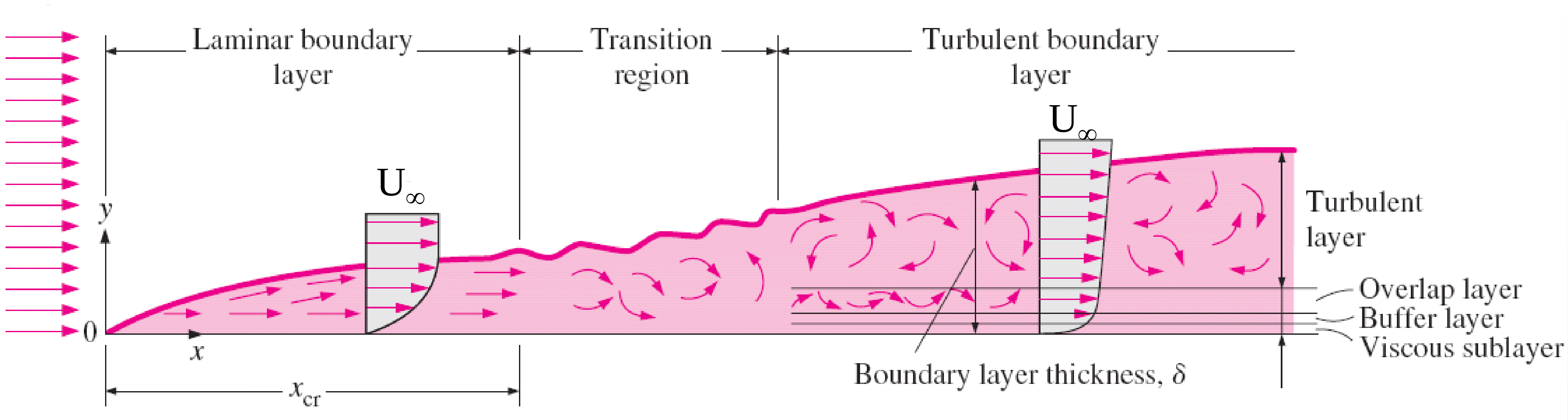
The *average friction and convection coefficients for* the entire surface can be determined by

$$C_D = \frac{1}{L}\int_0^L C_{D,x}\,dx \qquad h_{av} = \frac{1}{L}\int_0^L h_x\,dx$$

local

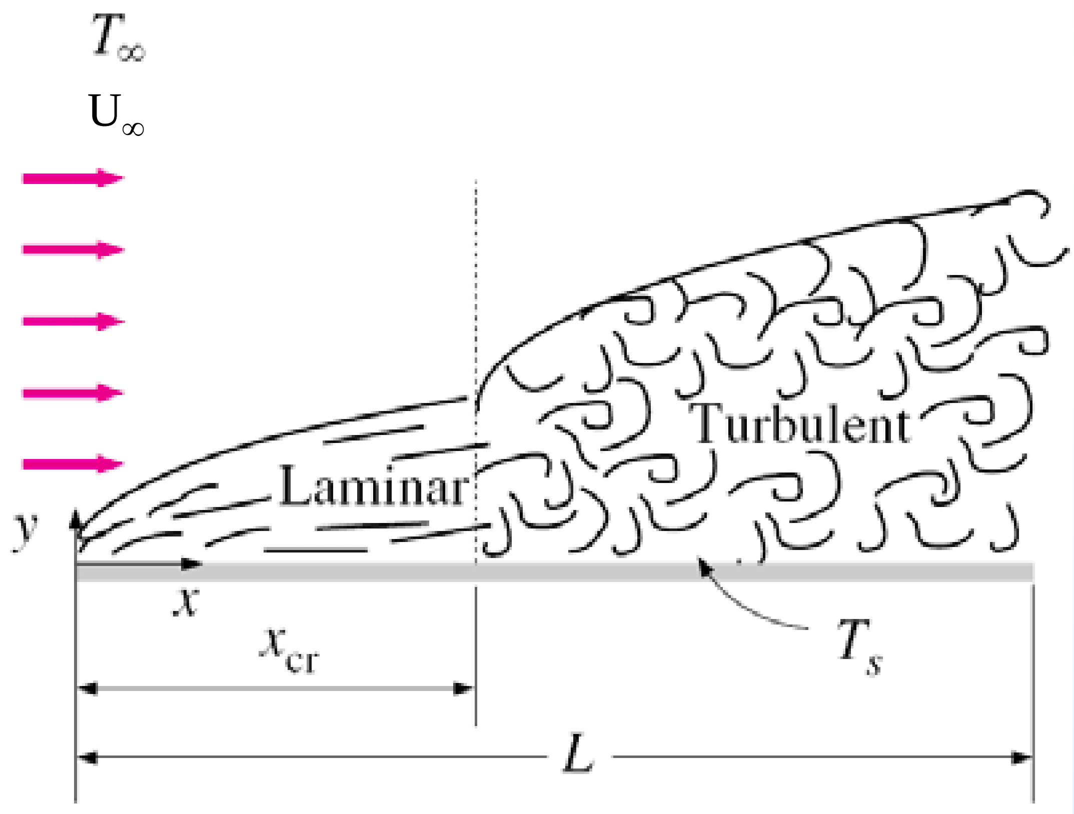
local

Parallel Flow Over Flat Plates



Critical Reynolds Number: $Re_{x,crit} = 5\ 10^5$ for a smooth surface

$$Re_x = \frac{\rho\, U_\infty\, x}{\mu} = \frac{U_\infty\, x}{\nu}$$



The actual value of the engineering critical Reynolds number may vary somewhat from 10^5 to $3\ 10^6$.

Local Friction Coefficient

The boundary layer thickness and the local friction coefficient at a location *x over a flat plate where*

$$Re_x = \frac{U_\infty x}{\nu}$$

Laminar Region, $Re_x < 5\ 10^5$:

$$\delta_{v,x}(x) = \frac{4.91\, x}{\sqrt{Re_x}}$$

$$C_{f,x} = \frac{\tau}{1/2\ \rho\ U_\infty^2} = \frac{0.664}{\sqrt{Re_x}}$$

Turbulent Region, $5\ 10^5 \le Re_x \le 10^7$:

$$\delta_{v,x}(x) = \frac{0.38\ x}{Re_x^{1/5}}$$

$$C_{f,x} = \frac{\tau}{1/2\ \rho\ U_\infty^2} = \frac{0.059}{Re_x^{1/5}}$$

Average Friction Coefficient

Laminar Region, $\mathrm{Re_L} < 5\ 10^5$:

$$C_f = \frac{1.328}{\sqrt{\mathrm{Re}_L}}$$

Turbulent Region, $5\ 10^5 \leq \mathrm{Re_L} \leq 10^7$:

$$C_f = \frac{0.074}{\mathrm{Re}_L^{1/5}}$$

When laminar and turbulent flows are significant:

$$C_f = \frac{1}{L}\left(\int_o^{x_{cr}} \mathrm{C}_{f,x \text{ laminar}}\ \mathrm{dx} + \int_o^{x_{cr}} \mathrm{C}_{f,x \text{ turbulent}}\ \mathrm{dx} \right)$$

$$C_f = \frac{0.074}{\mathrm{Re}_L^{1/5}} - \frac{1742}{\mathrm{Re}_L} \qquad 5\ 10^5 \leq \mathrm{Re}_L \leq 10^7$$

Heat Transfer Coefficient – Constant Wall Temperature, T_w

The local Nusselt number at location *x over a flat plate*

Laminar: $$Nu_x = \frac{h_x\ x}{k} = 0.332\ Re_x^{1/2}\ Pr^{1/3} \qquad Pr > 0.6\ ,\quad Re_x < 5\ 10^5$$

Turbulent: $$Nu_x = \frac{h_x\ x}{k} = 0.0296\ Re_x^{0.8}\ Pr^{1/3} \qquad 0.6 \le Pr \le 60 \quad \text{and}$$

$$5\ 10^5 \le Re_x \le 10^7$$

Note that h_x is infinite at the leading edge ($x = 0$) and decreases by a factor of $x^{0.5}$ *in the flow direction.*

Evaluate properties at $\dfrac{T_w + T_\infty}{2}$

Heat Transfer Coefficient – Constant Wall Temperature, T_w

Average Nusselt number over length L

Laminar: $Nu_L = \dfrac{h_{av}\ L}{k} = 0.664\ Re_L^{1/2}\ Pr^{1/3}$ $\quad Pr > 0.6 \quad , \quad Re_L < 5\ 10^5$

Turbulent: $Nu_L = \dfrac{h_L\ L}{k} = 0.037\ Re_L^{0.8}\ Pr^{1/3}$ $\quad 0.6 \le Pr \le 60$ and

$5\ 10^5 \le Re_x \le 10^7$

Laminar + Turbulent: $Nu_L = \dfrac{h_L\ L}{k} = (0.037\ Re_L^{0.8} - 871)\ Pr^{1/3}$ $\quad 0.6 \le Pr \le 60$ and

$5\ 10^5 \le Re_x \le 10^8$

Evaluate properties at $\dfrac{T_w + T_\infty}{2}$

Heat Transfer Coefficient – Constant Heat Flux, q_s

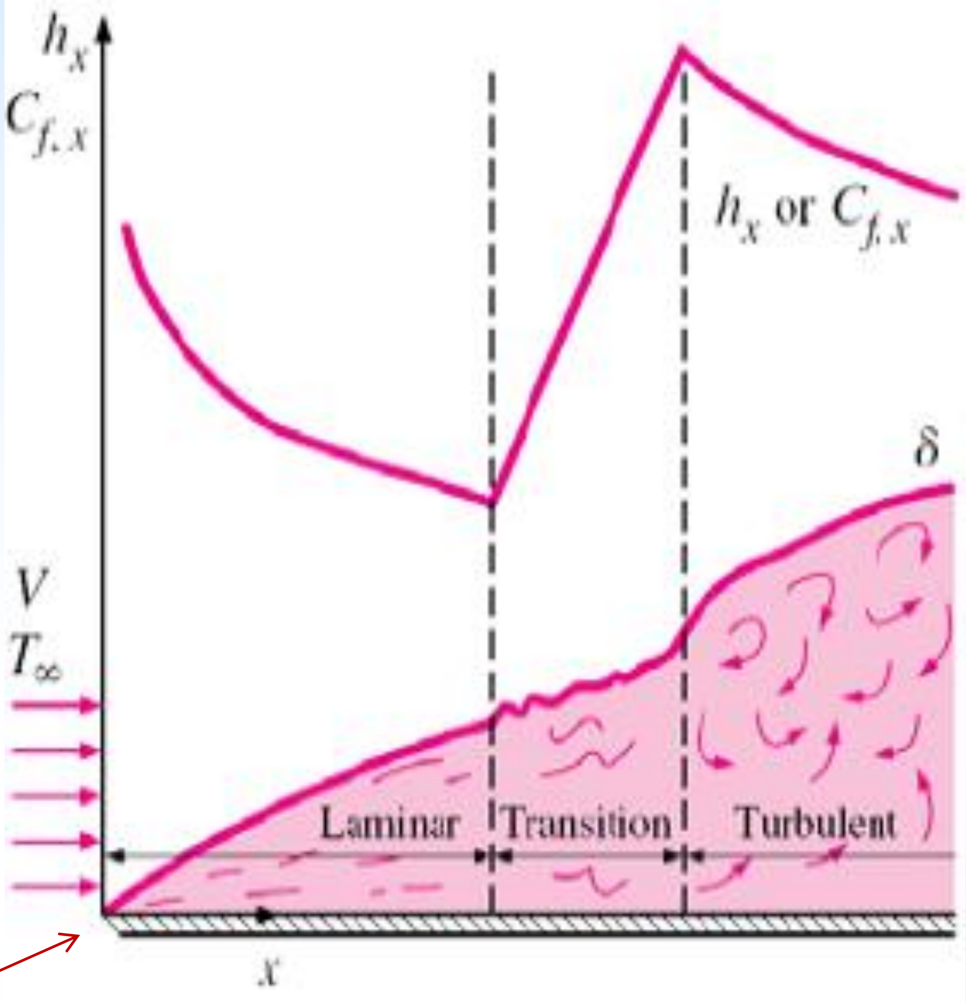
The local Nusselt number at location *x* over a flat plate:

Laminar: $$Nu_x = \frac{h_x\ x}{k} = 0.453\ Re_x^{1/2}\ Pr^{1/3} \qquad Pr > 0.6 \quad , \quad Re_x < 5\ 10^5$$

Turbulent: $$Nu_x = \frac{h_x\ x}{k} = 0.0308\ Re_x^{0.8}\ Pr^{1/3} \qquad 0.6 \le Pr \le 60 \quad \text{and}$$

$$5\ 10^5 \le Re_x \le 10^7$$

These relations give values that are 36 percent higher for laminar flow and 4 percent higher for turbulent flow relative to the isothermal plate case.



Leading Edge

For completely turbulent flow:

$$Nu_x = 0.0288\, Re_x^{0.8}\, Pr^{1/3} \quad \text{Local}$$

$$\overline{Nu} = 0.036\, Re_L^{0.8}\, Pr^{1/3} \quad \text{Average}$$

For turbulence starting at $Re_x = 5\ 10^5$:

$$\overline{Nu} = \left(0.037\, Re_L^{0.8} - 871\right) Pr^{1/3} \qquad Re < 10^8$$

Example 1

Air at atmospheric pressure and 40 °C flows with a velocity of 1 m/s along a flat plate kept at a uniform temperature of 100 °C.

(a) Determine the velocity boundary layer thickness and the local coefficient of friction at a distance 0.5 m from the leading edge.

(b) What are the average coefficient of friction over the length x = 0 to 0.5 m and the drag force acting on the plate over the same length per meter width of the plate?

(c) Determine the local heat transfer coefficient at x = 0.5 m, and the average heat transfer coefficient over the same length.

(d) Calculate heat flow rate over the same region.

