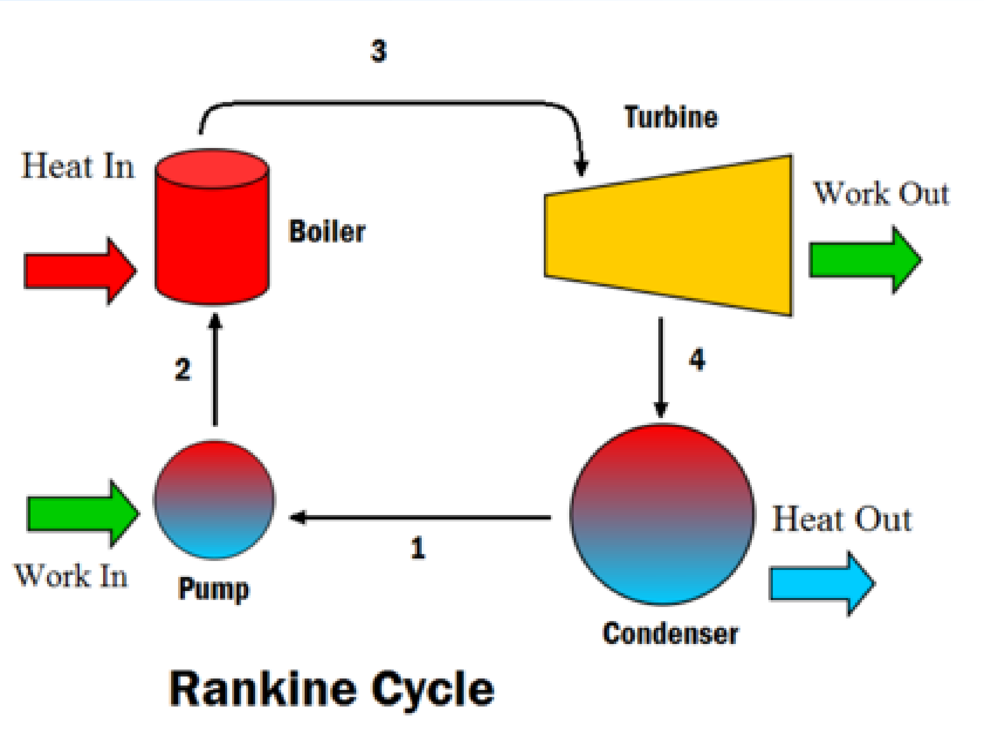
HEAT TRANSFER and THERMODYNAMICS



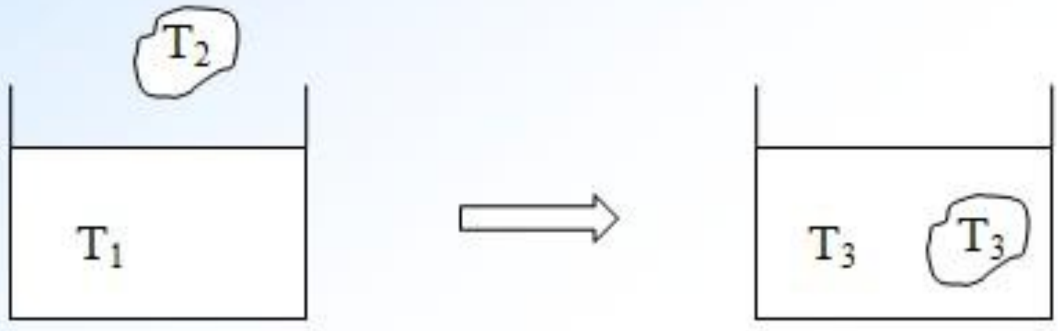
Water is the typical, but not the only, working fluid.

Turbine and pump are energy conversion devices.

Boiler (or evaporator) and condenser are energy (heat) exhange devices.

Thermodynamics tells us what Q and W and η are.

How should the boiler and condenser be designed? Geometry, size, material, etc.



Thermodynamics tells us what T_3 is

How long should you wait?

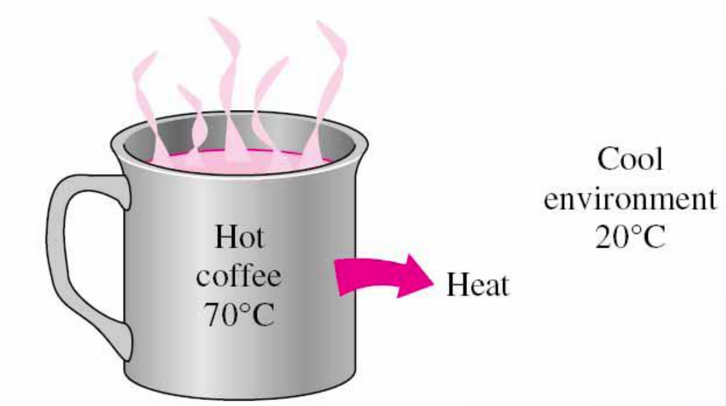
The relation between Mechanics and Strength of Materials is roughly the same as that between Thermodynamics and Heat Transfer: **Material and Properties**.

The science of **thermodynamics deals with the** amount of heat transfer as a system undergoes a process from one equilibrium state to another, and makes no reference to how long the process will take.

The science of **heat transfer deals with the** determination of the rates of thermal energy that can be transferred from one system to another as a result of temperature difference.

Thermodynamics deals with equilibrium states and changes from one equilibrium state to another. **Heat transfer,** on the other hand**, deals with systems that lack thermal equilibrium**, and thus it is a non-equilibrium phenomenon. Therefore, the study of heat transfer cannot be based on the principles of thermodynamics alone. However, the laws of thermodynamics lay the framework for the science of heat transfer.

The basic requirement for heat transfer is the presence of a temperature difference.



The second law requires that heat be transferred in the direction of decreasing temperature.

The temperature difference is the driving force for heat transfer.

