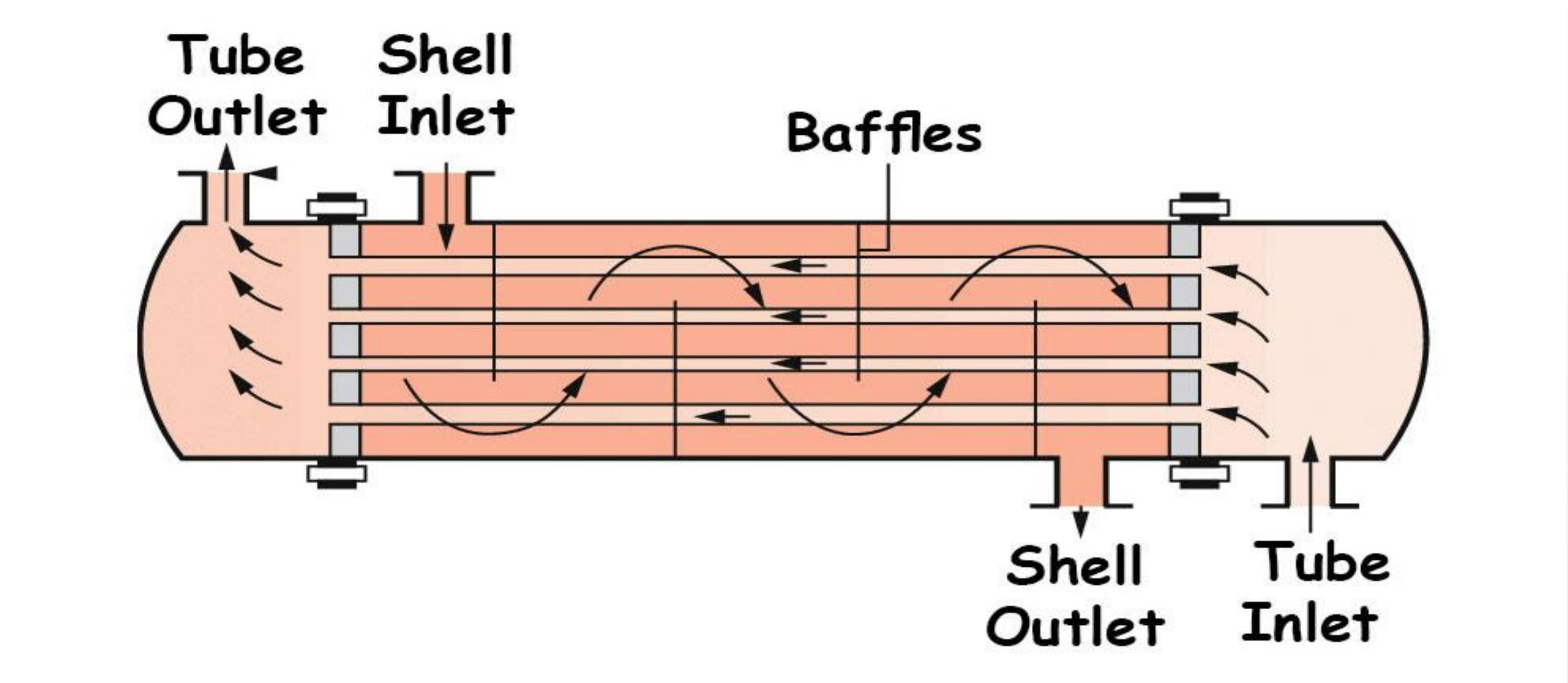
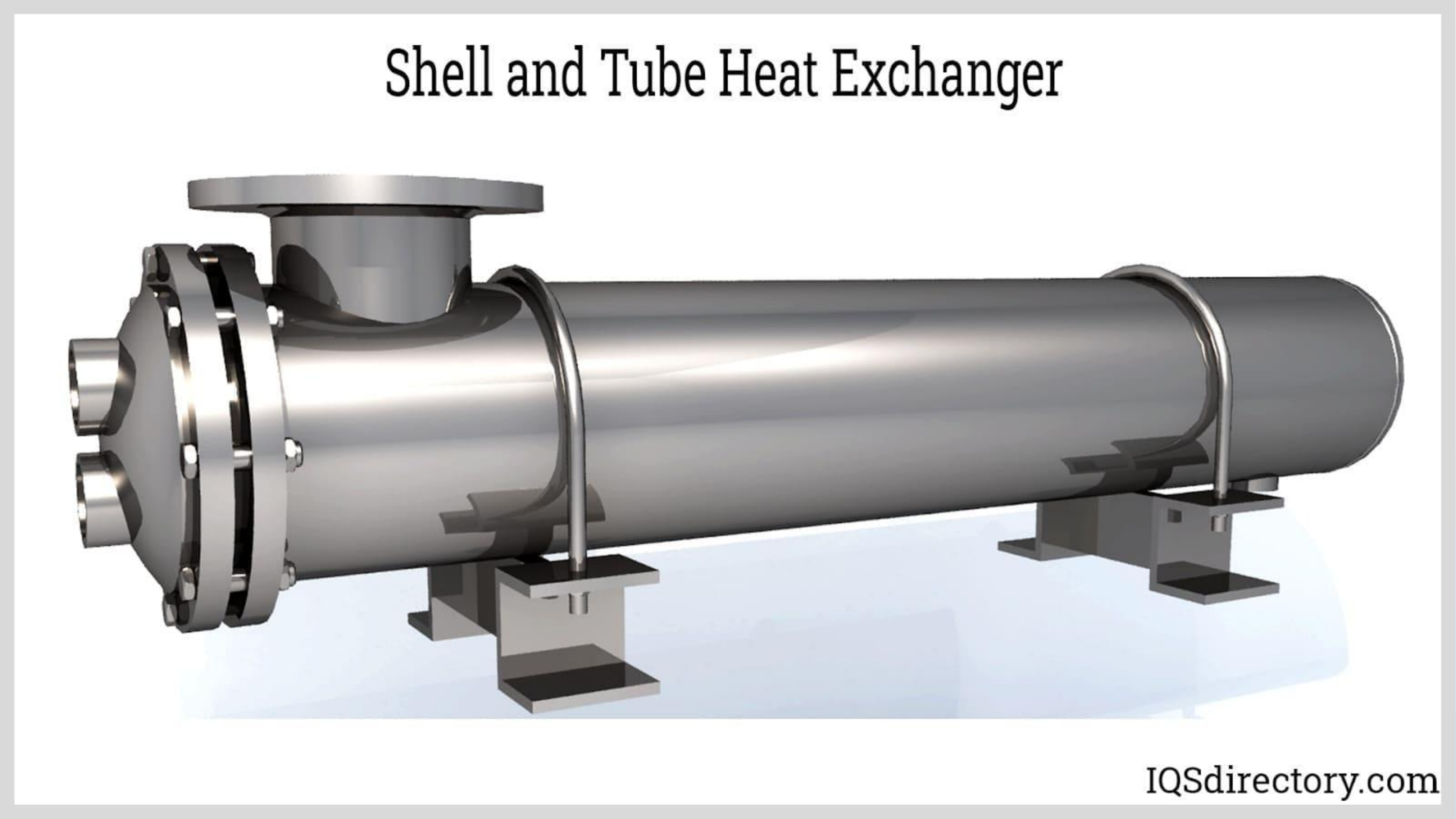
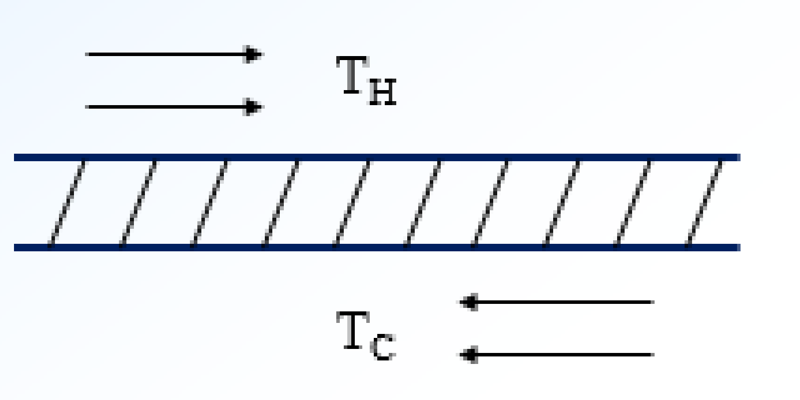
11 - HEAT EXCHANGERS