Solution

(a)

$$Re_x = \frac{U_\infty x}{\nu} \qquad \left.\begin{array}{l} T_w = 100\ °C \\ T_\infty = 40\ °C \end{array}\right\} \quad \frac{T_w + T_\infty}{2} = 70\ ^\circ C = 343\ K$$

Find the properties of air at 350 K $\left\{\begin{array}{ll} \rho = 0.998\ kg/m^3 & k = 0.03003\ W/m.K \\ \nu = 20.76\ 10^{-6}\ m^2/s & Pr = 0.697 \end{array}\right.$

$$Re_x = \frac{U_\infty x}{\nu} = \frac{(0.5)\,(1)}{20.76\ 10^{-6}} = 2.4\ 10^4 \quad => \quad \text{Laminar}$$

$$\delta(x) = \frac{4.91\,x}{\sqrt{Re_x}} = 0.016\ m \qquad C_{f,x} = \frac{0.664}{\sqrt{Re_x}} = 0.004$$

(b) $\bar{C}_f = 2\, C_x \cong 0.008$

$$\text{Drag Force} = w\, L\, \bar{c}_f \, \frac{\rho\, U_\infty^2}{2} = (1)\,(0.5)\,(0.008)\,\frac{(0.998)\, 1^2}{2} = 0.002 \text{ N}$$

(c) $$Nu_x = 0.332\, Re^{1/2}\, Pr^{1/3} = \frac{h_x\, x}{k}$$

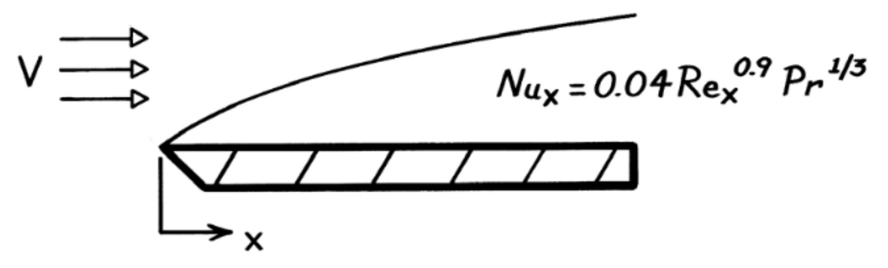
$$h_x = \frac{k}{x}\, 0.332\, Re^{1/2}\, Pr^{1/3} = \frac{0.03003}{0.5}\, 0.332\, (2.4\; 10^4)^{1/2}\, (0.697)^{1/3} = 2.74 \text{ W/m}^2.\text{K}$$

$$\bar{h} = 2\, h_x = 5.48 \text{ W/m}^2.\text{K}$$

$$Q = A\, \bar{h}\, (T_w - T_\infty) = (1)\,(0.5)\,(5.48)\,(100 - 40) = 164.62 \text{ W/m}$$

Such analysis is not vald for liquid metals where Pr < < 1 for laminar flow over a flat plate.

Example 2



Experimental results for heat transfer over a flat plate with an extremely rough surface were found to be correlated by an expression of the form

$$Nu_x = \frac{h_x\, x}{k} = 0.04\, Re_x^{0.9}\, Pr^{1/3}$$

where Nu_x is the local value of the Nusselt number at a position x measured from the leading edge of the plate. Obtain an expression for the ratio of average heat transfer coefficient h_{av} to the local coefficient h_x.

Hint: Use the definition of h_{av}

Solution

$$h_x = Nu_x \frac{k}{x} = 0.04 \frac{k}{x} Re_x^{0.9} Pr^{1/3}$$

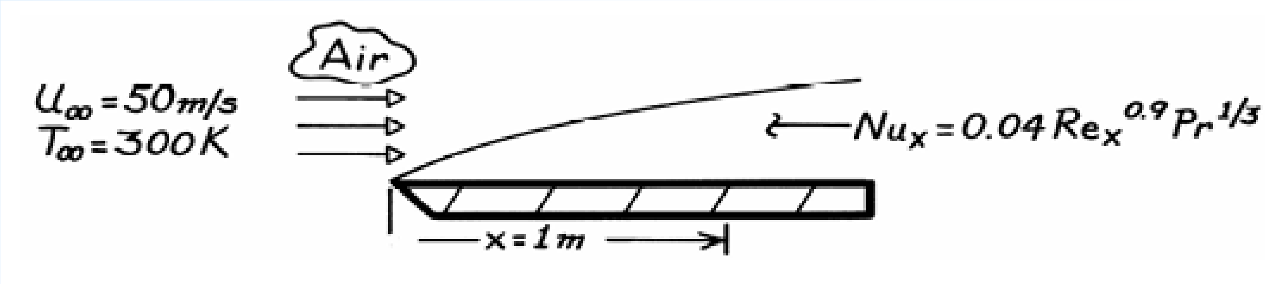
$$h_x = 0.04 \left(\frac{V}{\nu}\right)^{0.9} Pr^{1/3} \frac{x^{0.9}}{x} = c_L \; x^{-0.1}$$

$$h_{av} = \frac{1}{x} \int_0^x h_x \, dx = \frac{1}{x} c_L \int_0^x x^{-0.1} \, dx = 1.11 \, c_L \; x^{-0.1}$$

$$\frac{h_{av}}{h_x} = \frac{1.11 \, c_L \; x^{-0.1}}{c_L \; x^{-0.1}} = 1.11$$

Note that Nu_{av} / Nu_x is also equal to 1.11. However $\quad Nu_{av} \neq \frac{1}{x} \int_0^x Nu_x \, dx$

Example 3



For flow over a flat plate with an extremely rough surface, convection heat transfer effects are known to be correlated by the expression

$$Nu_x = \frac{h_x \, x}{k} = 0.04 \, Re_x^{0.9} \, Pr^{1/3}$$

For air flow at U_∞ = 50 m/s and T_∞ = 300 K, what is the surface shear stress at x = 1 m from the leading edge of the plate?

Hint: Use Chilton-Colburn analogy. $$C_f = \frac{\tau_s}{\rho \, U_\infty^2 \, / \, 2} = \frac{2}{Re_x} \, Nu_x \, Pr^{1/3}$$

Solution

Assumptions: - Modified Reynolds analogy is applicable

- Constant properties

Air properties at 300 K and 1 atm

$$\left.\begin{aligned} &Pr = 0.71 \\ &\rho = 1.16 \text{ kg/m}^3 \\ &\nu = 15.89\ 10^{-6} \text{ m}^2/\text{s} \end{aligned}\right\}$$

Apply Chilton-Colburn analogy:

$$\frac{C_f}{2} = St_x\ Pr^{2/3} = \frac{Nu_x}{Re_x\ Pr}\ Pr^{2/3} = \frac{0.04\ Re_x^{0.9}\ Pr^{1/3}}{Re_x\ Pr}\ Pr^{2/3} = 0.04\ Re_x^{-0.1}$$

$$\mathrm{Re}_x = \frac{U_\infty\ x}{\nu} = \frac{(50)\ (1)}{15.89\ 10^{-6}} = 3.15\ 10^6$$

$$C_f = 0.08\ (3.15\ 10^6)^{-0.1} = 0.0179 = \frac{\tau_s}{\frac{1}{2}\ \rho\ U_\infty^2}$$

Surface Shear Stress:

$$\tau_s = C_f\ \left(\frac{1}{2}\ \rho\ U_\infty^2\right) = (0.0179)\ \left(\frac{1}{2}\ (1.16)\ 50^2\right) = 25.96\ \mathrm{kg/m.s^2} = 25.96\ \mathrm{N/m^2}$$

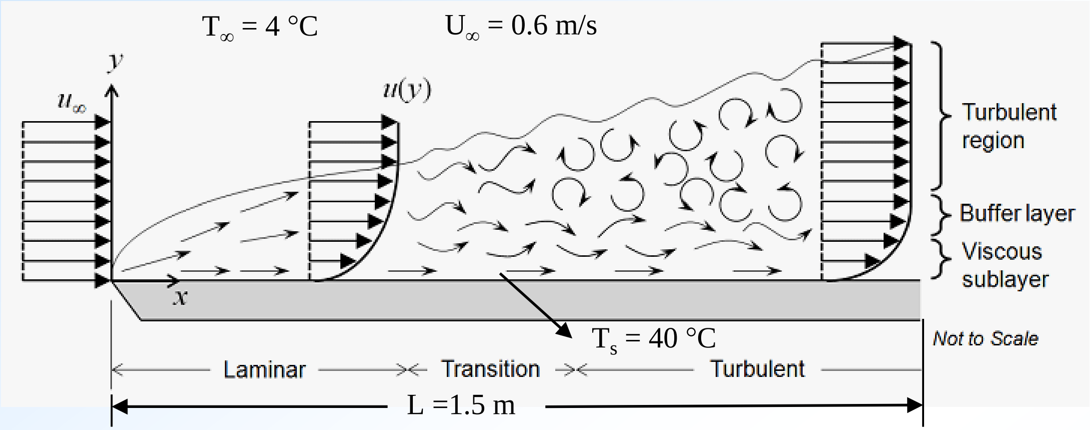
Note that turbulent flow will exist at the designated location

Note the following:

Reynolds Analogy: $$C_f = 2\,St = 2\,\frac{Nu}{Re\,Pr}$$

Colburn Analogy: $$C_f = 2\,St\,Pr^{1/3}$$

Example 4



The surface of a 1.5 m long flat plate is maintained at 40 °C, and water at a temperature of 4 °C and velocity of 0.6 m/s flows over the surface.

(a) Find heat flow rate per unit width of the plate, Q/d, using film temperature, $T_{film} = (T_w + T_\infty)/2$, for evaluating properties;

(b) Find the error in Q/d if T_∞ is used to evaluate the properties and the same correlations;

(c) Find Q/d if the flow is assumed to be turbulent all over the surface.

Solution

Properties of water at (40 + 4) / 2 = 22 °C

$\nu = 0.961\ 10^{-6}\ m^2/s$

$k = 0.606\ W/m.K$

$Pr = 6.62$

(a) $$Re_L = \frac{U_\infty\ L}{\nu} = \frac{(0.6)\ (1.5)}{0.961\ 10^{-6}} = 9.365\ 10^5 > 5\ 10^5 \quad => \quad \text{Flow is mixed}$$

$$\overline{Nu}_L = \frac{h_x\ x}{k} = \left(0.037\ Re_L^{4/5} - 871\right)\ Pr^{1/3} = 2522$$

$$\overline{h}_L = \overline{Nu}_L\ \frac{k}{L} = (2552)\ \frac{0.606}{1.5} = 1019\ W/m^2.K$$

$$\frac{Q}{d} = \overline{h}_L\ (L)\ (T_s - T_\infty) = (1.019)\ (1.5)\ (40 - 4) = 55\ kW/m$$

(b) $Re_L = \dfrac{U_\infty \; L}{\nu} = \dfrac{(0.6)\,(1.5)}{1560\;10^{-6}} = 5.769\;10^5 > 5\;10^5 \quad => \quad$ Flow is still mixed

$$\overline{Nu}_L = \frac{h_x \; x}{k} = \left(0.037\,(5.769\;10^5)^{4/5} - 871\right)(11.44)^{1/3} = 1424$$

$$\bar{h}_L = \overline{Nu}_L \; \frac{k}{L} = (1424)\;\frac{0.577}{1.5} = 575\ \text{W/m}^2\text{.K}$$

$$\frac{Q}{d} = \bar{h}_L \; (L)\;(T_s - T_\infty) = (0.575)\,(1.5)\,(40 - 4) = 31.1\ \text{kW/m}$$

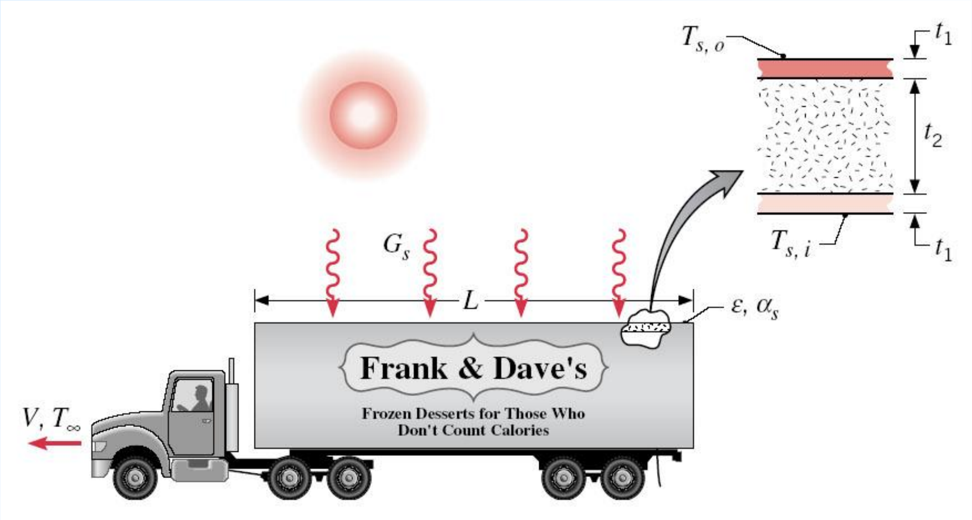
(c) Fully turbulent:

$$\overline{Nu}_L = 0.037\,(9.365\ 10^5)^{4/5}\,(6.62)^{1/3} = 4157$$

$$\overline{h}_L = (4157)\,\frac{0.606}{1.5} = 1679\ \mathrm{W/m^2.K}$$

$$\frac{Q}{d} = \overline{h}_L\,(L)\,(T_s - T_\infty) = 90.7\ \mathrm{kW/m}$$

Example 5



The roof of a refrigerated truck compartment is of composite construction, consisting of a layer of foamed urethane insulation (t_2 = 50 mm, k_i = 0.026 W/m.K) sandwiched between aluminum alloy panels (t_1 = 5 mm, k_p = 180 W/m.K).

The length and width of the roof are L = 10 m and w = 3.5 m, respectively, and the temperature of the inner surface is $T_{s,i}$ = - 10 °C.

Consider conditions for which the truck is moving at a speed of $V = U_\infty$ = 105 km/h, the air temperature is T_∞ = 32 °C, and the solar irradiation is G_s = 750 W/m^2. Turbulent flow may be assumed over the entire length of the roof.

(a) For equivalent values of the solar absorptivity and of the emissivity of the outer surface ($\alpha_s = \varepsilon = 0.5$), estimate the average temperature $T_{s,o}$, of the outer surface. What is the corresponding heat load imposed on the refrigeration system?