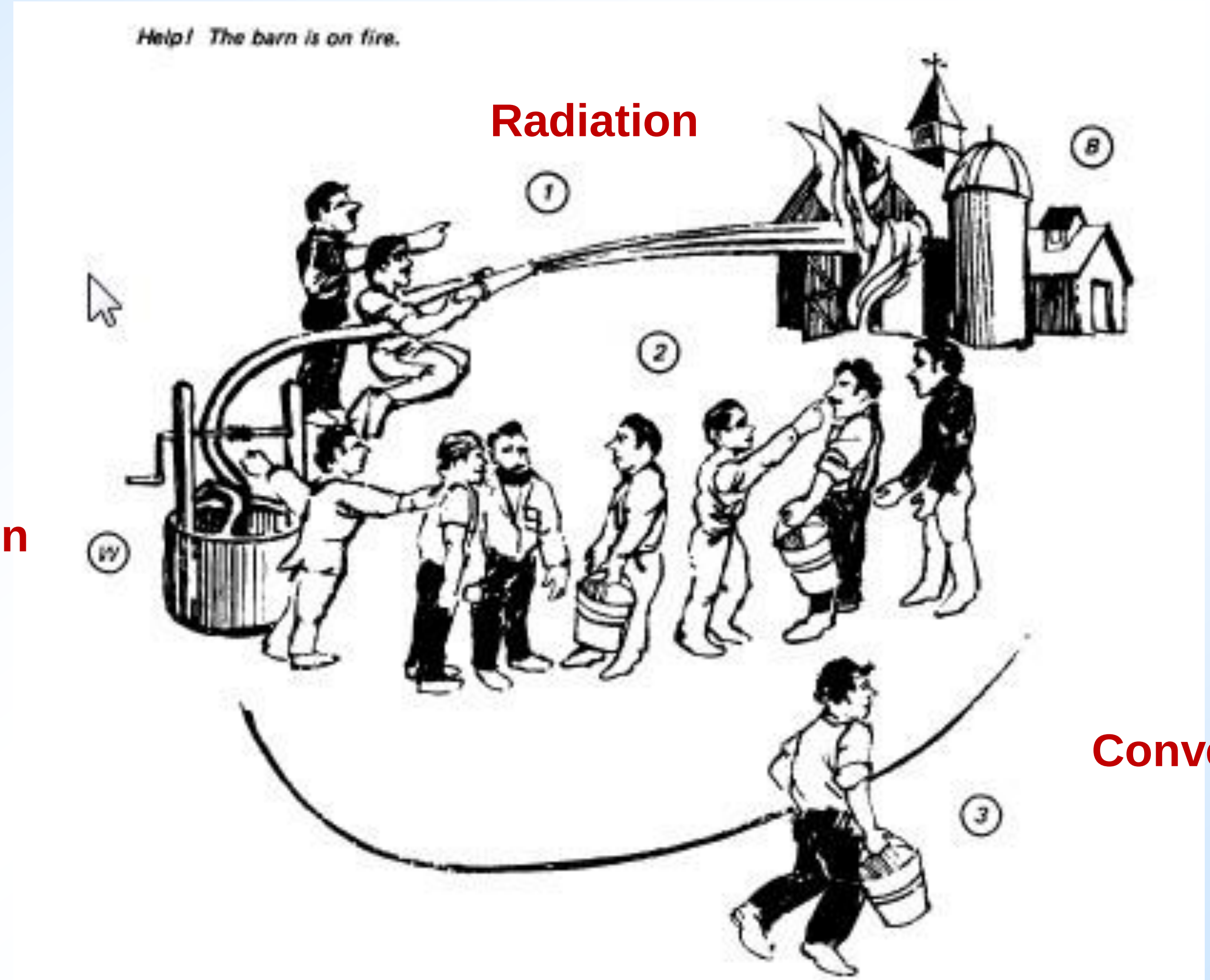
The rate of heat transfer in a certain direction depends on the magnitude of the temperature gradient in that direction.

The larger the temperature gradient, the higher the rate of heat transfer.

1. Few concepts

- Thermodynamics deals with the relations between different energy forms.
- Thermal Energy is a form of energy, and is part of internal energy defined in thermodynamics. Chemical and Nuclear energy are the other two.
- Heat transfer is concerned with the analysis of rate of heat transfer taking place in a system.
- Heat flow cannot be measured directly, but is related to the measurable quantity called temperature, or gradient of temperature, and hence temperature distribution.
- **Heat Flux** is the amount of heat transfer per unit area per unit time.
- Modes of Heat Transfer are **CONDUCTION**, **CONVECTION** and **RADIATION**. They are interrelated, but for simplicity in analysis, one may consider only one mode if and when the others are negligible.

Modes of Heat Transfer



First Law of Thermodynamics as a Rate Equation:

$$dKE + dPE + dU = \delta Q - \delta W$$

$$\frac{dKE}{dt} + \frac{dPE}{dt} + \frac{dU}{dt} = \frac{\delta Q}{dt} - \frac{\delta W}{dt}$$

$$\frac{dKE}{dt} + \frac{dPE}{dt} + \frac{dU}{dt} = \dot{Q} - \dot{W}$$

At steady state

i.e. no change with time:

$$Q = \int \dot{Q}\, dt = \dot{Q}\, \Delta t \qquad W = \int \dot{W}\, dt = \dot{W}\, \Delta t$$

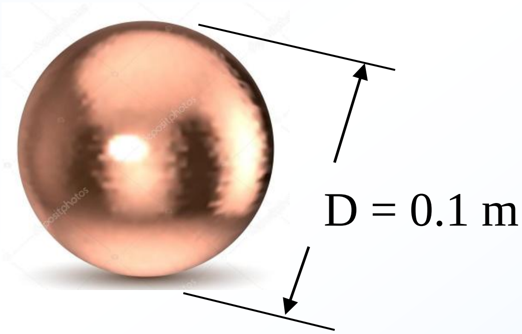
Example 1

A 10-cm-diameter cupper ball is to be heated from 100 °C to an average temperature of 150 °C in 30 minutes. Taking the average density and specific heat of copper in this temperature range to be $\rho = 8950$ kg/m^3 and $c_p = 0.395$ kJ/kg.K, respectively, determine

a) The total amount of heat transfer to the copper ball

b) The average rate of heat transfer to the ball; and

c) The average heat flux.



$$\left.\begin{array}{l} T_{initial} = 100\ ^\circ C \\ T_{final} = 150\ ^\circ C \end{array}\right\} \quad \Delta T = 50\ ^\circ C$$

$$t = 30 \text{ min} = 1800 \text{ s}$$

(a) $m = (\text{density})\ (\text{volume}) = (8950\ )\ \dfrac{\pi}{6}\ (0.1)^3 = 4.686\ \text{kg}$

$Q = m\ c\ \Delta T = (4.686)\ (0.395\ (150 - 100) = 92.6\ \text{kJ}$

Note that $Q = m\ c\ \Delta T$ is valid for solids, liquids and ideal gasses, but not for steam.