The device used to transfer thermal energy from a hotter fluid to a cooler fluid (or vice versa) is called a heat exchanger.



Shell-and-tube type heat exchanger


Shell and Tube Heat Exchanger
IQSdirectory.com



Basic heat exchange between two fluids with constant temperature separated by a solid wall is given by the expression:

$$\dot{Q} = \frac{T_H - T_C}{R_{tot}}$$

What if the fluid temperatures, T_H and T_C, do not remain constant?

Used in numerous applications such as:

- Evaporators and condensors in power plants
- Chemical processing plants and oil refineries
- Heating and air conditioning of buildings
- Household refrigerators
- Radiators in cars and houses
- Radiators for space vehicles

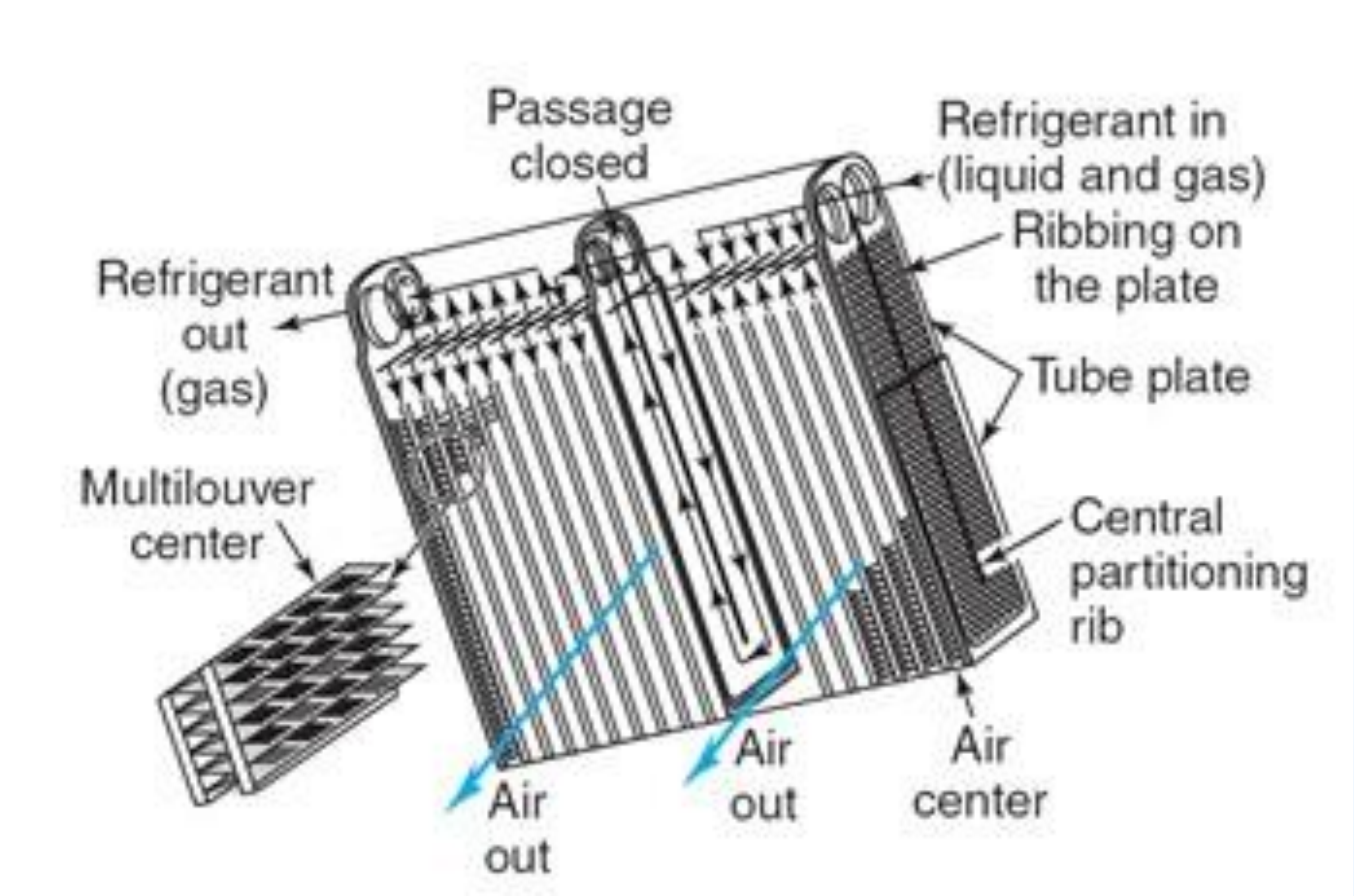
In car radiators: Heat exchange is between hot and cold fluids separated by a metal wall.