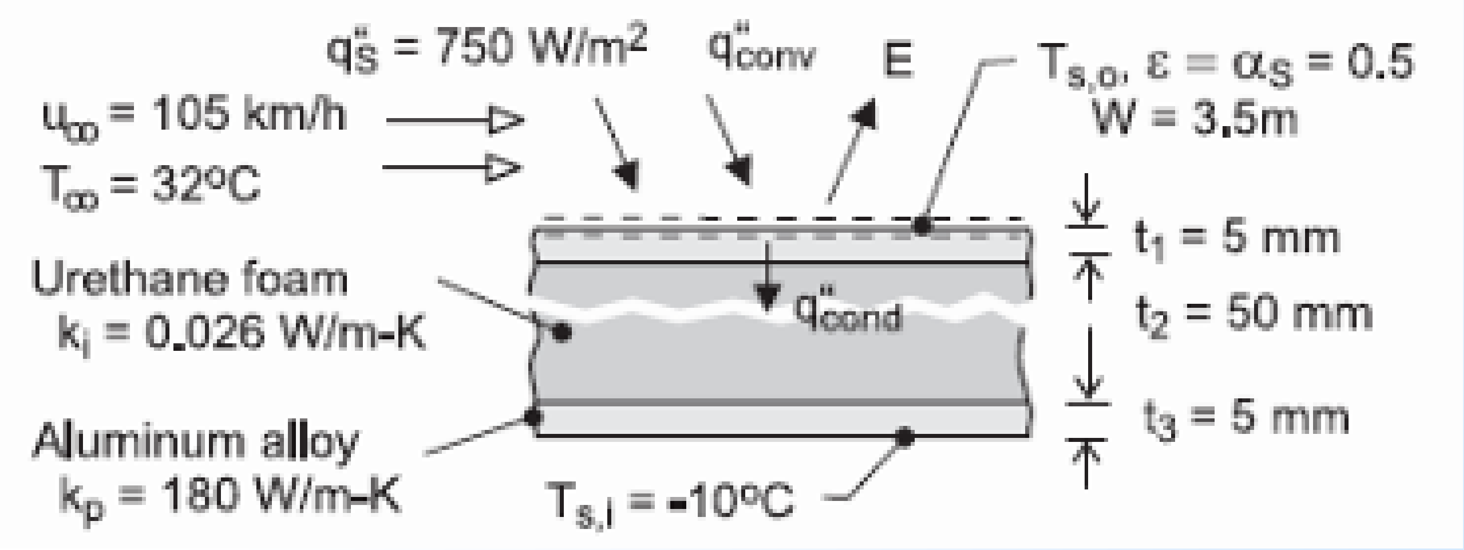
(b) A special finish ($\alpha_s = 0.15$, $\varepsilon = 0.5$) may be applied to the outer surface. What effects would such an application have on the surface temperature and the heat load?

(c) If, with $\alpha_s = \varepsilon = 0.5$, the roof is not insulated ($t_2 = 0$), what are the corresponding values of the surface temperature and the heat load?

Make the following assumptions: Negligible radiation from the sky; Turbulent flow over the entire surface; Constant properties.

Hint: Apply first law of thermodynamics (energy balance) to the outer surface:

$$\alpha_s\ G_s + h_{av}\ (T_\infty - T_{s,o}) - \varepsilon\ \sigma\ T_{s,o}^4 = \frac{T_{s,o} - T_{s,i}}{R_{tot}}$$



(a) Form an energy balance for the outer surface:

$$\alpha_s\ G_s + q''_{conv} - E = q''_{cond} = \frac{T_{s,o} - T_{s,i}}{R''_{tot}}$$

$$\alpha_s\ G_s + h_{av}\ (T_\infty - T_{s,o}) - \varepsilon\ \sigma\ T_{s,o}^4 = \frac{T_{s,o} - T_{s,i}}{2\ R''_p + R''_i}$$

where $R''_p = \frac{t_1}{k_p} = 2.78\ 10^{-5}\ m^2.K/W$ $\quad R''_i = \frac{t_2}{k_i} = 1.923\ 10^{-5}\ m^2.K/W$

and $Re_L = \frac{U_\infty\ L}{\nu} = \frac{(29.2)\ (10)}{15.89\ 10^{-6}} = 1.84\ 10^7$

$$\bar{h} = \frac{k}{L}\ 0.037\ Re^{4/5}\ Pr^{1/3} = \frac{0.0263}{10}\ 0.037\ (1.84\ 10^7)^{4/5}\ (0.707)^{1/3} = 56.2\ W/m^2.K$$

$$\alpha_s \; G_s + h_{av} \; (T_\infty - T_{s,o}) - \varepsilon \; \sigma \; T_{s,o}^4 = \frac{T_{s,o} - T_{s,i}}{2 \, R_p'' + R_i''}$$

$$(0.5)\;(750) + (56.2)\;(305 - T_{s,o}) - (0.5)\;(5.67\;10^{-8}) = \frac{T_{s,o} - 263}{5.67\;10^{-5} + 1.923}$$

Solve for $T_{s,o}$: $T_{s,o} = 306.8 \text{ K} = 33.8\ ^\circ\text{C}$

Heat Load: $Q = (W)\;(L)\;q''_{cond} = (3.5)\;(10)\;\dfrac{33.8 + 10}{1.923} = 797 \text{ W}$

With special surface finish (α_s = 0.15 and ε = 0.8):

$T_{s,o}$ = 301.1 K = 27.1 °C

Q = 675.3 W

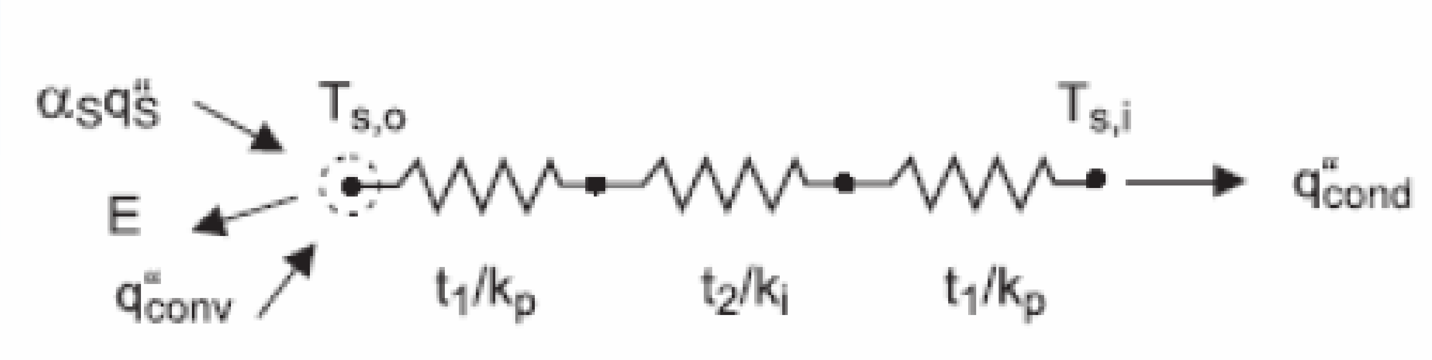
Without the insulation, t_2 = 0 and $\alpha_s = \varepsilon$ = 0.5

$T_{s,o}$ = 263.1 K = - 9.9 °C

Q = 90 630 W

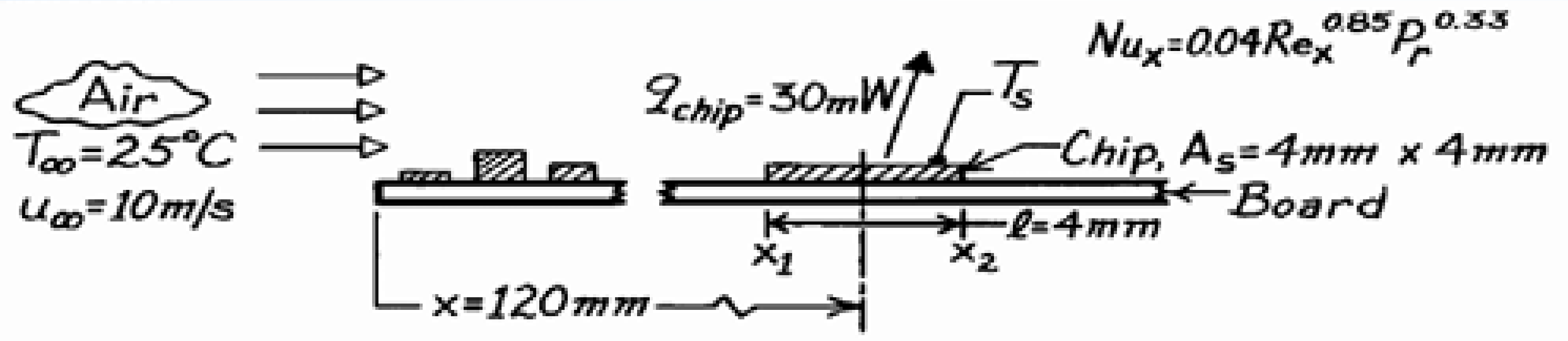
Comments:

1. Use of special surface finish reduces the solar input while increasing radiaton emission from the surface. The cumulative effect is to reduce the heat load by 15 %.
2. The thermal resistance of the aluminum panels is negligible and without the insulation the heat load is enoumous.



Example 6

A chip mounted on a circuit board experiences forced air cooling with prescribed temperature and velocity as shown in the Figure. The chip with the given area is located at a given distance from the leading edge.



Find the surface temperature of the chip when the heat dissipation rate is 30 mW.

Assume 1. Parallel flow over a flat plate with the given correlation

2. Heat is lost from the top surface of the chip only

Assume that the surface temperature is T_s = 45 °C. This is to be checked (and iterated if necessary).

Properties of air at (45 + 25) °C / 2 = 310 K

$$\left.\begin{array}{l} k = 0.027 \text{ W/m.K} \\ \nu = 16.90\ 10^{-6} \text{ m}^2\text{/s} \\ Pr = 0.706 \end{array}\right\}$$

Heat flow rate at the upper surface of the chip:

$$q_{chip} = \bar{h}_{chip}\, A_s\, (T_s - T_\infty) \quad \text{or} \quad T_s = T_\infty + \frac{q_{chip}}{\bar{h}_{chip}\, A_s}$$

Assume that the average convection coefficient over the chip length is equal to the local value at the cntre of the chip:

$$\bar{h}_{chip} \cong h_x = \frac{Nu_x\, k}{x} \quad \text{at } x = 120 \text{ cm}$$

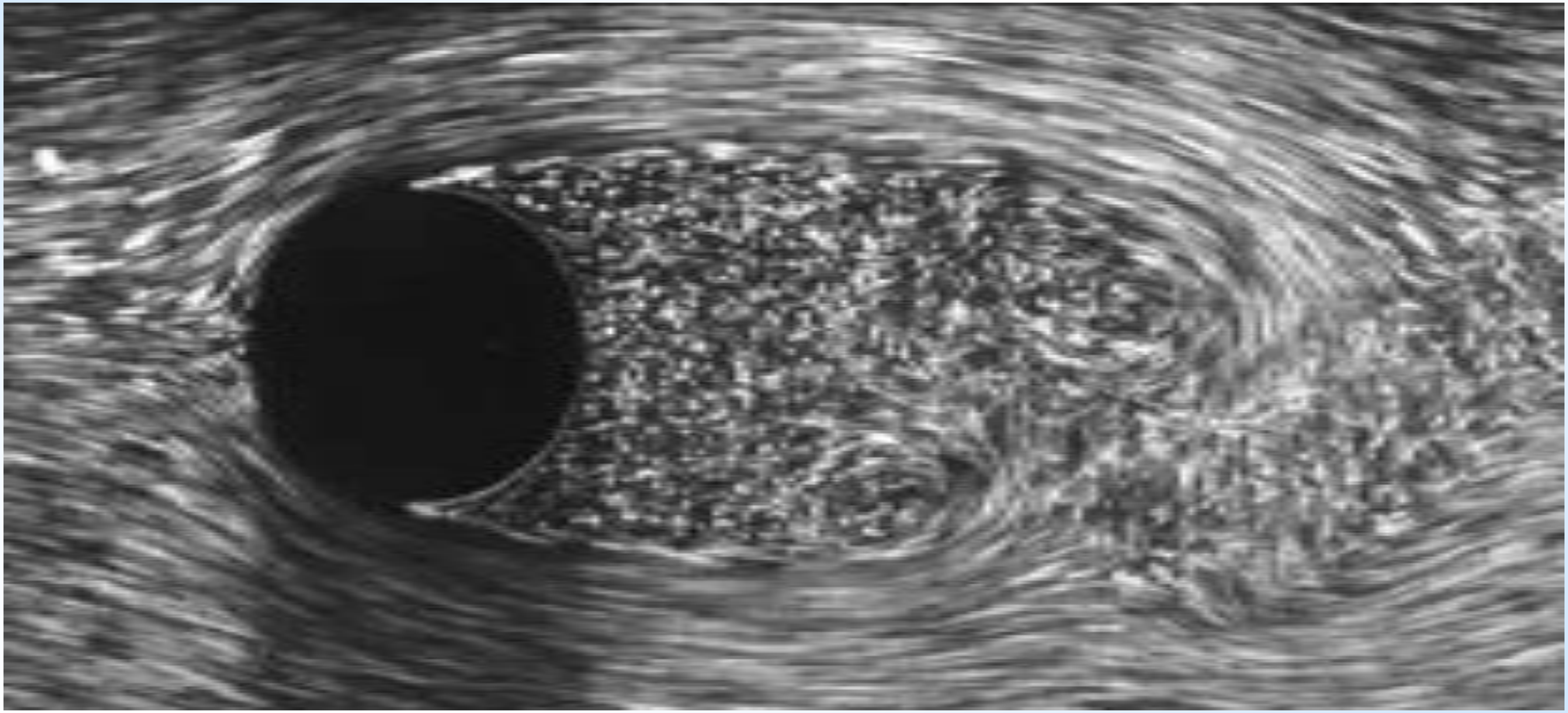
$$Nu_x = 0.04\ Re_x^{0.85}\ Pr^{0.33} = 0.04\ \frac{(10)\ (0.12)}{16.90\ 10^{-6}}\ (0.706)^{0.33} = 473.4$$