(b) $\dot{Q}_{av} = \dfrac{Q}{t} = \dfrac{92.6}{1800} = 0.0514\ \text{kJ/s (or kW)}$

(c) $\dot{q}_{av} = \dfrac{\dot{Q}_{av}}{A} = \dfrac{51.4}{(\pi)\ (0.1)^2} = 1636\ \text{W/m}^2$ **Heat Flux**

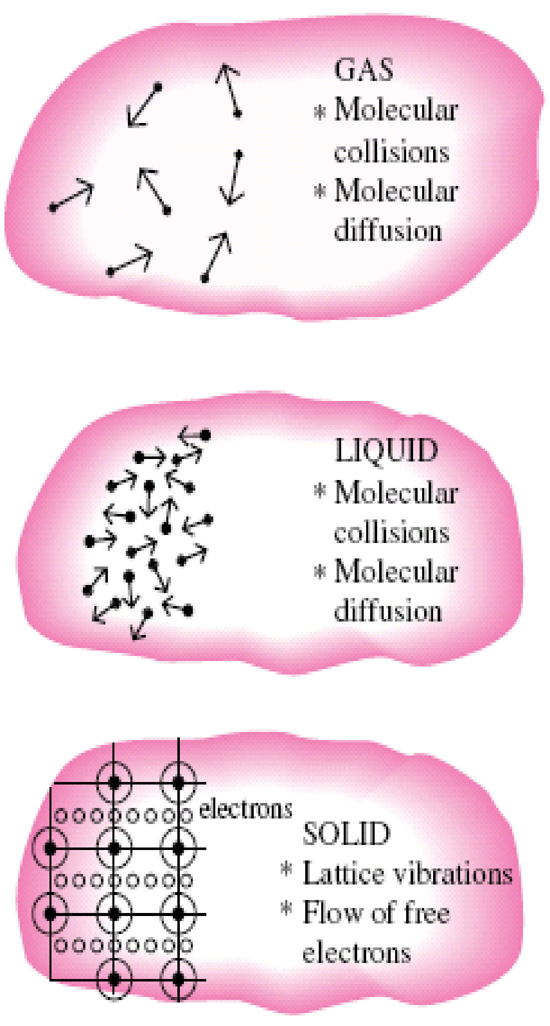
1.1. Conduction

Conduction is the transfer of energy from the more energetic particles of a substance to the adjacent, less energetic ones as a result of interactions between the particles.

Conduction can take place in solids, liquids, or gases

– In **solids,** conduction is due to the combination of vibrations of the molecules in a lattice and the energy transport by free electrons.

– In **gases and liquids** conduction is due to the collisions and diffusion of the molecules during their random motion.



Materials that are good electric conductors are generally good heat conductors (Cu and Ag).

Basic law of heat conduction based on experimental observations originates from Biot, but generally named after the French mathematical physicist, Fourier.

John Baptiste Joseph Fourier
1768 - 1830

Jean-Baptiste Biot
1774 - 1862

Fourier's Law: Rate of heat flow by conduction in a given direction x is proportional to the area normal to the direction of heat flow, and to the gradient of temperature in that direction.

$$\dot{Q}_{cond,x} = -k\,A\,\frac{dT}{dx} \qquad \text{in J/s or W}$$

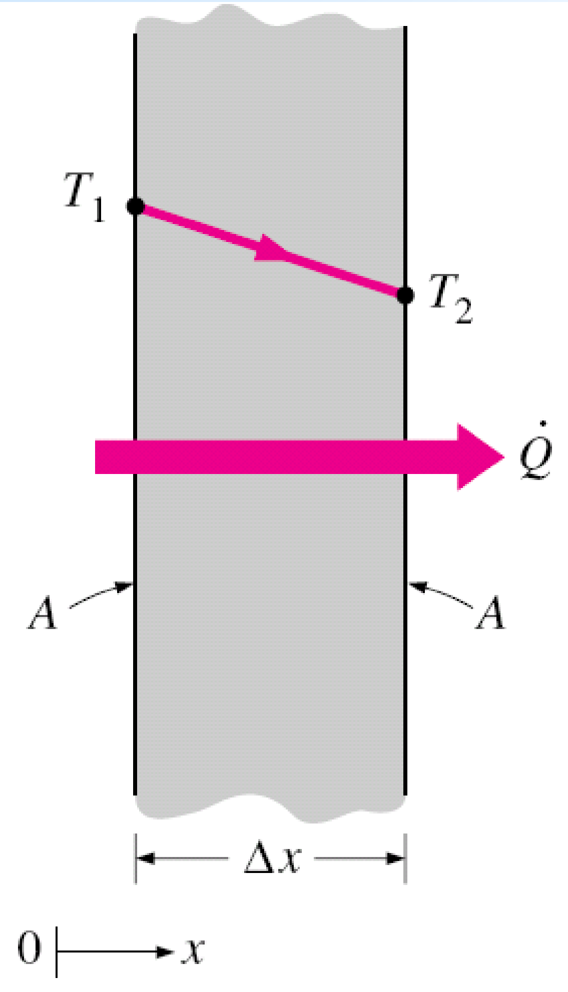
$$\frac{\dot{Q}_{cond,x}}{A} = q''_{cond,x} = -k\,\frac{dT}{dx} \qquad \text{in W/m}^2$$

This is Fourier's Law of Heat Conduction (Steady State)

$\dot{Q}_{cond}$: Rate of heat flow through perpendicular area A, in the positive x-direction

$q''_{cond,x}$: Heat flux in the positive x-direction

k : Constant of proportionality, called **thermal conductivity**, in W/m.˚C



Rate of Heat Conduction ∝

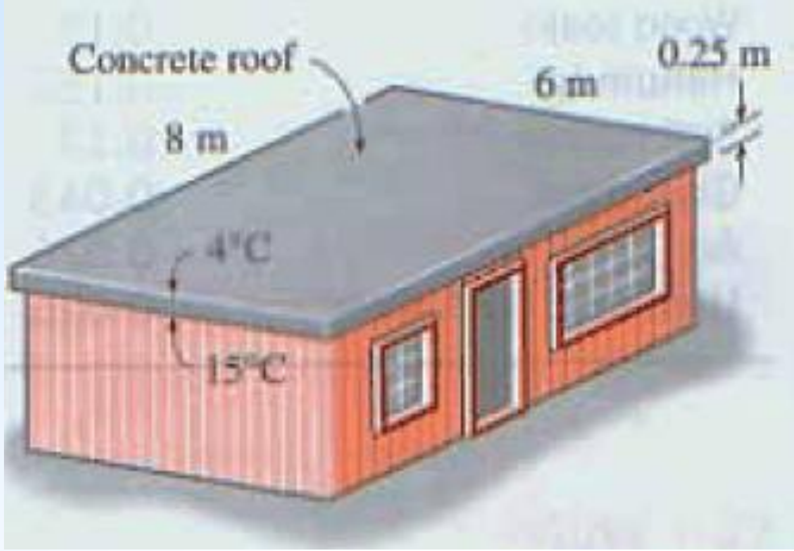
$$\frac{(Area)(Temperature\ Difference)}{Thickness}$$

$$\dot{Q}_{\mathrm{cond,\,x}} = -k\,A\,\frac{\dot{d}T}{dx} = k\,A\,\frac{T_1 - T_2}{\Delta x}$$

where the constant of proportionality ***k is the* thermal conductivity of the material.**

Note the negative sign.