In boilers and condensers: Heat exchange is by boiling and condensation.

In cooling towers: Hot fluid (steam) is cooled by direct mixing with the cold fluid (air).

Quite a large variety of designs exits depending on the application, types of fluids, pressure and temperature levels, modes of heat flow, maintenance and cost requirements, size, weight, construction material, so on.

Therefore, complete design problems will not be discussed here. We will see thermal analysis of some common types and some basic design considerations.



Automotive evaporator

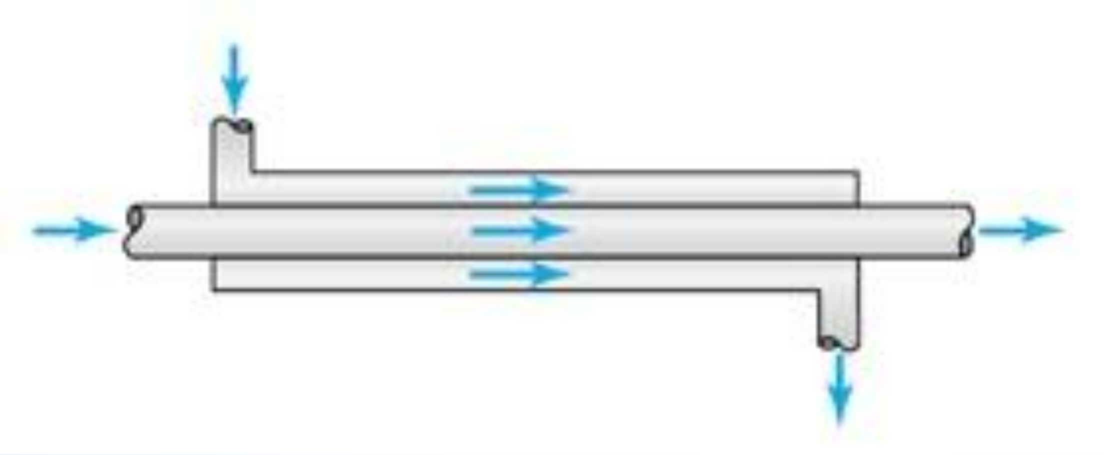
11.1 Types of heat exchangers

Classifiction according to:
1. Configuration of fluid paths through the heat exhanger
2. Intended application

Classification by fluid-flow arrangement

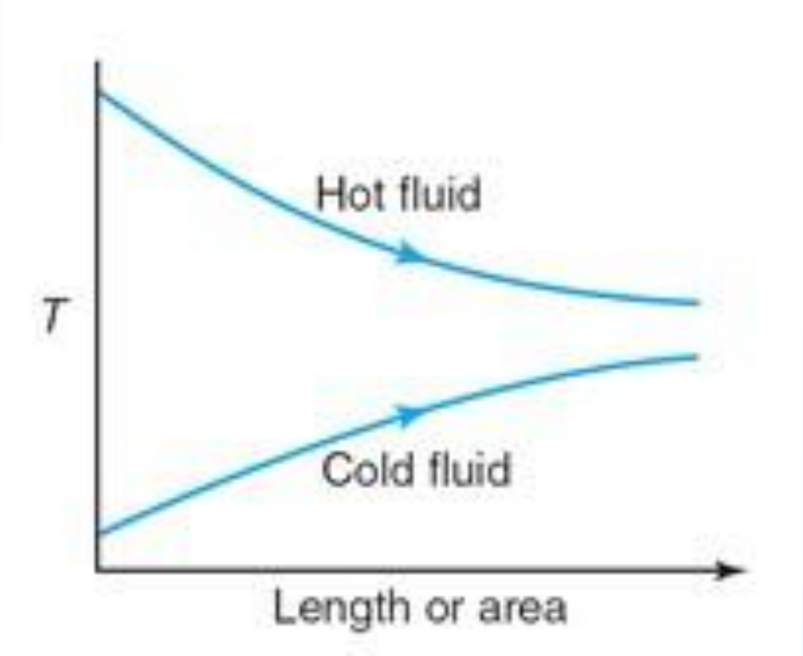
1. Parallel-flow heat exchanger
2. Counter-flow heat exchanger
3. Cross-flow, single pass heat exchanger
4. Cross-flow, multi pass heat exchanger