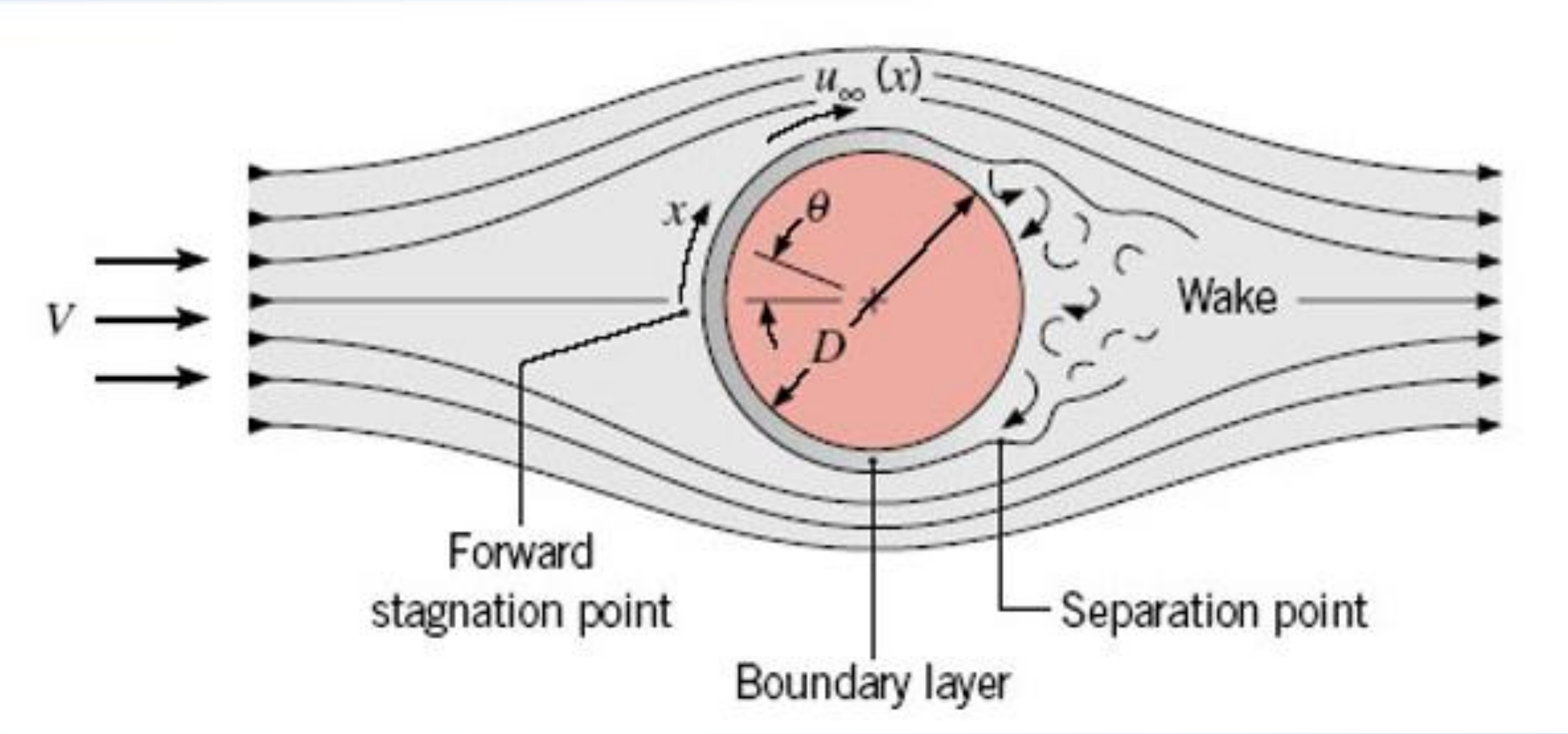
$$\bar{h}_{chip} \cong h_x = \frac{Nu_x\ k}{x} = \frac{(473.4)\ (0.027}{0.120} = 107\ W/m^2.K$$

$$T_s = T_\infty + \frac{q_{chip}}{\bar{h}_{chip}\ A_s} = 25 + \frac{30\ 10^{-3}}{(107)\ (4\ 10_{-3})^2} = 25 + 17.5 = 42.5\ ^\circ C$$

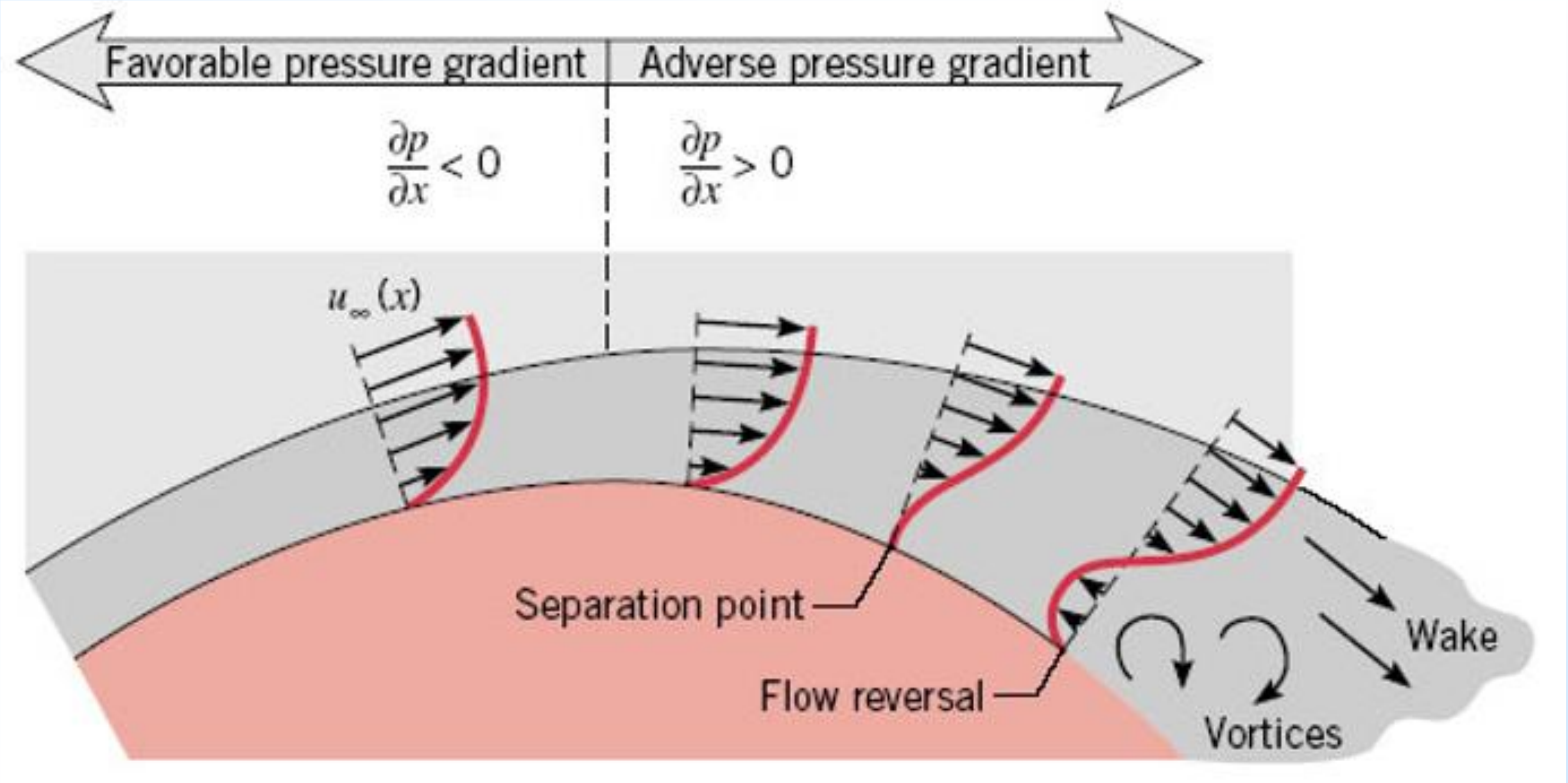
Note that the assumed value of the average surface temperature is reasonable.

Flow Across Cylinders and Spheres





Boundary layer formation on a circular cylinder in cross flow



Velocity profile associated with **separation** on a circular cylinder in cross flow

Flow across cylinders and spheres is frequently encountered in many heat transfer systems

– shell-and-tube heat exchangers;

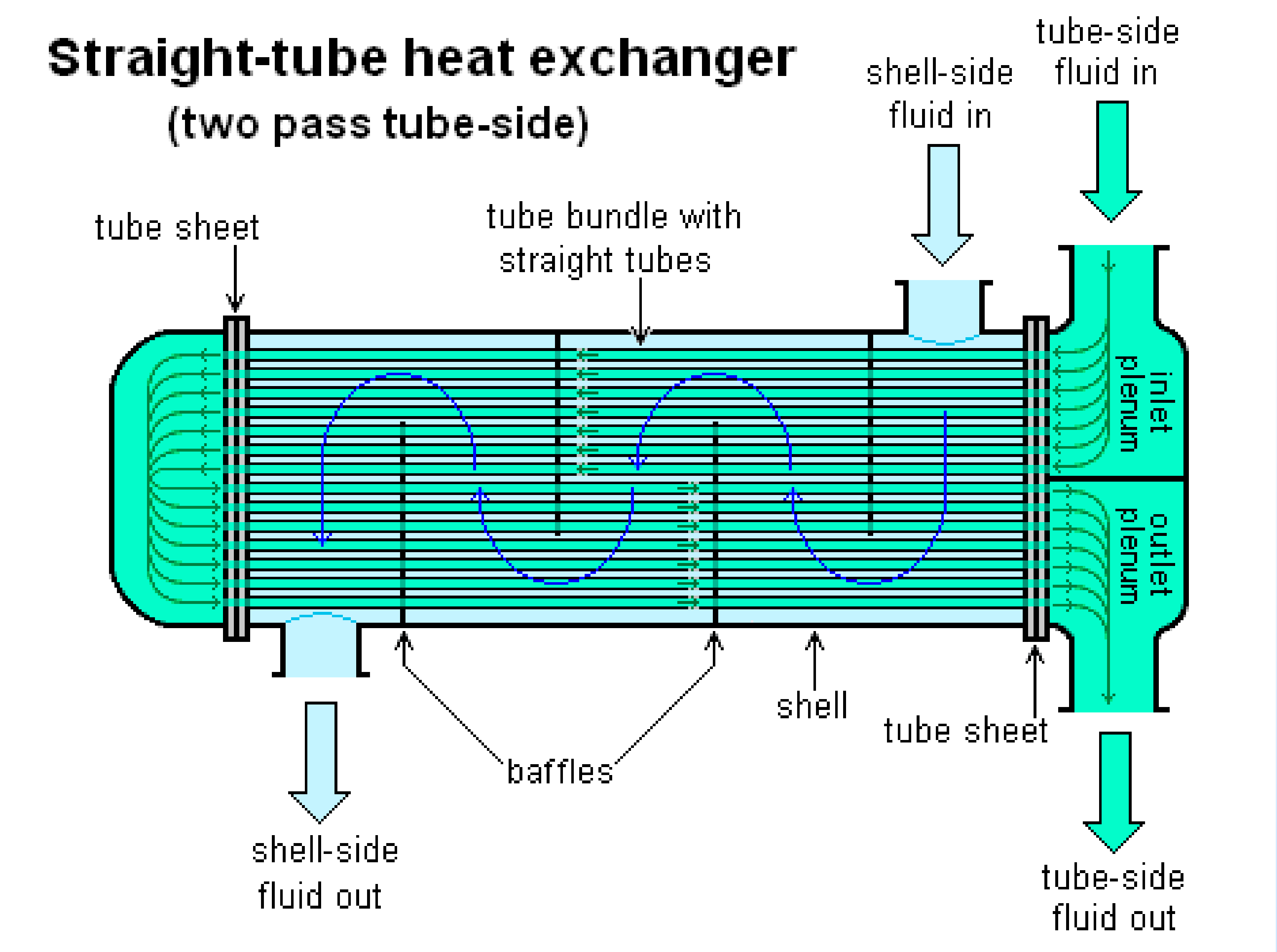
– Pin-fin heat sinks for electronic cooling.

The characteristic length for a circular cylinder or sphere is taken to be the *external (outer) diameter, D.*

$$Re_D = \frac{U_\infty D}{\nu} = \frac{\rho U_\infty D}{\mu}$$

The **critical Reynolds number for flow across a circular cylinder** or sphere is about $Re_{cr} = 2\ 10^5$.

Cross-flow over a cylinder exhibits complex flow patterns depending on the Reynolds number.


Straight-tube heat exchanger
(two pass tube-side)
shell-side fluid in
tube-side fluid in
tube sheet
tube bundle with straight tubes
inlet plenum
outlet plenum
shell
tube sheet
baffles
shell-side fluid out
tube-side fluid out

Pin Fin Heat Sinks

At very low upstream velocities (Re ≤ 1), the fluid completely wraps around the cylinder.

At higher velocities the boundary layer detaches from the surface, forming a separation region behind the cylinder.

Flow in the wake region is characterized by periodic vortex formation and low pressures.

The nature of the flow across a cylinder or sphere strongly affects the total drag coefficient C_D.

At low Reynolds numbers (Re < 10) — friction drag dominates.

At high Reynolds numbers (Re > 5000) — pressure drag dominates.

At intermediate Reynolds numbers — both pressure and friction drags are significant.