Example 2

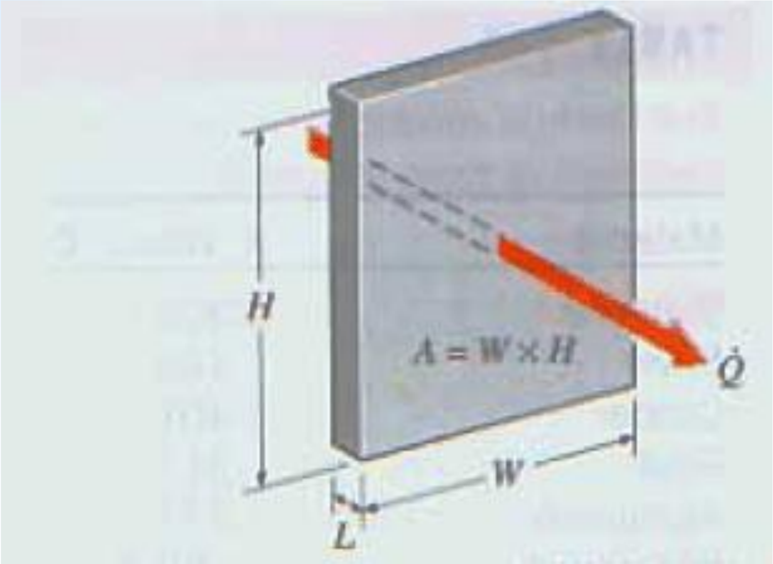


The roof of an electrically heated home is 6 m long, 8 m wide, and 0.25 m thick, and is made of a flat layer of concrete whose thermal conductivity is k = 0.8 W/m^2.K. The temperatures of the inner and the outer surfaces of the roof one night are measured to be 15 °C and 4 °C, respectively, for a period of 10 hours.

Determine

(a) the rate of heat loss through the roof that night, and

(b) (b) the cost of that heat loss to the home owner if the cost of electricity is 0.3 TL/kWh.

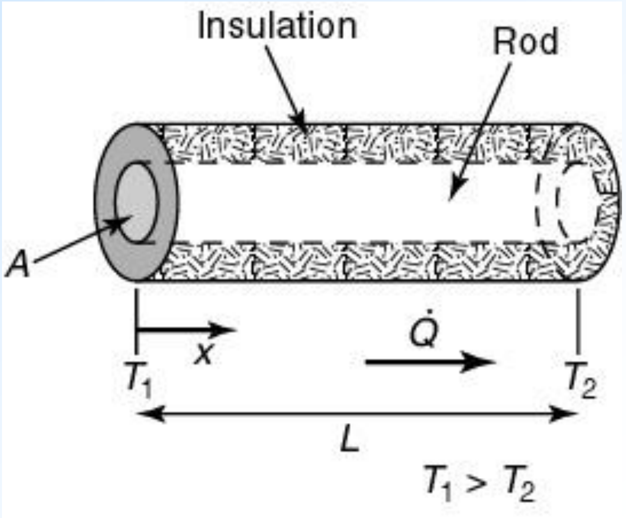


(a) $\dot{Q} = k\,A\,\dfrac{\Delta T}{\Delta x} = (0.8)\,(6)\,(8)\,\dfrac{15-4}{0.25} = 1690\text{ W} = 1.69\text{ kW}$

(b) $Q = \dot{Q}\,\Delta t = (1.69)(10) = 16.9\text{ kWh}$

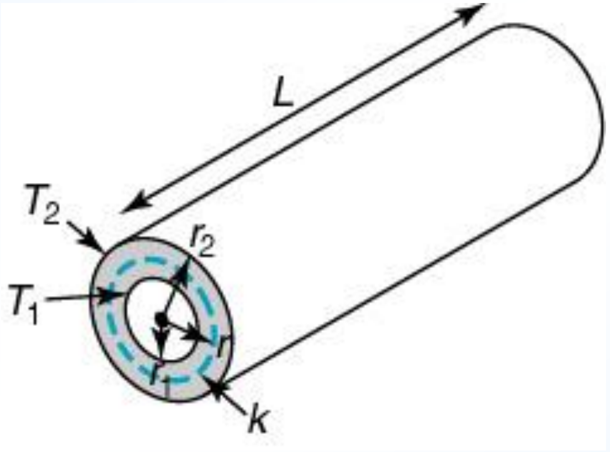
Cost = (16.9) (0.1) = 1.69 TL from the roof only

Steady, one-dimensional heat conduction



$$\dot{Q}_{cond,x} = -k\,A(r)\frac{dT}{dx} = -k\,(\pi\, r^2)\,\frac{dT}{dx}$$

$$\dot{Q}_{cond,x} = k\,(\pi\, r^2)\,\frac{T_1 - T_2}{L}$$

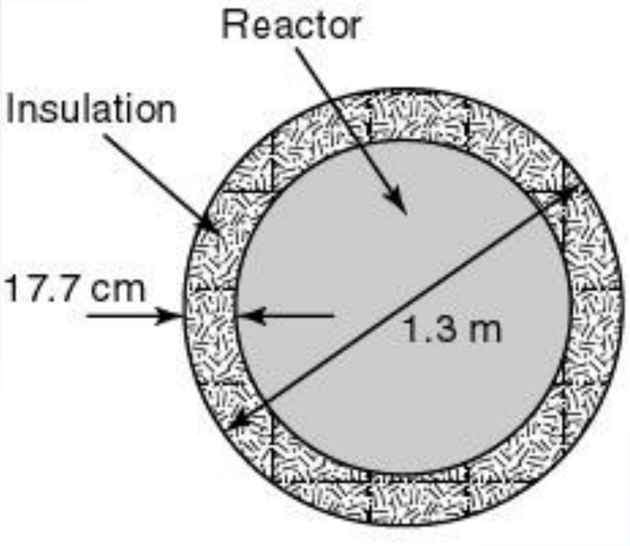


$$\dot{Q}_{cond,r} = -k\,A(r)\,\frac{dT}{dr} = -k\,(2\,\pi\,r\;L)\,\frac{dT}{dr} =$$

$$\dot{Q}_{cond,r} = k\,(2\,\pi\,L)\,\frac{T_1 - T_2}{\ln\left(\frac{r_2}{r_1}\right)}$$