Others



1. Parallel flow

heat exchanger

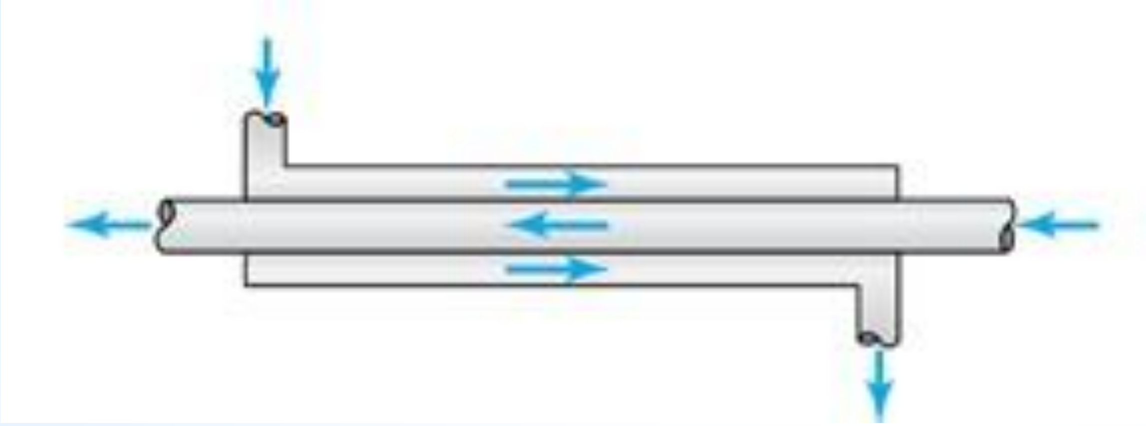


$$\dot{Q} = \dot{m}_H \; c_{p,H} \; (T_{H,i} - T_{H,o})$$

$$= \dot{m}_C \; c_{p,C} \; (T_{C,o} - T_{C,i})$$

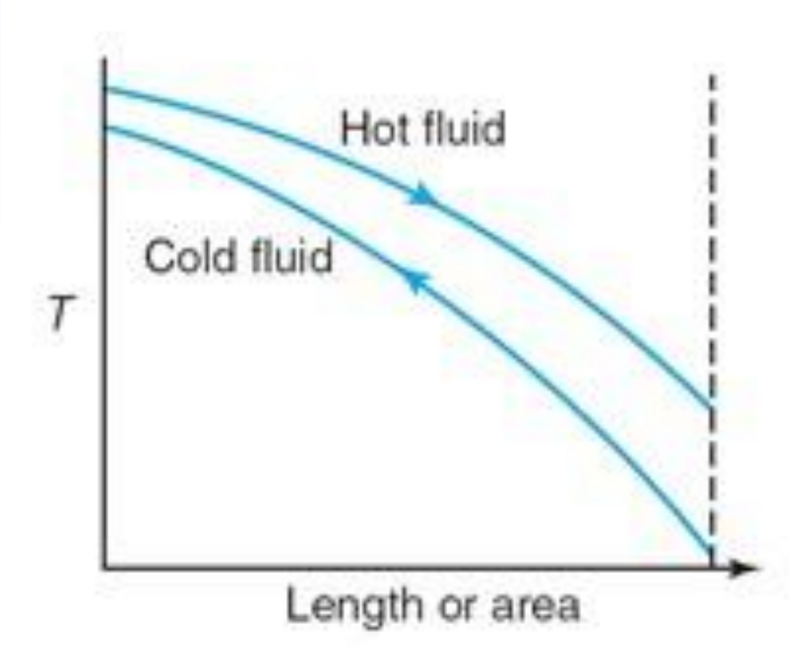
$$= A \; U \; (LMTD)$$

$$= \varepsilon \; \dot{Q}_{max}$$



2. Counter flow

heat exchanger

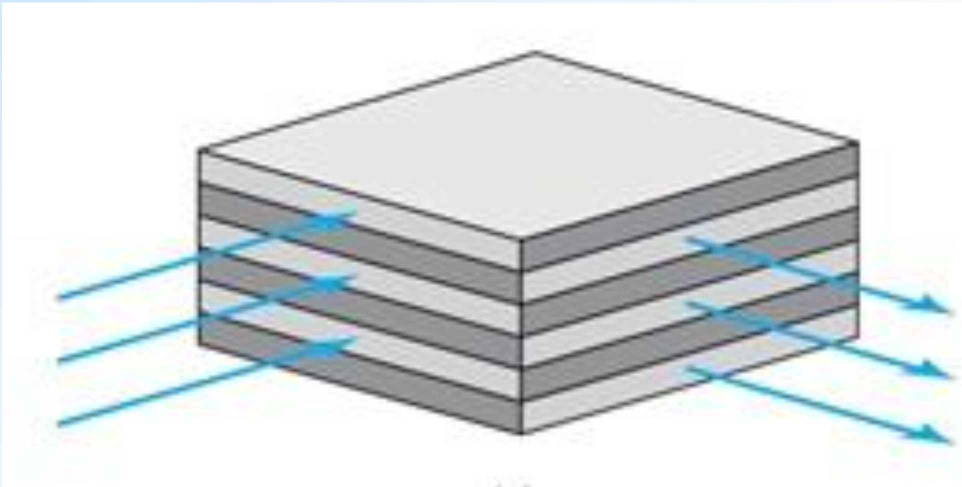


$$\dot{Q} = \dot{m}_H \; c_{p,H} \; (T_{H,i} - T_{H,o})$$

$$= \dot{m}_C \; c_{p,C} \; (T_{C,o} - T_{C,i})$$

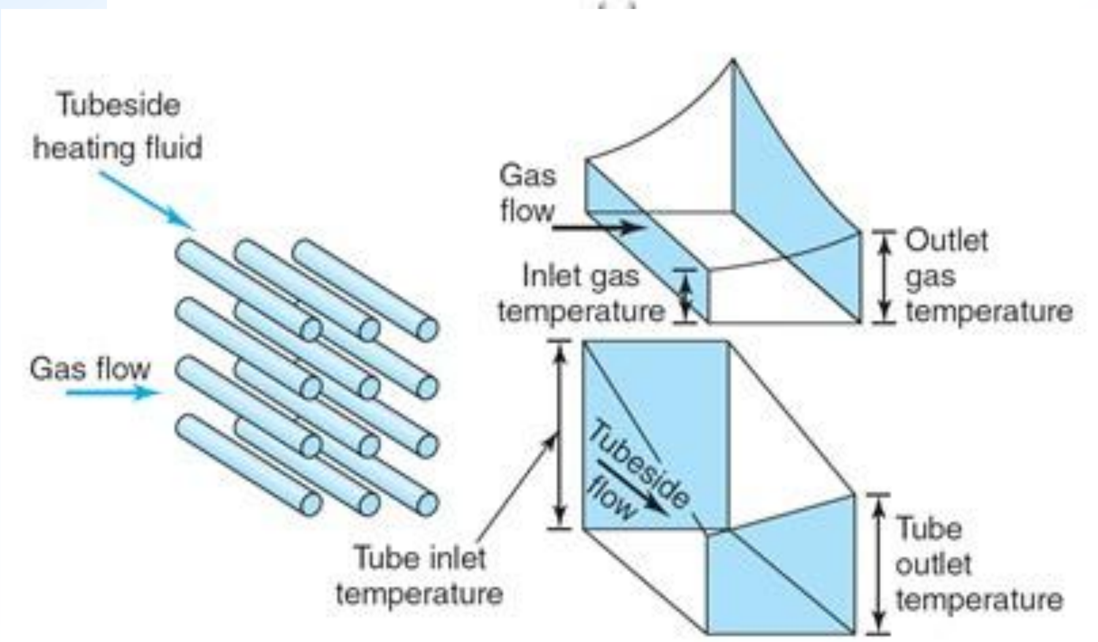
$$= A \; U \; (LMTD)$$

$$= \varepsilon \; \dot{Q}_{max}$$



3. Cross flow

heat exchanger



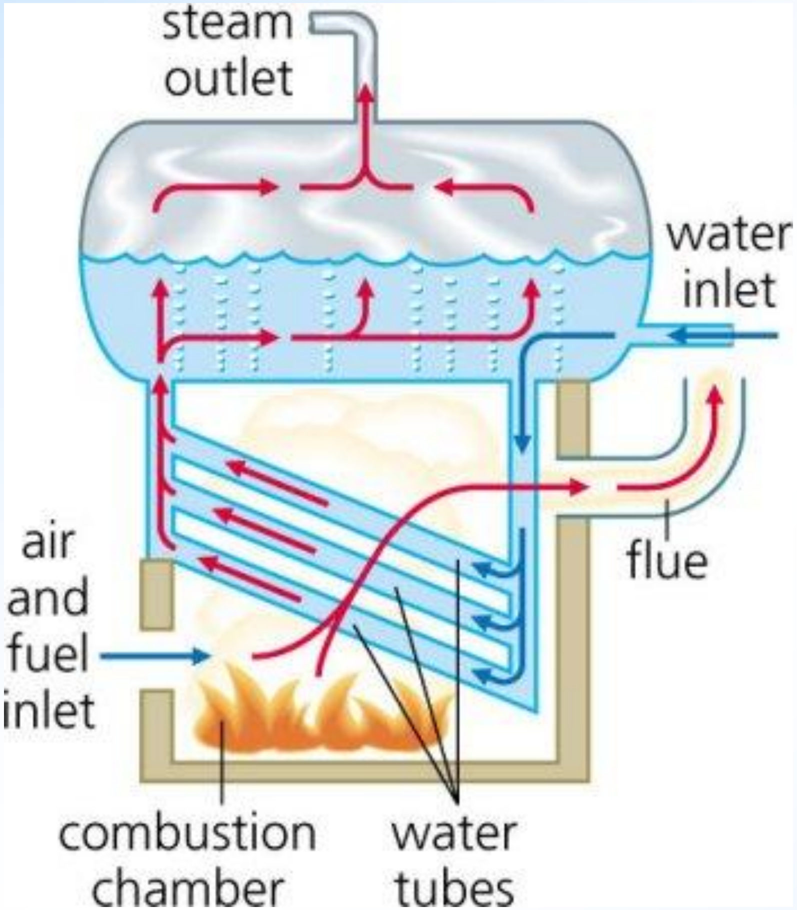
$$\dot{Q} = \dot{m}_H \; c_{p,H} \; (T_{H,i} - T_{H,o})$$
$$= \dot{m}_C \; c_{p,C} \; (T_{C,o} - T_{C,i})$$