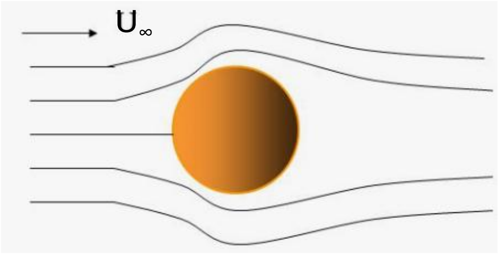
The Drag Coefficient, C_D, is a dimensionless quantity that is used to quantify the drag or resistance of an object in a fluid environment, such as air or water.

$$C_D = \frac{\text{Drag Force} / A}{\frac{1}{2}\rho U_\infty^2}$$

ρ is the density of the fluid

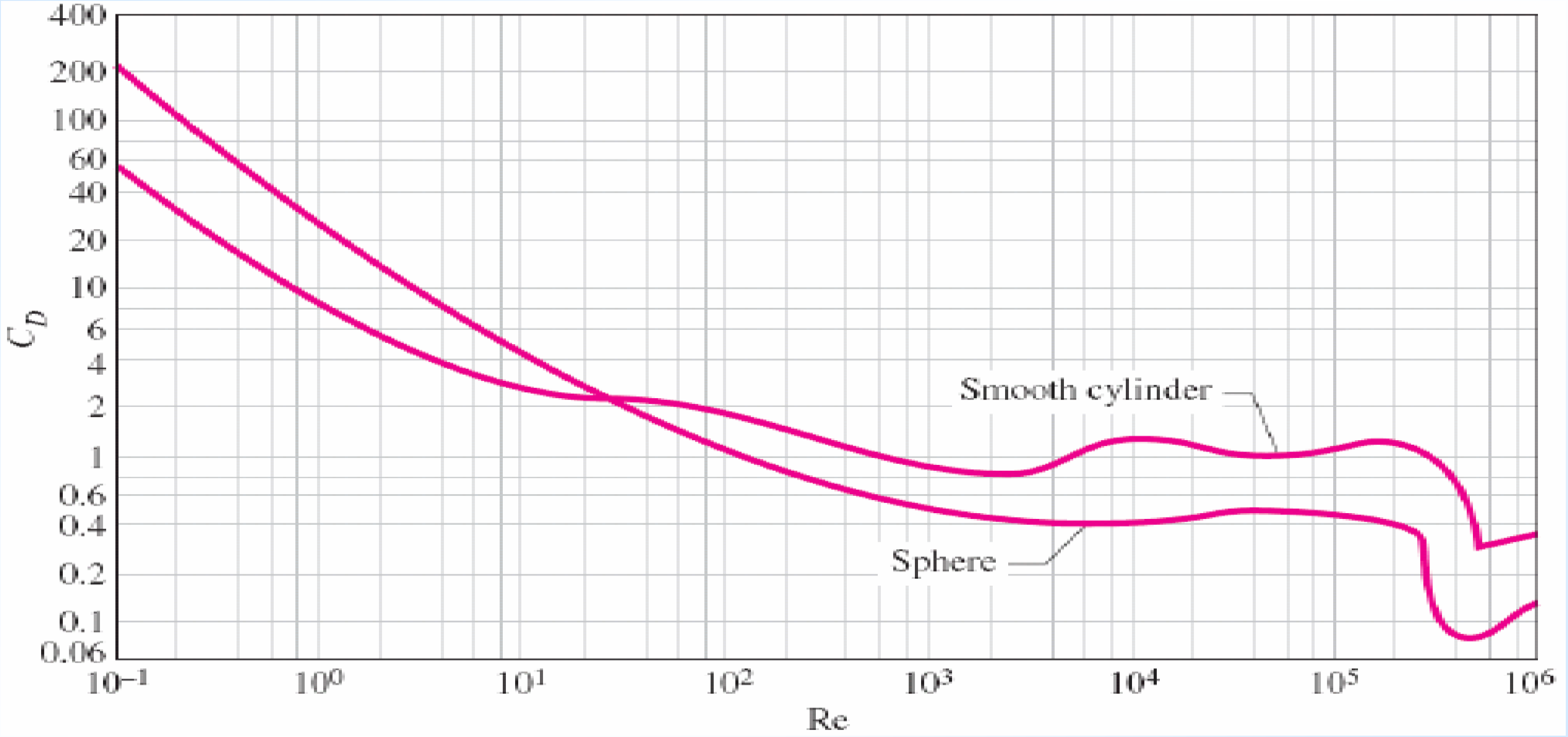
U_∞ is the fluid velocity away from the object

A is the reference area



The reference area, A, depends on what type of drag coefficient is being measured. For many objects, the reference area is the projected frontal area, $\pi D^2/4$ for a sphere, D L for a cyclinder, where pressure drag is dominant. It can be the suface area when friction drag is dominant such as an air foils.

Average C_D for circular cylinders and spheres



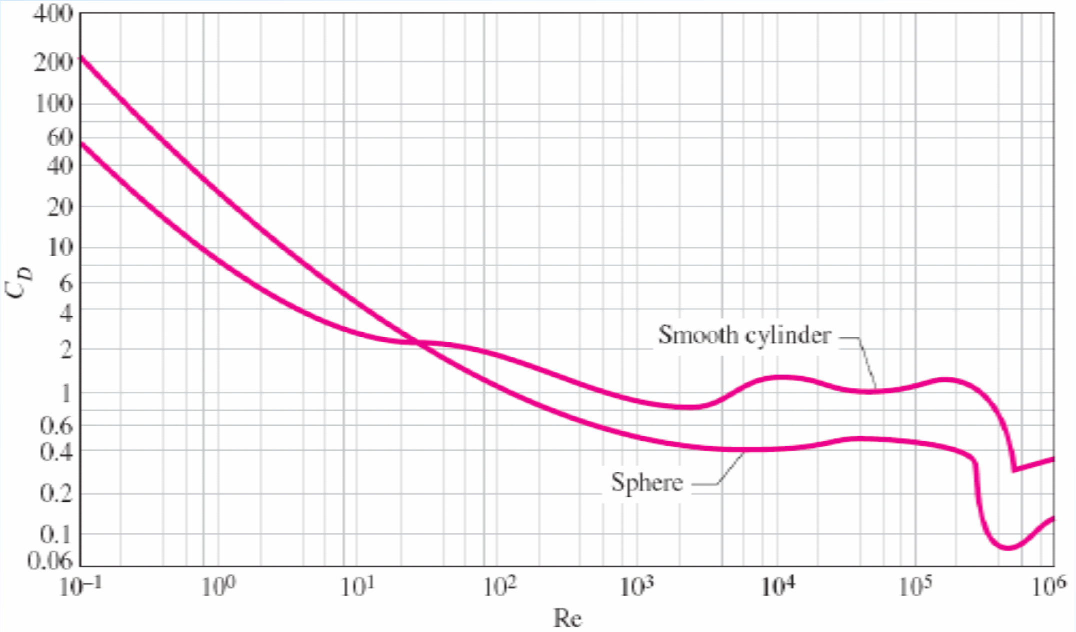
Completely Laminar
Drag $\rightarrow U_\infty$

Vortices form

Inertial force dominates
Drag $\rightarrow U^2_\infty$

Turbulence
Separation

Average C_D for circular cylinders and spheres



Re ≤ 1 — creeping flow

Re ≈ 10 — separation starts

Re ≈ 90 — vortex shedding starts.

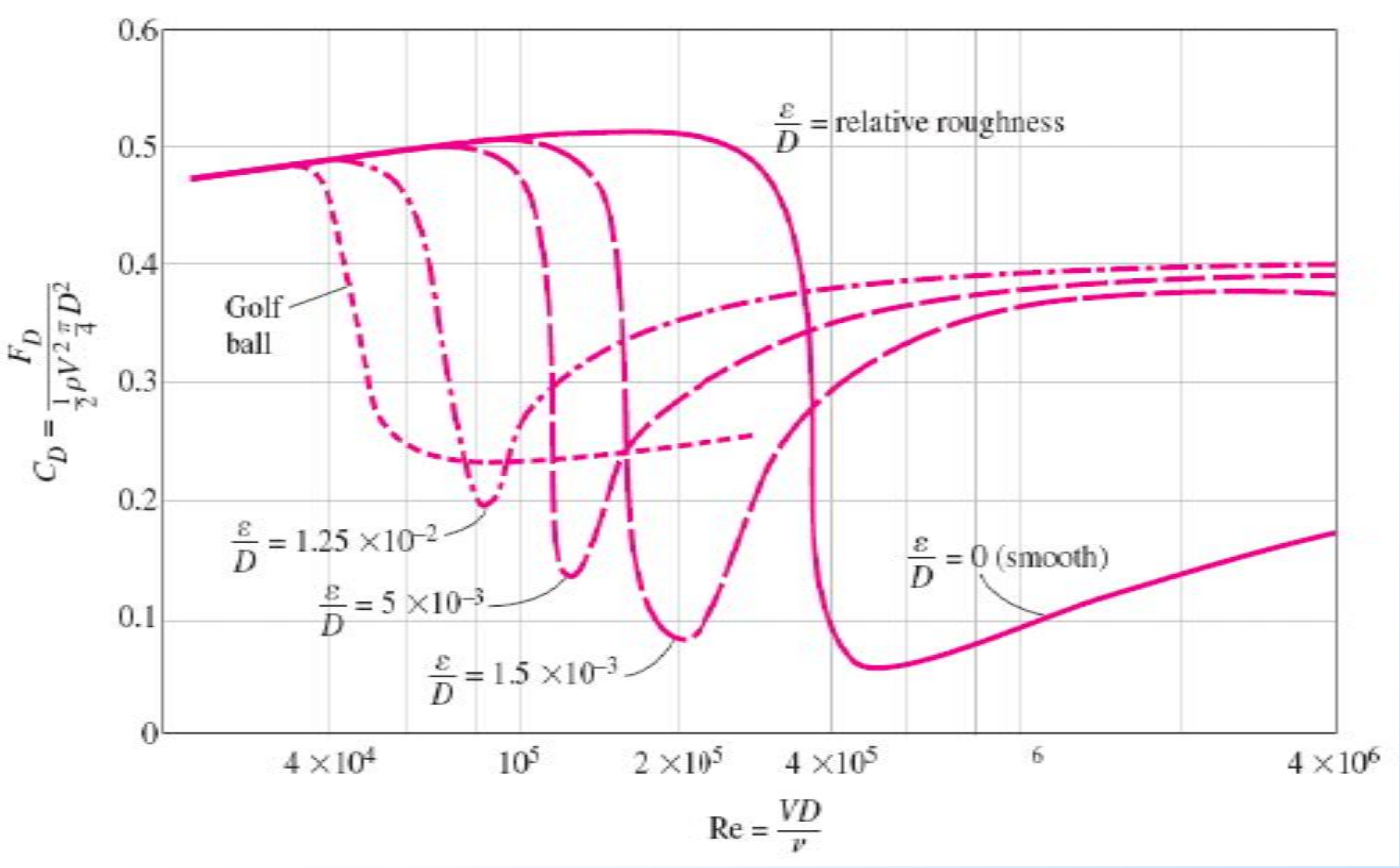
10^3 < Re < 10^5

- – in the boundary layer, flow is laminar
- – in the separated region, flow is highly turbulent

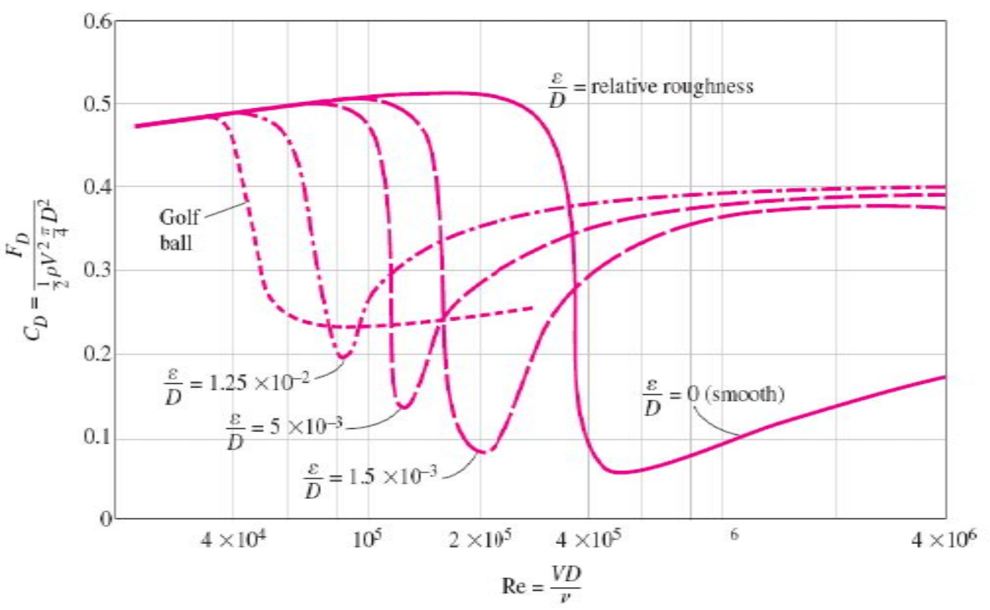
10^5 < Re < 10^6

- — turbulent flow

Effect of Surface Roughness (sphere)



Effect of Surface Roughness



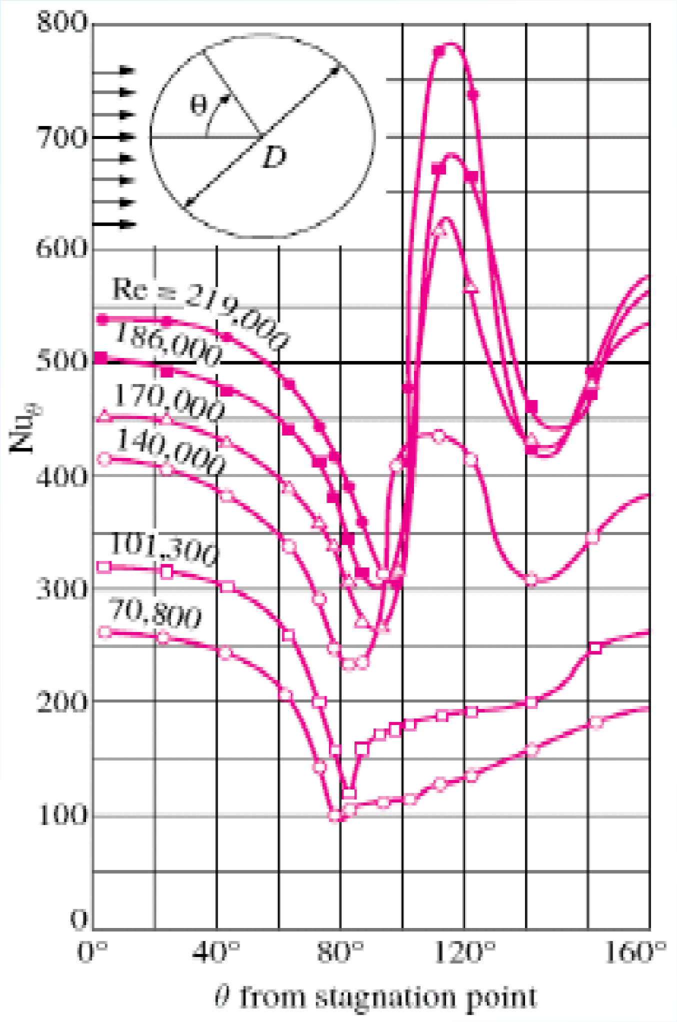
Surface roughness, in general, increases the drag coefficient in turbulent flow.