Example 3

A chemical reactor is in the shape of a long cylinder as shown in the Figure. The reactor is covered with a layer of insulation 17.7 cm thick. The reactor loses heat through the insulation at a rate of 15.3 W per meter length. The thermal conductivity of the insulation is 0.04 W/m.K



If the temperature at the inner surface of the insulation is 45.5 °C, what is the the temperature at the outer surface of the insulation?

$$\dot{Q}_{cond,r} = k\,(2\,\pi\,L)\,\frac{T_1 - T_2}{\ln\left(\dfrac{r_2}{r_1}\right)}$$

$$\frac{\dot{Q}_{cond,r}}{L} = k\,(2\,\pi)\,\frac{T_1 - T_2}{\ln\left(\dfrac{r_2}{r_1}\right)} \quad => \quad 15.3 = (0.04)\,(2\,\pi)\,\frac{45.5 - T_2}{\ln\left(\dfrac{0.65}{0.65 - 0.17}\right)}$$

$$T_2 = 45.5 - \frac{15.3}{(0.04)\,(2\,\pi)}\,\ln\left(\frac{0.65}{0.65 - 0.17}\right) = 27\ ^{\circ}C$$

Thermal conductivity, k, W/m.K:

The thermal conductivity of a material is a measure of the ability of the material to conduct heat.

High value for thermal conductivity $\Rightarrow$ good heat conductor

Low value $\Rightarrow$ poor heat conductor or **insulator**

The thermal conductivities of gases such as air vary by a factor of 10^4 from those of pure metals such as copper

Pure crystals and metals have the highest thermal conductivities, and gases and insulating materials the lowest

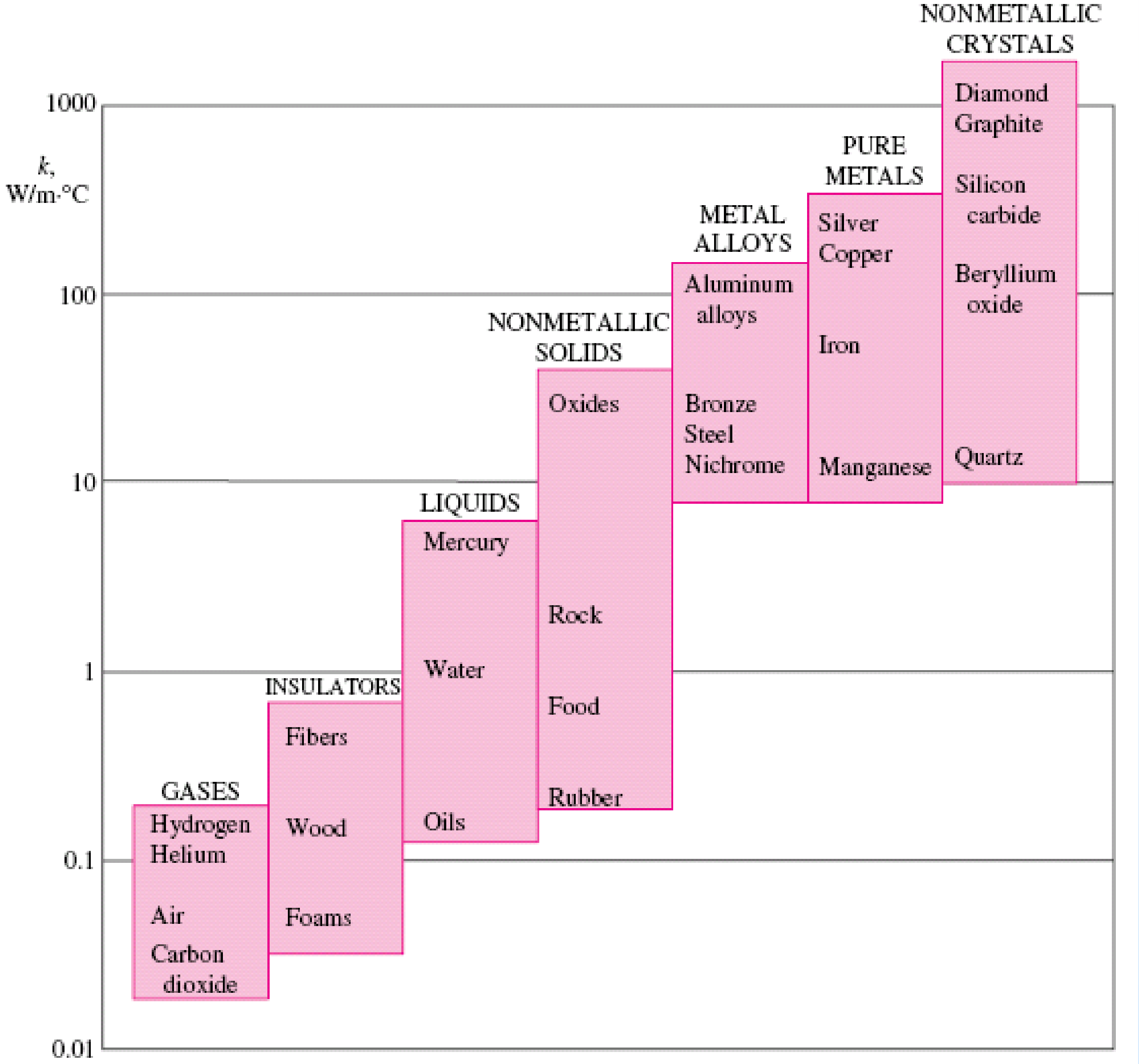

NONMETALLIC CRYSTALS
1000
k, W/m·°C
Diamond
Graphite
PURE METALS
Silicon carbide
METAL ALLOYS
Silver
Copper
Beryllium oxide
Aluminum alloys
100
NONMETALLIC SOLIDS
Iron
Oxides
Bronze
Steel
Nichrome
Manganese
Quartz
10
LIQUIDS
Mercury
Rock
1
Water
INSULATORS
Food
Fibers
GASES
Rubber
Hydrogen
Helium
Wood
Oils
0.1
Air
Foams
Carbon dioxide
0.01

TABLE A2 - A7			
Thermo physical Properties			
Composition	At 300 K		
	ρ, kg/m^3	c_p, J/kg.K	k, W/m.K
Silver	10500	235	429
Pure Copper	8933	385	401
Carbon steel	7854	434	60.5
Stainless steel	8055	480	15.1
Plate glass	2500	750	1.4
Concrete	2300	880	1.4
Water	998	4182	0.60
Wood (Oak)	545	2385	0.17
Leather	998		0.16
Fiberglass	105	795	0.036
Air	1.18	1005.7	0.026

Thermal Diffusivity:

$$\alpha = \frac{Heat\ Conducted}{Heat\ Stored} = \frac{k}{\rho\, c_{\mathrm{p}}} \quad in\ m^2/s$$

The thermal diffusivity represents how fast heat diffuses through a material.