$$= A \; U \; F \; (LMTD)$$
$$= \varepsilon \; \dot{Q}_{max}$$

Classification by types of application

1. Boilers (or steam generators)
2. Condensers
3. Shell-and-tube heat exchangers
4. Cooling towers
5. Compact heat exchangers
6. Radiators for space power plants
7. Regenerators

Others

Boilers



It is one of the earlier types.

It is typically used to generate steam.

In conventional types, hot gases of combustion are used.

In nuclear reactors, hot pressurized water is used (PWRs) or liquid metal is used (LMFBRs), or direct contact with a hot surface is used (BWRs)

Condensers (Condensors)

The condensor coil of a refrigerator

It is typically used in power plants.

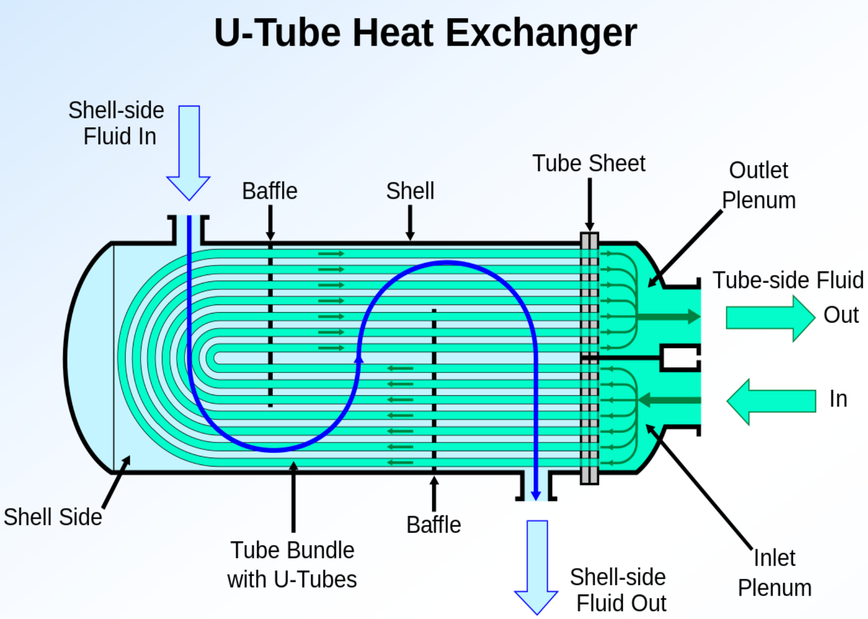
Major types include surface, jet, and evaporative condensers.