This is especially the case for streamlined bodies.

For blunt bodies such as a circular cylinder or sphere, however, an increase in the surface roughness may actually *decrease the* drag coefficient.

This is done by tripping the boundary layer into turbulence at a lower Reynolds number, causing the fluid to close in behind the body, narrowing the wake and reducing pressure drag considerably.

Heat Transfer Coefficient



Flows across cylinders and spheres, in general, involve *flow separation,* which is difficult to handle, analytically.

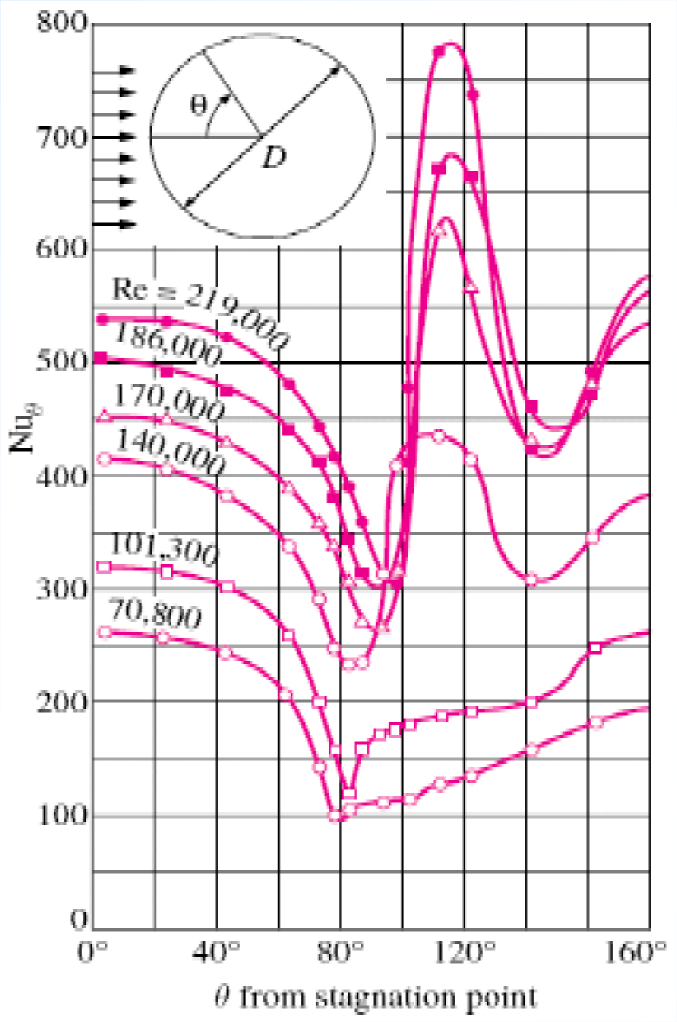
The local Nusselt number, Nu_θ, around the periphery of a cylinder subjected to cross flow varies considerably.

Small θ — Nu_θ decreases with increasing θ as a result of the thickening of the laminar boundary layer.

80º < θ < 90º — Nu_θ reaches a minimum

- low Reynolds numbers — due to separation in laminar flow
- high Reynolds numbers — transition to turbulent flow.

Heat Transfer Coefficient



θ > 90º laminar flow –

Nu_θ increases with increasing θ due to intense mixing in the separation zone.

90º < θ < 140º turbulent flow –

Nu_θ decreases due to the thickening of the boundary layer.

θ ≈ 140º turbulent flow –

Nu_θ reaches a second minimum due to flow separation point in turbulent flow.

Average Heat Transfer Coefficient

For flow over a cylinder (Churchill and Bernstein):

$$Nu_{cyl} = \frac{h\,D}{k} = 0.3 + \frac{0.62\,Re^{1/2}\,Pr^{1/3}}{\left[1 + \left(0.4/Pr\right)^{2/3}\right]^{1/4}} \left[1 + \left(\frac{Re}{282000}\right)^{5/8}\right]^{4/5}$$

Evaluate fluid properties at the film temperature: $T_f = \dfrac{T_w + T_\infty}{2}$

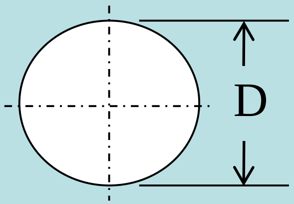
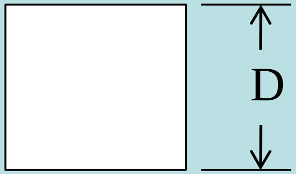
For flow over a sphere (Whitaker):

$$Nu_{sph} = \frac{h\,D}{k} = 2 + \left[0.4\,Re^{1/2} + 0.06\,Re^{2/3}\right] Pr^{1/3} \left(\frac{\mu_\infty}{\mu_w}\right)^{1/4}$$

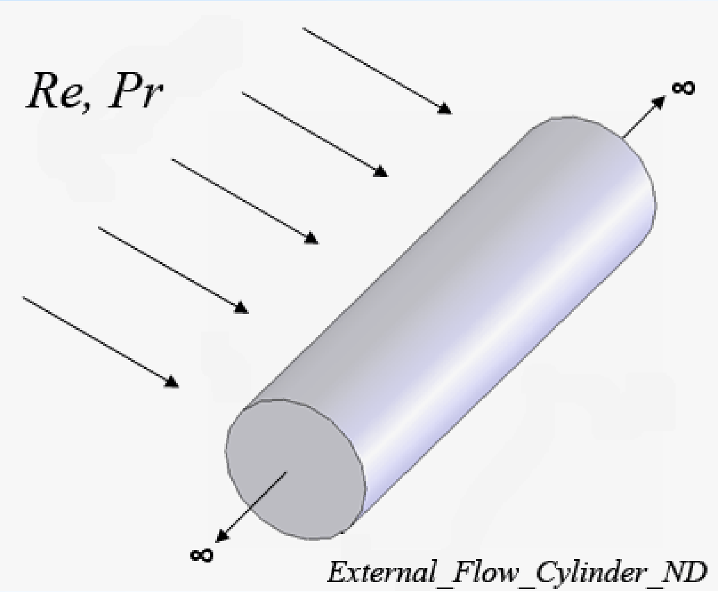
Both correlations are accurate within ± 3 %.

A more compact correlation for flow across cylinders $Nu_{cyl} = \dfrac{h\,D}{k} = C\,Re^{m}\ Pr^{1/3}$

Experimentally determined constants are given in the Table.

Cross section of the cylinder	Fluid	Range of Re	C	m
Circle 	Gas or Liquid	0.4 – 4	0.989	0.33
		4 – 40	0.911	0.385
		40 – 4000	0.683	0.466
		4000 – 40 000	0.193	0.618
		40 000 – 400 000	0.027	0.805
Square 	Gas	5000 – 100 000	0.102	0.675

Example 7



Water at 10 °C with a free stream velocity U_∞ = 1.5 m/s flows across a single cylinder of 2.5 cm outer diameter (OD) whose surface is kept at 60 °C.

(a) Find the drag force per unit length of the tube;

(b) Determine h_{av} and Q per unit length of the tube.

Solution

(a) Drag Force = $F_D = C_D \dfrac{1}{2} \rho U_\infty^2 A_F$

Properties of water at $\dfrac{T_w + T_\infty}{2} = \dfrac{60 + 10}{2} = 35\ ^\circ C$

$$\left.\begin{aligned} c_p &= 4.174\ kJ/kg.^\circ C \\ \rho &= 994\ kg/m^3 \\ \mu &= 7.2\ 10^{-4}\ kg/m.s \\ k &= 0.626\ W/m.^\circ C \\ Pr &= 4.8 \end{aligned}\right\}$$

$$Re_D = \frac{U_\infty D}{\mu / \rho} = \frac{(1.5)\,(0.025)\,(994)}{7.2\ 10^{-4}} = 5.18\ 10^4$$

For this Reynolds number, read from the Figure in the text: $C_D \approx 1.3$

$$\frac{F_D}{L} = C_D\ \frac{1}{2}\ \rho\ U_\infty^2\ D = (1.3)\ \frac{1}{2}\ (994)\ (1.5)^2\ (0.025) = 36.3\ N/m$$

(b) $\overline{Nu} = \dfrac{\bar{h}\, D}{k_f} = C\, Re^n\, Pr^{1/3}$

For this Nusselt number, read from the Table in the text: $\left.\right\}$ $C = 0.027$, $n = 0.805$

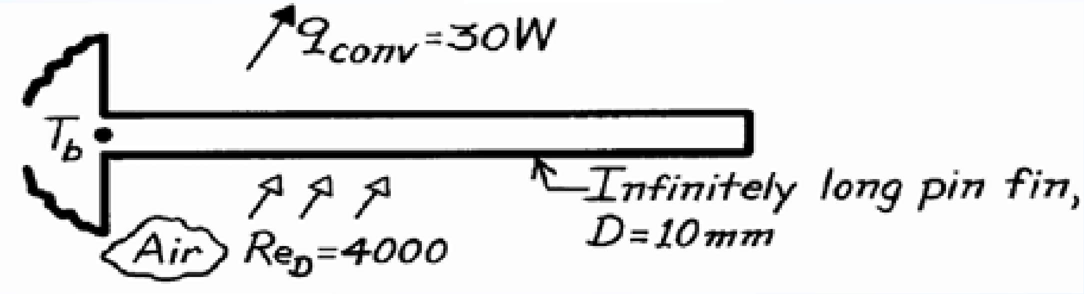
$$\bar{h} = \frac{k_f}{D}\, C\, Re^n\, Pr^{1/3} = \frac{0.626}{0.025}\,(1.3)\,(5.18\ 10^4)^{0.805}\,(4.8)^{1/3}$$

$$\cong 7000\ W/m^2.K$$

$$\frac{Q}{L} = \bar{h}\,\pi\, D\,(T_w - T_\infty) = (7000)\,\pi\,(0.025)\,(60 - 10) \cong 27500\ W/m$$

Example 8

A pin fin of 10 mm diameter dissipates 30 W by forced convection to air in cross flow with a Reynolds number of 4000. If the diameter of the fin is doubled and all other conditions remain the same, estimate the heat flow rate from the fin. Assume that the fin is infinitely long.



Assumptions: - Pin behaves as infinitely long fin

- Conditions of flow as well as base and air temperatures remain the same for both situations
- Negligible radiation heat heat transfer

For an infinitely long pin fin, the fin heat rate is:

$$Q_{fin} = Q_{conv} = (\bar{h}\, P\, k\, A_c)^{1/2}\ \theta_b \qquad \text{where } P = \pi D \text{ and } A_c = \pi D^2/2$$

Hence $\quad Q_{conv} \propto (\bar{h}\, D\, D^2)^{1/2}$

For forced convection cross-flow over a cylinder, an approximate correlation for estimating the dependence of h on the diameter is:

$$\overline{Nu}_D = \frac{\bar{h}\, D}{k} = C\, Re_D^m\, Pr^{1/3} = C \left(\frac{V\, D}{\upsilon}\right)^m Pr^{1/3}$$

Using a table in the text book, for $Re_D = 4000$, find $m = 0.466$

Therefore $\quad \bar{h} \propto D^{-1}\, D^{0.466} = D^{-0.534}$

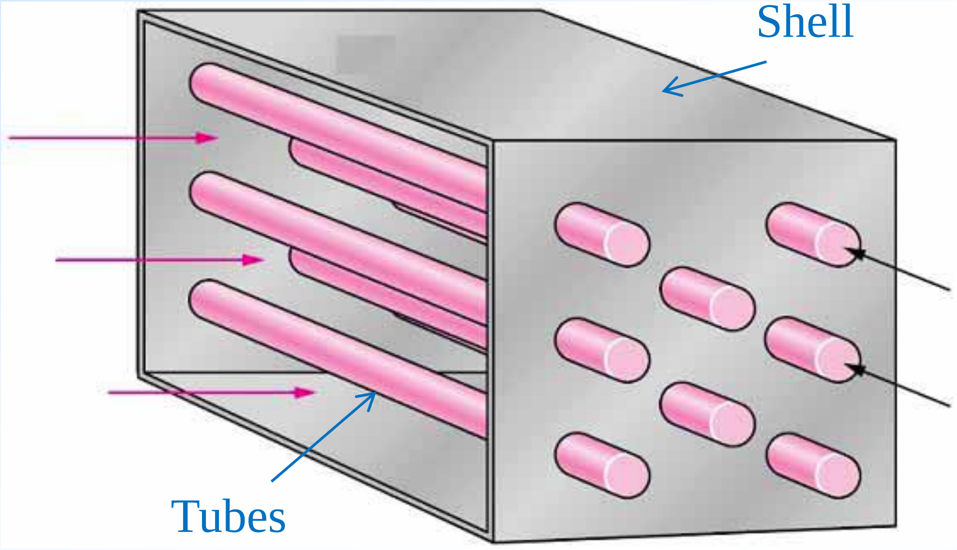
Substitute: $Q_{conv} \propto (\bar{h}\, D\, D^2)^{1/2} = (D^{-0.534}\, D\, D^2)^{1/2} = D^{1.23}$

Hence, with Q_1 -> D_1 = 10 mm and Q_2 -> D_2 = 20 mm find

$$Q_2 = Q_1 \left(\frac{D_2}{D_1}\right)^{1.23} = 30 \left(\frac{20}{10}\right)^{1.23} = 70.4 \text{ W}$$

The effect of doubling the diameter, with all other conditions remaining the same, is to increase the fin rate heat by a factor of 2.35. The effect is nearly linear, with enhancements due to the increase in surface and cross-sectional areas ($D^{1.5}$) exceeding the attenuation due to a decrease in the heat transfer coefficient ($D^{-0.267}$). Note that, with increasing Reynolds number, the exponent m increases and there is greater heat transfer enhancement due to increasing the diameter.