A material that has a high thermal conductivity or a low heat capacity will have a large thermal diffusivity.

The larger the thermal diffusivity, the faster is the propagation of heat into the medium.

Material	Average Temp., ˚C	Thermal Diffusivity, m^2/s
Cu	0	114.1 10^{-6}
Fe (pure)	0	18.1 10^{-6}
Ag	0	170.4 10^{-6}
Zn	0	41.3 10^{-6}
Asbestos	0	0.258 10^{-6}
Brick, Fire Clay	204.4	0.516 10^{-6}
Granite	0	1.291 10^{-6}
Ice	0	1.187 10^{-6}
Rubber	0	0.077 10^{-6}
Water	0	0.129 10^{-6}

1.2. Convection

The mechanism of heat transfer between a fluid and a solid surface in relative motion is called convection.

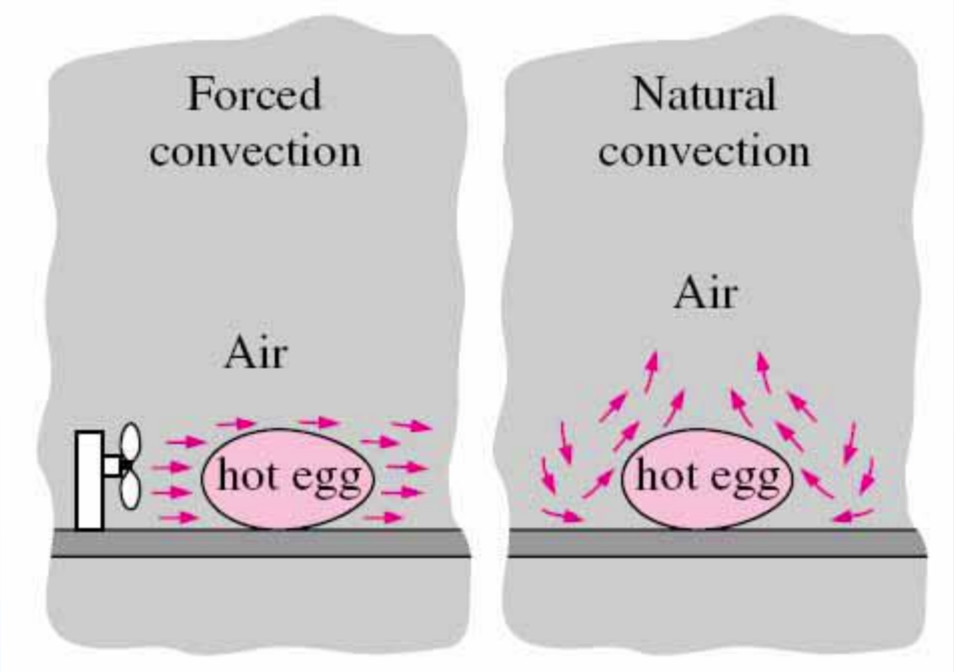
Convection = Conduction + Advection (fluid motion)

Convection is commonly classified into three sub-modes:

– Forced convection,

– Natural (or free) convection,

– Change of phase (liquid/vapor, solid/liquid, etc.)



Newton's Law of Cooling – Definition of convective heat transfer coefficient:

$$\dot{Q}_{conv} = h\,A_s\,\left(T_s - T_f\right)$$

$\dot{Q}_{conv}$: Heat flow rate at the wall, in W

T_s: Surface temperature of the wall, in ˚C

T_f: Mean temperature of the fluid, in ˚C

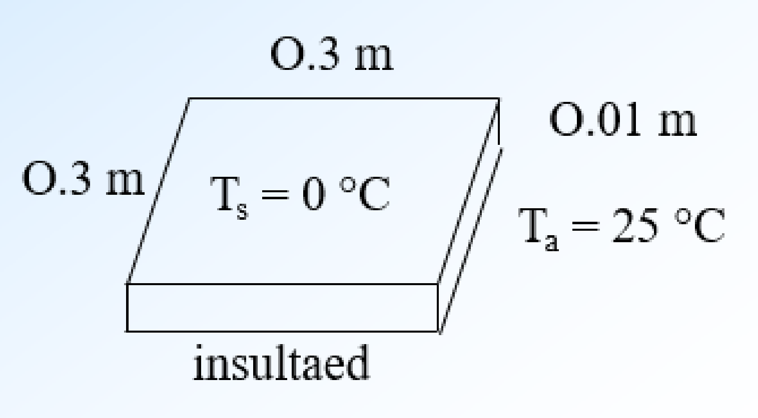
h: Convective Heat Transfer Coefficient, in W/m^2.˚C

h depends on variables such as the surface geometry, the nature of fluid motion, the properties of the fluid, and the bulk fluid velocity.

Type of Flow	h , W/m^2.˚C
Free convection of gases	2 – 25
Free convection of liquids	10 – 1000
Turbulent forced convection of air (gases) inside pipes	25 – 250
Turbulent forced convection of water inside pipes	50 – 20 000
Boiling of water	2 500 – 60 000
Condensation of steam	5 000 – 100 000

Example 4

A slab of ice in a thin-walled container, 1 cm thick and 30 cm on each side, is placed on a well-insulated pad. At its top surface, the ice is exposed to ambient air for which T_a = 25 ˚C and the convective heat transfer coefficient is 25 W/m^2.K. Neglecting heat transfer from the sides and assuming the ice-water mixture remains at 0 ˚C, how long will it take to completely melt the ice? The density and latent heat of fusion of ice are 920 kg/m^3 and 334 kJ/kg, respectively.