Surface condenser is the most common type.

Condenser: Condences steam to liquid water

Condensor: Condenses a refrigerant gas to liquid state.

Shell-and-tube heat exchangers

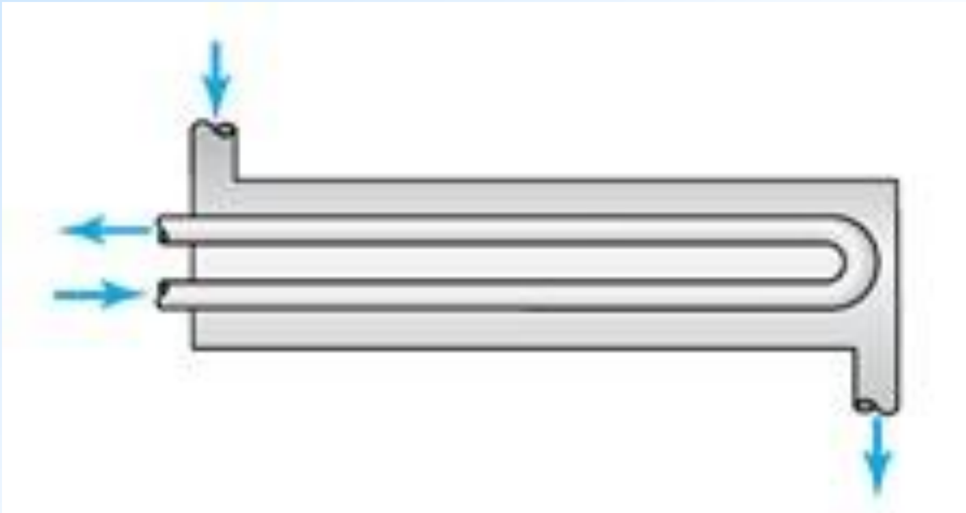


Liquid-to-liquid and in some cases gas-to-gas heat exchangers are of this type.

For liquids, h values on each side are within a factor of 2 to 3 of each other.

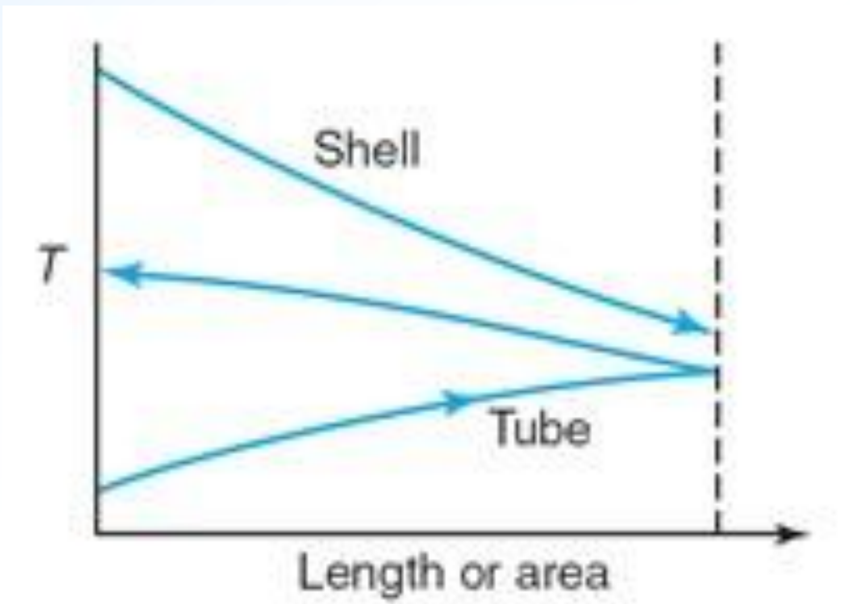
For gas-to gas types, h values are within 3 to 4 of wach other. But, much larger volume of heat transfer matrix is required because absolute values of h's are 10 to 100 times lower.

The number of flow passes on the tube side is determined mostly by the allowable pressure drop.

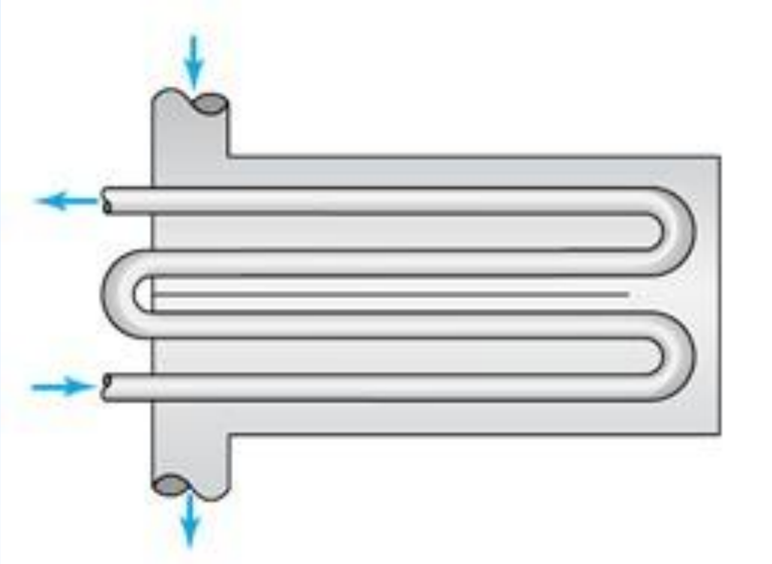


Shell-and-tube type

1 shell, 2 tube passes



$$\dot{Q} = \dot{m}_H \, c_{p,H} \, (T_{H,i} - T_{H,o})$$
$$= \dot{m}_C \, c_{p,C} \, (T_{C,o} - T_{C,i})$$
$$= A \, U \, F \, (LMTD)$$
$$= \varepsilon \, \dot{Q}_{max}$$



Shell-and-tube type

1 shell, 4 tube passes

Two analyses of heat transfer in a heat exchanger are commonly used:

Log-mean-temperature difference (LMTD) method, mainly for design

and

Effectiveness-NTU (ε-NTU) method, mainly for rating.