Flow Across Tube Banks

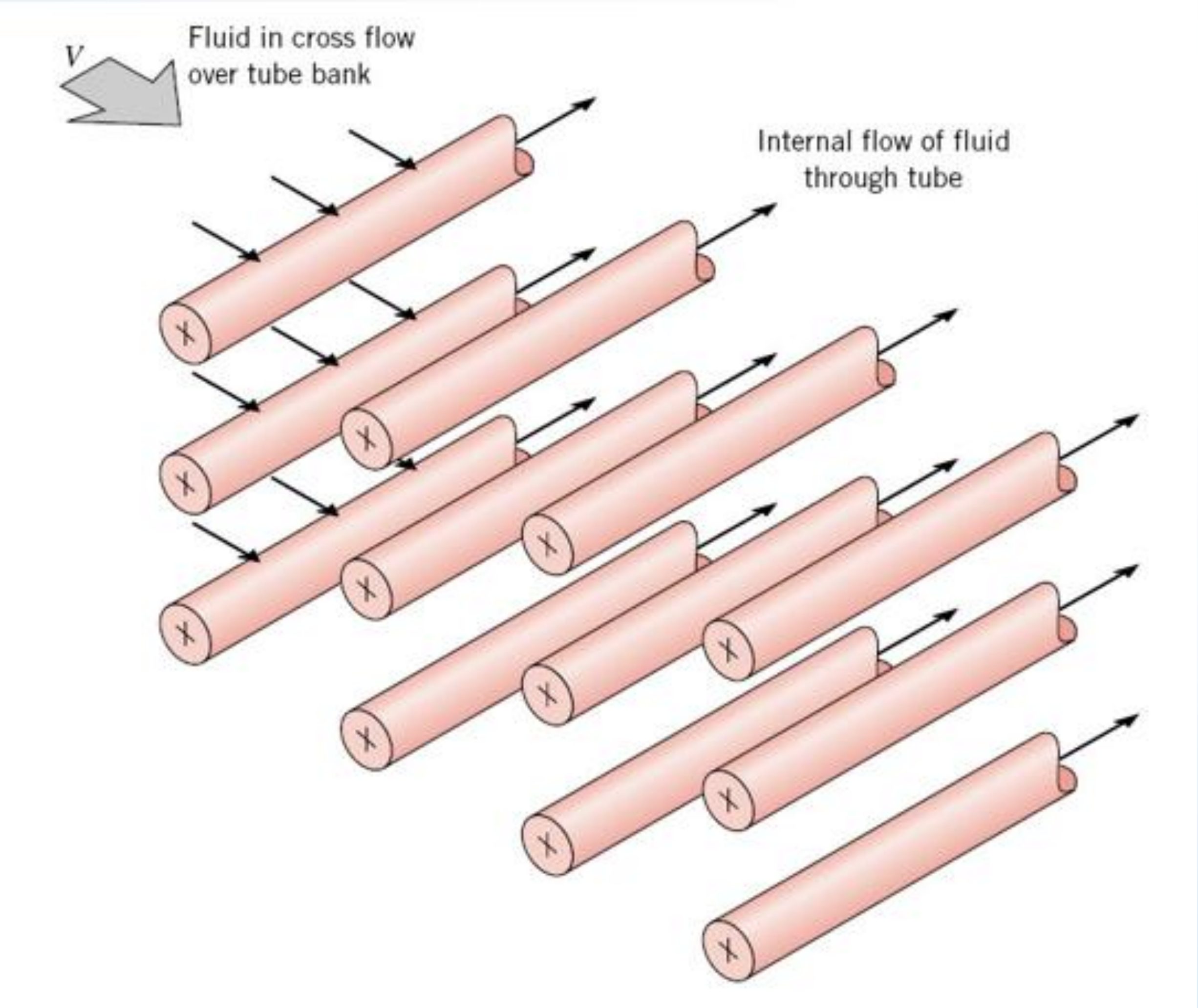


Cross-flow over tube banks is commonly encountered in practice in heat transfer equipment such as heat exchangers.

In such equipment, one fluid moves through the tubes while the other moves over the tubes in a perpendicular direction.

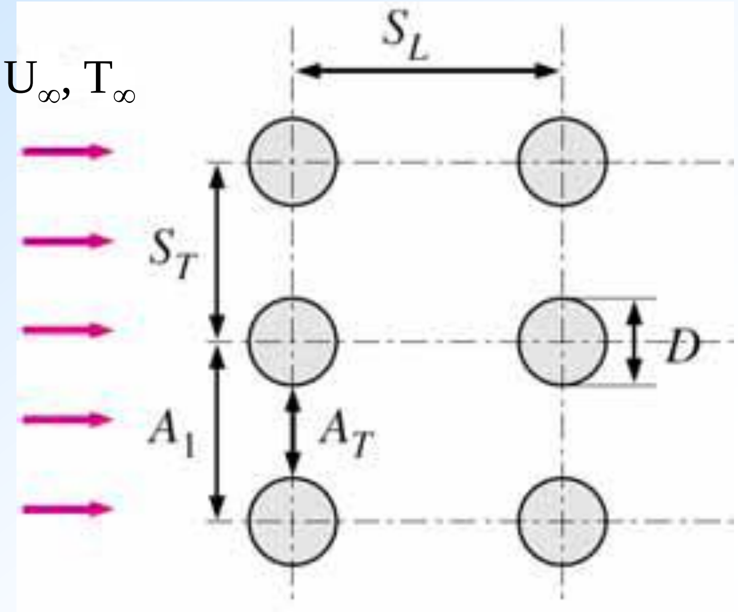
Flow *through the tubes can be analyzed by considering* flow through a single tube, and multiplying the results by the number of tubes.

For flow *over the tubes the tubes affect the flow pattern* and turbulence level downstream, and thus heat transfer to or from them are altered.

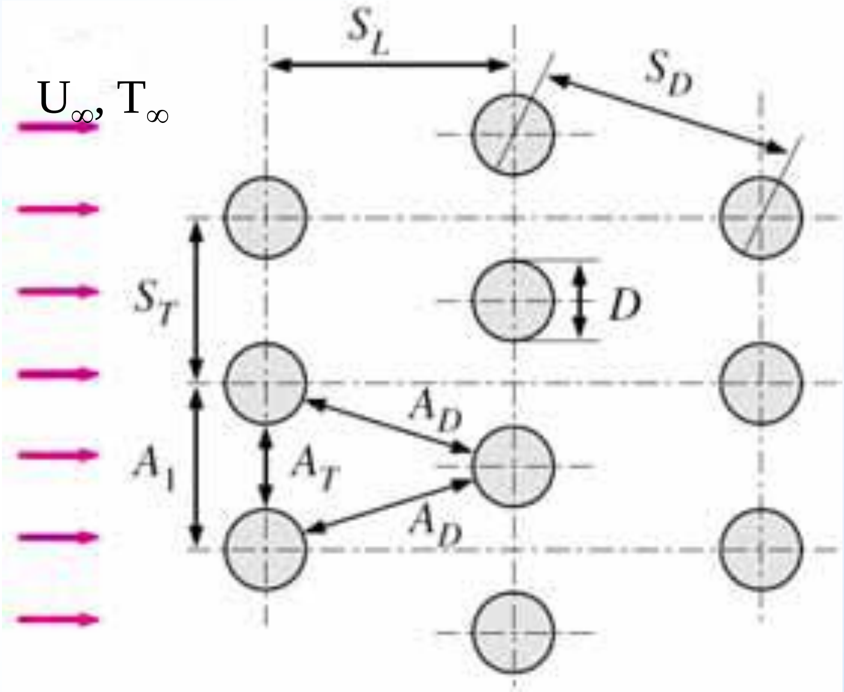


Schematic of a tube bank in cross flow

Typical Arrangement of Tubes



In-line



Staggered

D is the characteristic length

S_L : Longitudinal pitch S_T : Transverse pitch S_D : Diagonal pitch

As the fluid enters the tube bank, the flow area decreases from $A_1 = S_T L$ *to* $A_T = (S_T - D) L$ *between the tubes, and thus* flow velocity increases.

In tube banks, the flow characteristics are dominated by the maximum velocity U_{max}.

The Reynolds number is defined on the basis of maximum velocity as

$$Re_D = \frac{\rho \, U_{max} \, D}{\mu} = \frac{U_{max} \, D}{\nu}$$

For ***in-line arrangement, the maximum velocity occurs at*** the minimum flow area between the tubes

$$U_{max} = \frac{S_T}{S_T - D} U_\infty$$

In ***staggered arrangement***

For $S_D > \dfrac{S_T + D}{2}$ $\qquad U_{max} = \dfrac{S_T}{S_T - D} U_\infty$

For $S_D < \dfrac{S_T + D}{2}$ $\qquad U_{max} = \dfrac{S_T}{2\left(S_D - D\right)} U_\infty$

The nature of flow around a tube in the first row resembles flow over a single tube.

The nature of flow around a tube in the second and subsequent rows is very different.

The level of turbulence, and thus the heat transfer coefficient, increases with row number.

There is no significant change in turbulence level after the first few rows, and thus the heat transfer coefficient remains constant.

Zukauskas has proposed correlations whose general form is

$$Nu_D = \frac{h_{av}\ D}{k} = C\ Re^m\ Pr^n \left(\frac{Pr}{Pr_s}\right)^{0.25}$$

where the values of the constants *C, m, and n depend on* Reynolds number.

The average Nusselt number relations given in the Table are for tube banks with N_L = 16 or more rows.

Those relations can also be used for tube banks with N_L < *16 provided* that they are modified as

$$Nu_{D,N_L} = F\ Nu_D$$

The correction factor, F values, are given in the following table.

Nussel number correlations for cross flow over tube banks for N > 16 and

0.7 < Pr < 500 (from Zukauskas, 1987)

Arrangement	Range of Re	C	m	n
In-line	0 – 100	0.9	0.4	0.36
	100 – 1000	0.52	0.5	0.36
	1000 – 2 10^5	0.27	0.63	0.36
	2 10^5 – 2 10^6	0.033	0.8	0.4
Staggered	0 – 500	1.04	0.4	0.36
	500 - 1000	0.71	0.5	0.36
	1000 – 2 10^5	0.35	0.6	0.36
	2 10^5 – 2 10^6	0.031	0.8	0.36
All properties except Pr are to be evaluated at $(T_{in}+T_{out})/2$. Pr is to be evaluated at T_s. Last two equations has a another factor $(S_T/S_L)^{0.2}$				

Correction factor, F, to be used for N_L < 16 and Re_D > 1000

(from Zukauskas, 1987)

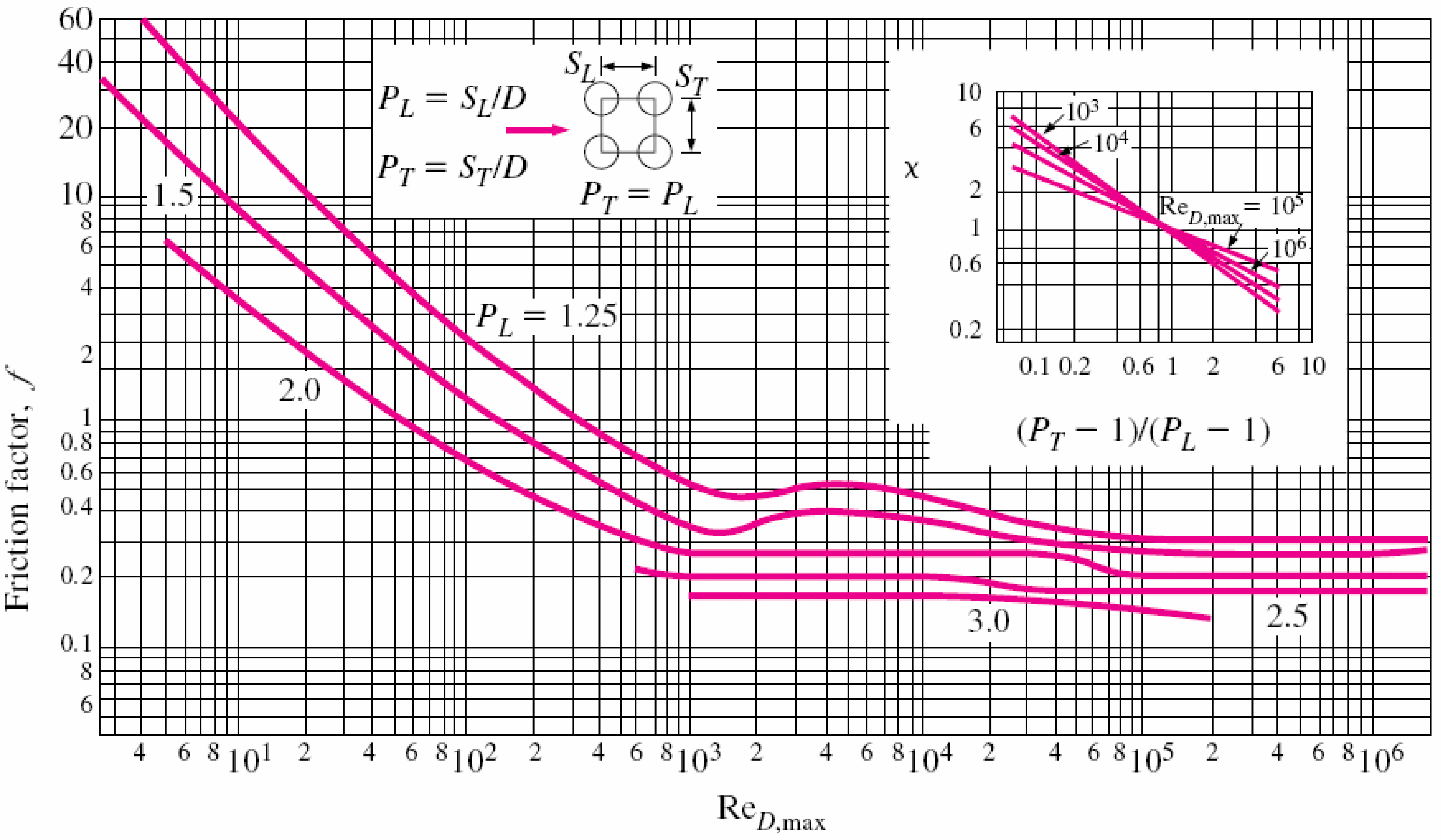
N_L	1	2	3	4	5	7	10	13
In,line	0.7	0.8	0.86	0.90	0.93	0.96	0.98	0.99
Staggered	0.64	0.76	0.84	0.89	0.93	0.96	0.98	0.99