$h = 25$ W/m^2.K

$\rho_s = 920$ kg/m^3

$e_f = 334$ kJ/kg

$t = ?$

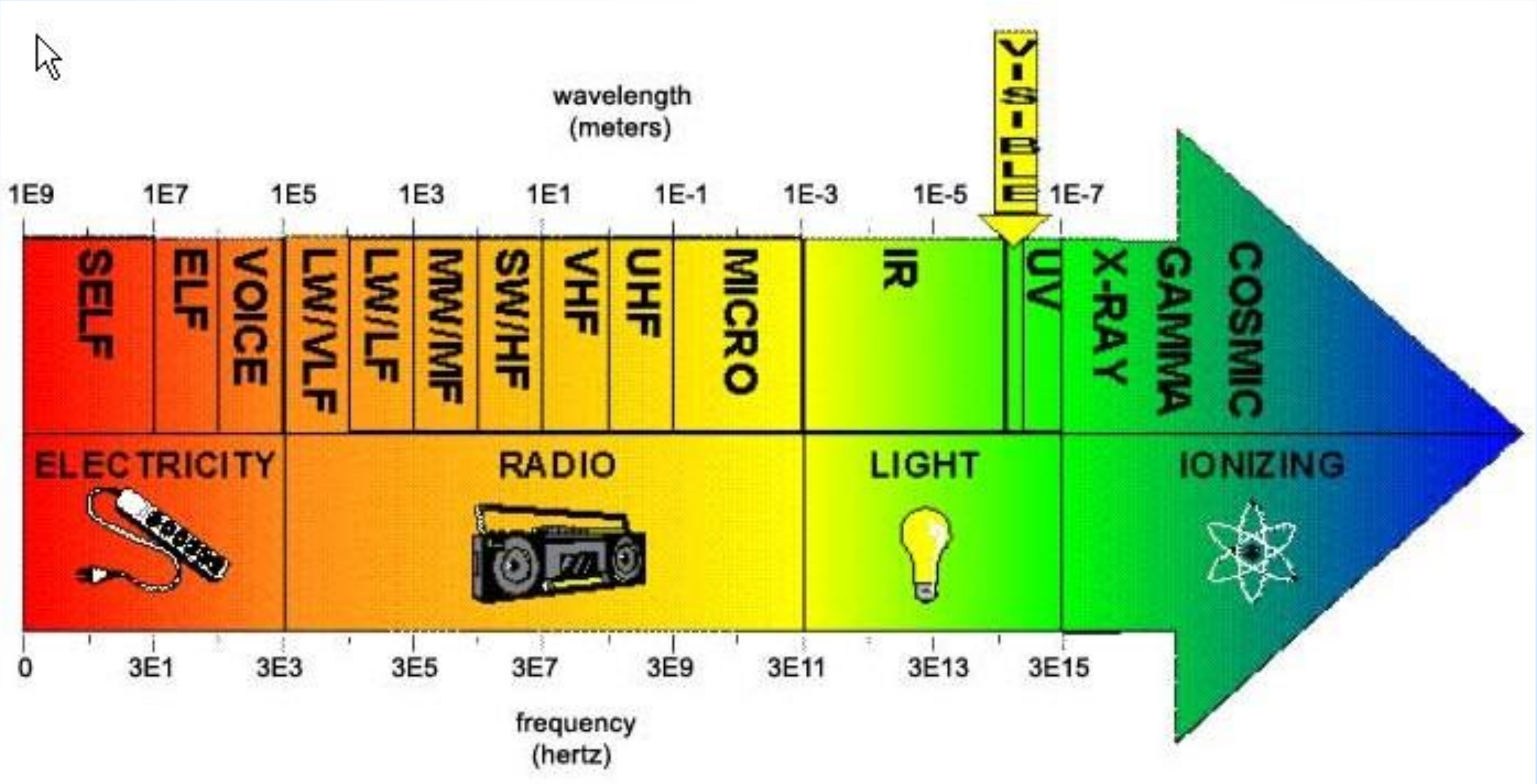
$$t = \frac{E}{Q} = \frac{e\,m}{A_s\ h\,\Delta T} = \frac{e\,V\,\rho}{A_s\ h\,(T_a - T_s)} = \frac{e\,t_h\ \rho}{h\,(T_a - T_s)}$$

$$t = \frac{e\,t_h\ \rho}{h\,(T_a - T_s)} = \frac{(334\ 10^3)\ (0.01)\ (920)}{(25)\ (25 - 0)} = 4916.5 \text{ s} = 82 \text{ minutes}$$

1.3. Radiation

- Radiation is the energy emitted by matter in the form of *electromagnetic waves (or photons) as a result of the* changes in the electronic configurations of the atoms or molecules.
- Heat transfer by radiation does not require the presence of an *intervening medium.*
- In heat transfer studies we are interested in *thermal radiation (radiation emitted by bodies because of their* temperature).
- Radiation is a *volumetric phenomenon. However, radiation* is usually considered to be a *surface phenomenon for* solids that are opaque to thermal radiation.

All objects with a temperature above 0 Kelvin emit radiation (electromagnetic waves), mostly invisible to the human eye.