In both method of analysis, **overall heat transfer coefficient, U,** is used.

Cooling Towers

They are used to dispose waste heat from industrial processes or power plants by rejecting heat into the atmosphere rather than rives, lakes, or oceans.

Two types exist: natural convection and forced convection.

Compact heat exchangers

For aircraft, marine, and aerospace vehicles.

Weight and size are important parameters.

So, they are made compact and with fins.

Compact heat exchangers are characterized by a high surface area per unit volume, which can result in a higher efficiency than conventional heat exchangers, in a significantly smaller volume (typically Compact heat exchangers can achieve efficiencies of over 95% cf. 80% for non-compact heat exchangers). Compact heat exchangers transfer more energy in a cost-effective manner than other heat exchangers and save more energy when compared to standard technology.

Radiators for space power plants

The waste heat from condensers of space vehicles are by radiators that dissipate heat by radiation. They run red hot.

Regenerators

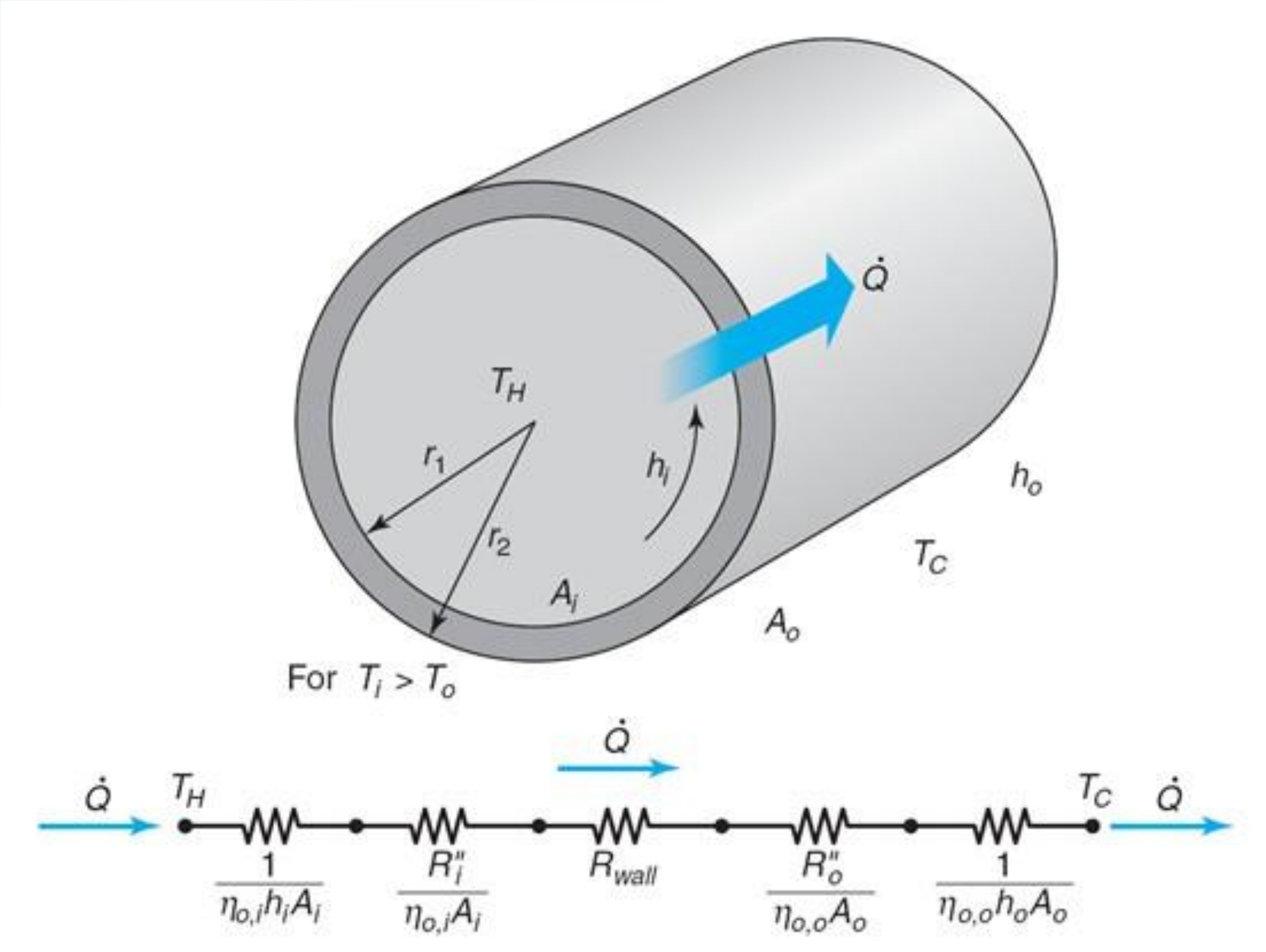
A regenerative heat exchanger, or more commonly a regenerator, is a type of heat exchanger where heat from the hot fluid is intermittently stored in a thermal storage medium before it is transferred to the cold fluid. To accomplish this the hot fluid is brought into contact with the heat storage medium, then the fluid is displaced with the cold fluid, which absorbs the heat.

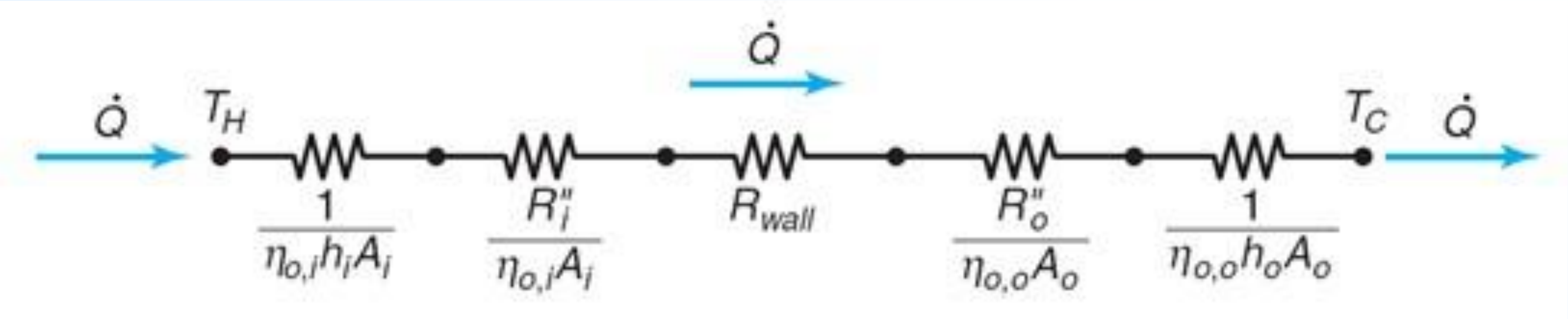
They are generally used to preheat air for steam power plants, furnaces, etc.