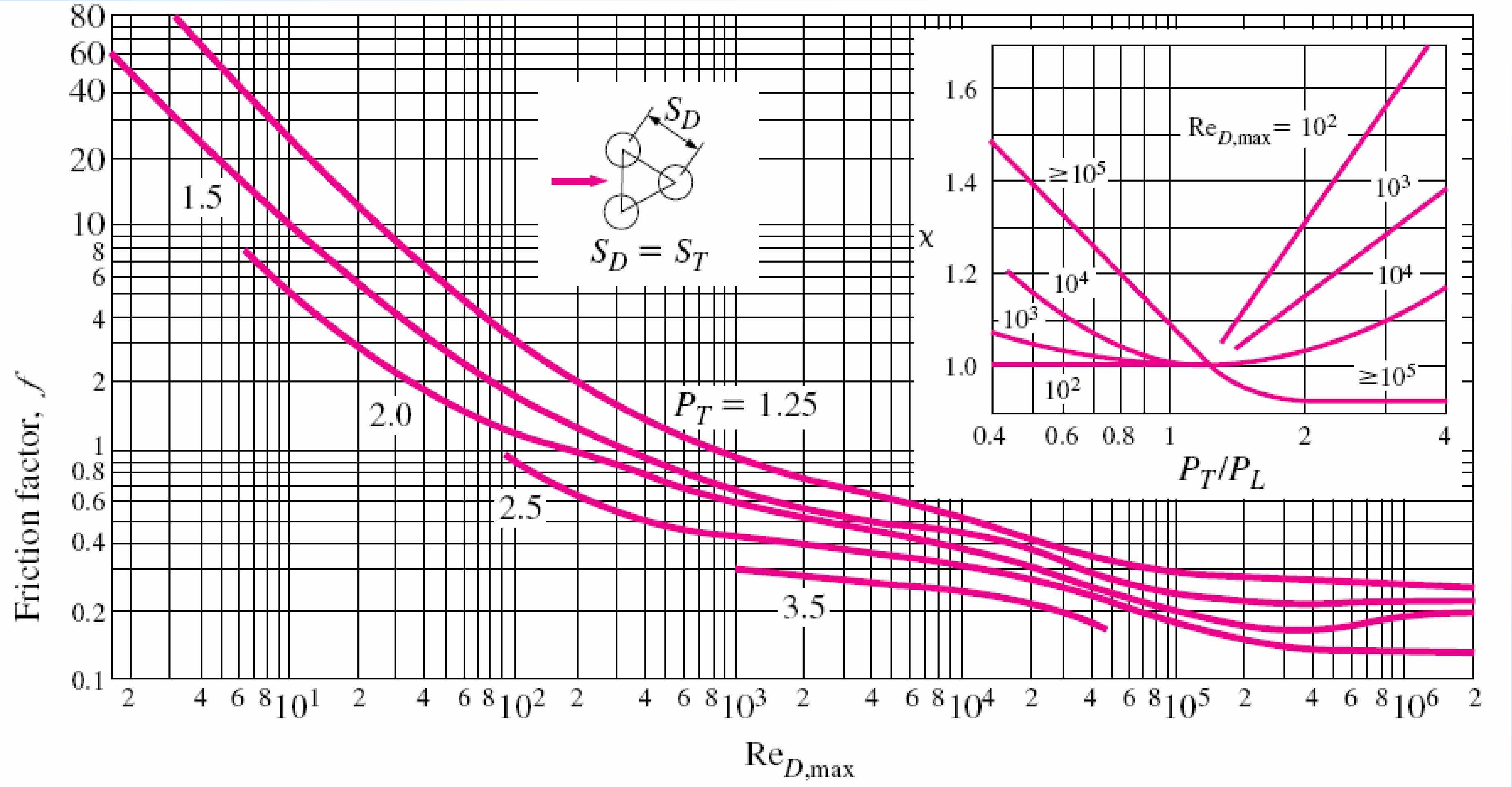
Pressure Drop

The *pressure drop over tube banks is expressed as:*

$$\Delta P = Nu_L \ f \ \varsigma \ \frac{\rho \ U_{max}^2}{2}$$

f is the friction factor. ς is the correction factor given in the insert and is used to account for the effects of deviation from square arrangement (in-line) and from equilateral arrangement (staggered).


$P_L = S_L/D$
$P_T = S_T/D$
S_L
S_T
$P_T = P_L$
$P_L = 1.25$
1.5
2.0
3.0
2.5
Friction factor, f
$Re_{D,max}$
χ
$(P_T - 1)/(P_L - 1)$
$Re_{D,max} = 10^5$
10^3
10^4
10^6


Friction factor, f
$Re_{D,max}$
$P_T = 1.25$
1.5
2.0
2.5
3.5
S_D
$S_D = S_T$
χ
P_T/P_L
$Re_{D,max} = 10^2$
$\geq 10^5$
10^3
10^4
10^2

Example 9

Air at atmospheric pressure and temperature T_∞ = 40 °C flows across a tube bank consisting of D = 2 cm outer diameter tubes at temperature T_w = 100 °C and in-line arrangement with transverse and longitudinal pitches $S_T = S_L$ = 2 D. The bank consists of L = 0.5 m long tubes arranged in N_L = 10 rows deep in the direction of flow and N_T = 20 rows high perpendicular to the flow. The velocity of air just before it enters the tube bank is U_∞ = 10 m/s.

(a) Determine the pressure drop across the tube bank;

(b) Determine the average heat transfer coefficient and the total heat flow rate from the tubes to the air.

Properties of air at $\dfrac{T_w + T_\infty}{2} = \dfrac{40 + 100}{2} = 70\ ^\circ C$

$$\left.\begin{aligned} c_p &= 1.009\ \text{kJ/kg.}^\circ\text{C} \\ \rho &= 0.9980\ \text{kg/m}^3 \\ \mu &= 2.075\ 10^{-5}\ \text{kg/m.s} \\ k &= 0.03003\ \text{W/m.}^\circ\text{C} \\ \nu &= 20.76\ 10^{-6}\ \text{m}^2\text{/s} \\ Pr &= 0.697 \end{aligned}\right\}$$

Maximum flow velocity: $U_{max} = U_\infty \dfrac{S_T}{S_T - D} = (1) \dfrac{2\ D}{(2\ D) - D} = 20\ \text{m/s}$

Reynolds number: $Re_{max} = \dfrac{U_{max}\ D}{\nu} = \dfrac{(20)\ (0.02)}{20.76\ 10^{-6}} = 1.9\ 10^4$

(a) Neglect variation of μ by temperature: $\Delta P = 2\, f'\, \rho\, U_{max}^2\, N_L$

Calculate f' using the equation given in the text book: f' = 0.0465

$$\Delta P = 2\, f'\, \rho\, U_{max}\, N_L = (2)\,(0.045)\,(0.998)\,(20)^2\,(10) = 371.5\ N/m^2$$

(b) $\overline{Nu} = C\, Re_{max}^{n}\, Pr^{1/3}$

$\frac{S_T}{D} = \frac{S_L}{D} = 2$

In-line arrangement

Use the Table in the text book:

C = 0.254

n = 0.632

$$\overline{Nu} = C\, Re_{max}^{n}\, Pr^{1/3} = (0.254)\,(1.9\ 10^4)^{0.632}\,(0.697)^{1/3} = 113.96$$

$$\bar{h} = Nu\,\frac{k}{D} = 113.96\,\frac{0.03}{0.02} \cong 171\ W/m^2.K$$

Heat flow rate: $Q = (\bar{h})\,(\text{Total Surface Area})\,(T_w - T_\infty)$

Total Surface Area = π D L (Number of Tubes)

$= \pi\,(0.02)\,(0.5)\,(20)\,(10) = 6.28\ m^2$

T_∞ is not known => Trial-and-error solution

=> Start with an assumption for T_∞, and iterate

Assume T_∞ = 40 °C remains constant (nor quite true) as a first approximation

$Q_1 = (\bar{h})\,(\text{Total Surface Area})\,(T_w - T_\infty) = (171)\,(6.28)\,(100 - 40) = 64465.5\ W$

This should be the same as the rate of energy taken up by the air:

$$Q_1 = \dot{m}\, c_p\, \Delta T = \rho\, U_\infty\, A_\infty\, c_p\, (T_{out} - T_{in})$$

where A_∞ is the flow area without tubes

U_∞ is the air velocity without tubes

$$64465.5 = (0.998)\,(10)\,(1009)\,(L\, N_T\, S_T)\,(T_{out} - T_{in})$$
$$= (0.998)\,(10)\,(1009)\,(0.5)\,(20)\,(2)\,(0.02)\,(T_{out} - 40)$$

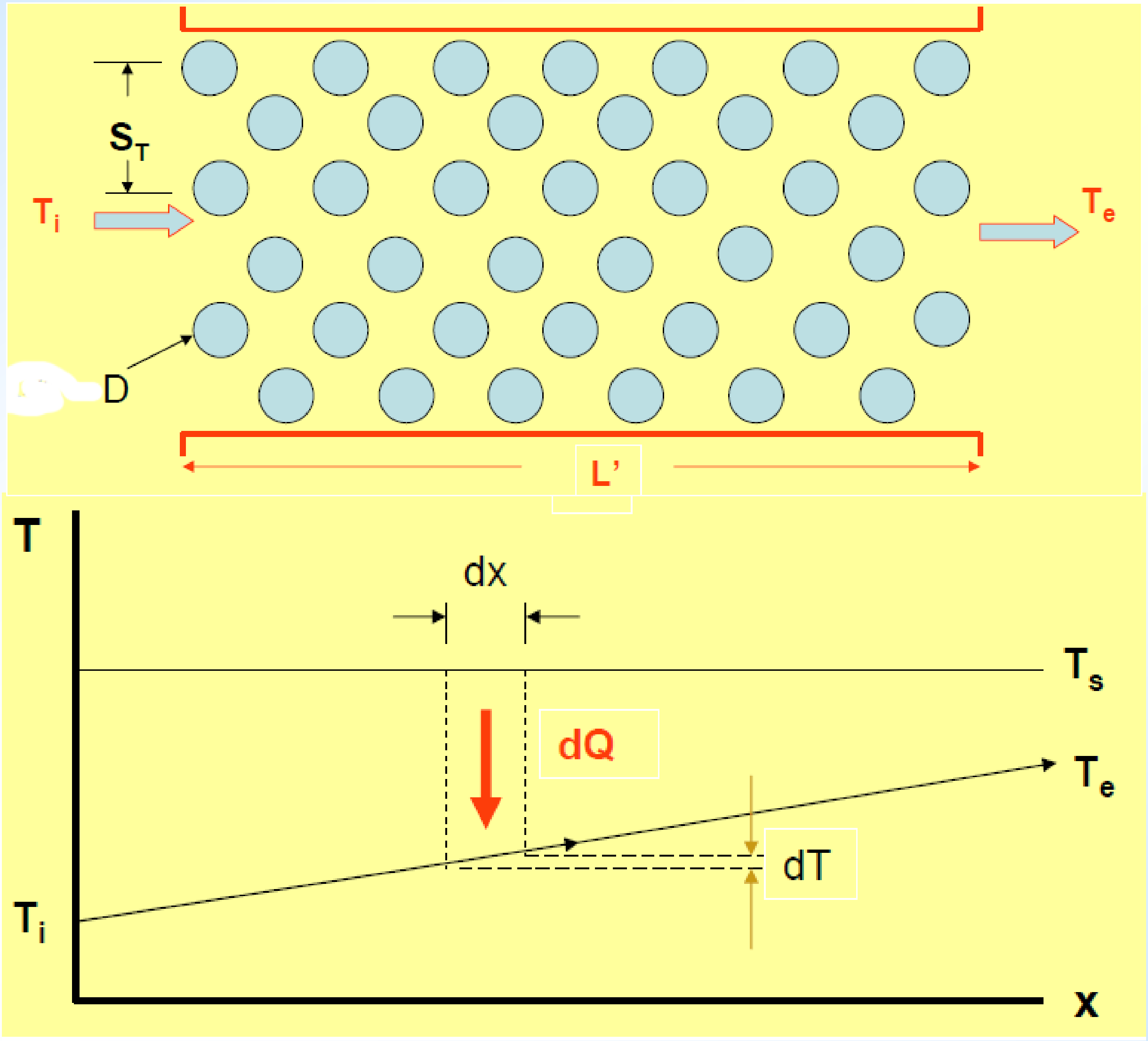
Solve for $T_{out} = 56\ ^\circ C$

$$T_\infty = \frac{T_{in} + T_{out}}{2} = \frac{40 + 56}{2} = 48\ ^\circ C$$

Use this T_∞ as the second estimate and recalculate Q, and repeat

For better accuarcy, use logarithmic mean temperature difference instead of ΔT

Log-mean Temperature Difference (LMTD)



Total heat flow rate: $Q = \left(\dot{m}\ c_p\right)_c \left(T_e - T_i\right)$

Energy balance (cold fluid): $dQ = \left(\dot{m}\ c_p\right)_c\ dT$

Heat transfer (tube surface to fluid): $dQ = h_{av}\ \left(dA_s\right)\ \left(T_s - T\right)$

Combine equations: $$-\int_{T_i}^{T_e} \frac{dT}{\left(T - T_s\right)} = \int_0^{A_s} \frac{h_{av}}{\left(\dot{m}\ c_p\right)_c}\ dA_s$$

$$\ln\left(\frac{T_s - T_i}{T_s - T_e}\right) = \frac{h_{av}\ A_s}{\left(\dot{m}\ c_p\right)_c}$$

A_s= Total Heat transfer area = π D L N

L = Length of each tube

N = Total number of tubes

$$\ln\left(\frac{T_s - T_i}{T_s - T_e}\right) = \frac{h_{av}\ A_s}{\left(\dot{m}\ c_p\right)_c} \qquad Q = \left(\dot{m}\ c_p\right)_c \left(T_e - T_i\right)$$

$$Q = h_{av}\ A_s\ \frac{\left(T_e - T_i\right)}{\ln\left(\frac{T_s - T_i}{T_s - T_e}\right)} = h_{av}\ A_s\ \frac{\left(T_s - T_i\right) - \left(T_s - T_e\right)}{\ln\left(\frac{T_s - T_i}{T_s - T_e}\right)} = h_{av}\ A_s\ LMTD$$

$$LMTD = \frac{\left(T_s - T_i\right) - \left(T_s - T_e\right)}{\ln\left(\frac{T_s - T_i}{T_s - T_e}\right)}$$

Log mean temperature difference