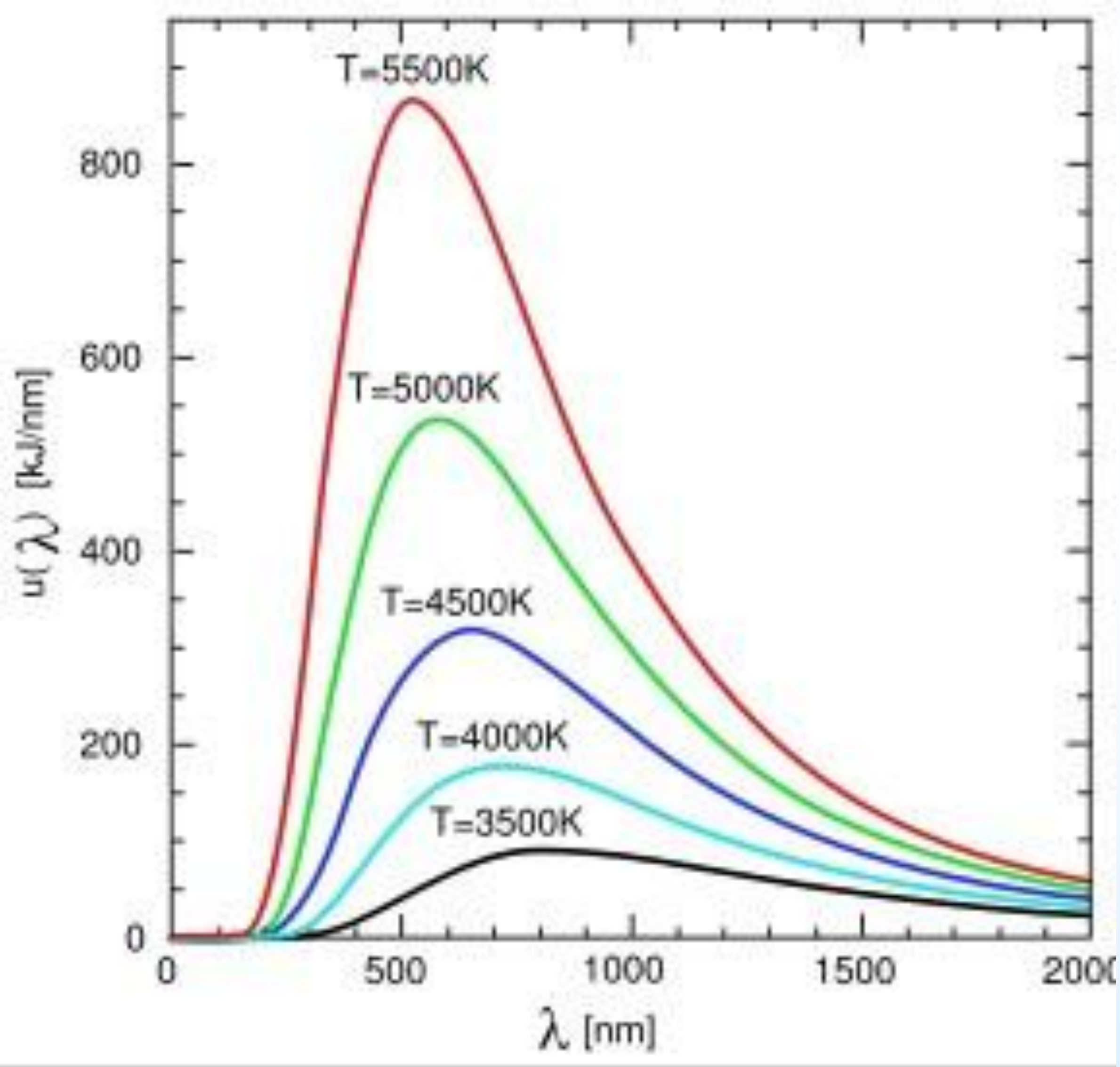


Radiative energy emitted by a body because of its temperature is transmitted in space in the form of electromagnetic waves (Maxwell’s classical wave theory) or in the form of discrete-energy photons (Planck’s hypothesis). Both concepts are utilized.

Two bodies at different temperatures in vacuum ⇨ no conduction or convection heat transfer is possible. In such cases, heat transfer occurs by thermal radiation only.

The emission or absorption of radiative energy by a body is a bulk process, i.e., the radiation originated from the interior of the body is emitted through its surface; conversely, the radiation incident on the surface of a body penetrates into the medium where it is attenuated. (Example: gases)

If a large proportion of the radiation is attenuated within a very short distance in the material, we may speak of radiation being absorbed or emitted by the surface. (Example: metals)



This diagram shows how the peak wavelength and the total radiated amount vary with temperature. Although very high temperatures are shown here, the same relationship holds true down to absolute zero.

Stefan-Boltzmann's law: $\dot{Q}_{rad} = \sigma A_s T_s^4$

$\dot{Q}_{rad}$: Rate of radiative energy leaving a black body (a perfect emitter or absorber), in W

A_s: Surface area of black body, in m^2

T_s: Absolute temperature of black body, in K

σ: Stefan-Boltzmann constant = 5.6697 10^{-8} $W/m^2.K^4$

The radiation emitted by all real surfaces is less than the radiation emitted by a blackbody at the same temperature, and is expressed as

$$\dot{Q}_{rad} = \varepsilon \sigma A_s T_s^4$$

ε is the emissivity of the surface.

Radiation Heat Exchange: $\dot{Q}_{rad} = \varepsilon\, \sigma\, A_s \left(T_s^{\,4} - T_\infty^{\,4}\right)$

$\dot{Q}_{rad}$: Rate of net radiative energy leaving a body, in W

A_s: Surface area of the body, in m^2

T_s: Absolute temperature of the body, in K

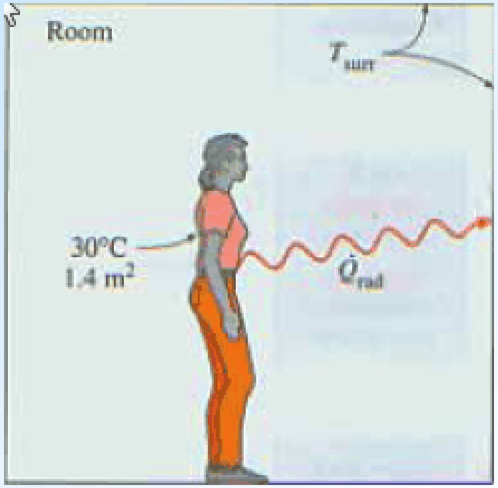
T_∞: Absolute temperature of the enclosure (surroundings), in K

σ: Stefan-Boltzmann constant = $5.6697\ 10^{-8}$, in $W/m^2.K^4$

If the two bodies are not perfectly black or the surface A is not completely enclosed by the other body, then

$$\dot{Q}_{rad} = F\, \varepsilon\, \sigma\, A_s \left(T_s^{\,4} - T_\infty^{\,4}\right)$$

F is a factor for geometry, called the view factor (< 1).



Example 5:

It is a common experience to feel chilly in winter and warm in summer in our homes even when the thermostat setting is the same. This is due to the so called radiation effect resulting from radiation heat exchange between our bodies and the surrounding surfaces of the walls and the ceiling.

Consider a person standing in a room maintained at 22 °C at all times. The inner surfaces of the walls, floors and the ceiling of the house are observed to be at an average temperature of 10 °C in winter and 25 °C in summer. Determine the rate of radiation heat transfer between this person and the surrounding surfaces if the exposed surface area and the average surface temperature of the person are 1.4 m^2 and 30 °C, respectively (Emissivity = 0.95).

$$\dot{Q}_{rad} = \varepsilon\ \sigma\ A_s\ (T^4_{person} - T^4_{\infty})$$

$$\dot{Q}_{rad,wiinter} = (0.95)\ (5.67\ 10^{-8})\ (1.4)\ \left[(30 + 273)^4 - (10 + 273)^4\right]$$

$$= 152\ W$$

$$\dot{Q}_{rad,summer} = (0.95)\ (5.67\ 10^{-8})\ (1.4)\ \left[(30 + 273)^4 - (25 + 273)^4\right]$$

$$= 41\ W$$

When heat transfer by convection and radiation are of the same order of magnitude and occur simultaneously, a proper analysis is complicated.

Heat transfer processes involving change of phase such as condensation, evaporation, ablation, etc. are equally complicated to analyze.

Radiation heat transfer coefficient, h_r

The radiative heat exchange between a surface at T and its environment at T_∞ is generally calculated from the simplified equation, which is the definition of h_r.

$$\dot{Q}_{rad} = \varepsilon\, \sigma\, A_s\, (T^4 - T_\infty^4) = h_r\, A_s\, (T - T_\infty)$$

$$h_r = \varepsilon\, \sigma\, (T^2 + T_\infty^2)\,(T + T_\infty)$$