The same space is alternatively occupied by hot and cold gases between which heat is exchanged. They are usually rotary types.

In regenerative heat exchangers, the fluid on either side of the heat exchanger can be the same fluid. The fluid may go through an external processing step, and then it is flowed back through the heat exchanger in the opposite direction for further processing. Usually the application will use this process cyclically or repetitively.

11.2 Overall Heat Transfer Coefficient, U





$$R_{tot} = R_{conv,i} + R_{fouling,i} + R_w + R_{fouling,o} + R_{conv,o}$$

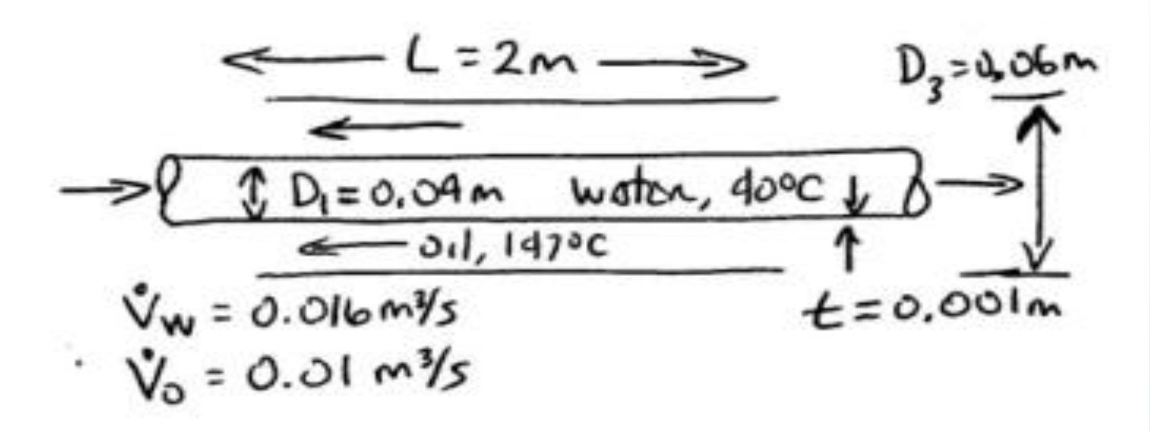
$$= \frac{1}{\eta_{f,i}\, h_i\, A_i} + \frac{R''_i}{\eta_{f,i}\, A_i} + \frac{\ln(r_o/r_i)}{2\pi k L} + \frac{R''_0}{\eta_{f,o}\, A_o} + \frac{1}{\eta_{f,o}\, h_o\, A_o}$$

$$R_{tot} = \frac{1}{U A} = \frac{1}{U_i\, A_i} = \frac{1}{U_o\, A_o}$$

Type of Fluid	Fouling Factor, R'', $m^2 \cdot K/W$
Water	
Seawater	0.000275–0.00035
Treated cooling tower water	0.000175–0.00035
River water	0.00035–0.00053
Treated boiler feedwater	0.00009
Liquids	
No. 6 fuel oil	0.0009
Engine lube oil	0.000175
Refrigerants	0.000175
Ethylene glycol solutions	0.00035
Kerosene	0.00035–0.00053
Heavy fuel oil	0.00053–0.00123
Gas or Vapor	
Steam (non-oil-bearing)	0.0009
Exhaust steam (oil-bearing)	0.00026–0.00035
Compressed air	0.000175
Natural gas flue gas	0.0009

Example 1 (Kaminsky P. 13-8)

A double-piped, cross-flow heat exchanger consists of a 4 cm pipe inside a 6 cm pipe. It is 2 m long. The water inside the inner pipe has an average temperature of 40 ºC and a flow rate of 0.016 m^3/s. In the annulus (between the inner and outer pipes) unused engine oil has an average temperature of 147 ºC and a flow rate of 0.01 m^3/s. The inner tube has a wall thickness of 1 mm and is made of 304 stainless steel.



a) Determine the overall heat transfer coefficient based on outside surface area of the inner tube (in $W/m^2.K$).

b) Determine the overall heat transfer coefficient based on outside surface area of the inner tube if the water and the oil sides are fouled (in $W/m^2.K$).

Assumptions

1. The overall heat transfer coefficient is uniform over the heat exchanger.
2. The flows are fully developed.

Calculate the overall heat transfer coefficient:

$$\frac{1}{U_o\ A_o} = \frac{1}{\eta_{o,i}\ h_i\ A_i} + \frac{R_i''}{\eta_{o,i}\ A_i} + \frac{\ln\left(D_o/D_i\right)}{2\ \pi\ k\ L} + \frac{R_o''}{\eta_{o,o}\ A_o} + \frac{1}{\eta_{o,o}\ h_o\ A_o}$$

There are no fins, so $\eta_{o,i} = \eta_{o,o} = 1$

The two areas are $A_i = \pi\, D_i\, L$ and $A_o = \pi\, D_o\, L = \pi\, \left(D_i + 2\,t\right) L$

Substituting in these expressions and simplifying:

$$\frac{1}{U_o} = \frac{1}{h_i}\left(\frac{D_i + 2\,t}{D_i}\right) + R_i''\left(\frac{D_i + 2\,t}{D_i}\right) + \frac{\left(D_i + 2\,t\right)\,\ln\left[\left(D_i + 2\,t\right)/D_i\right]}{2\,k} + R_o'' + \frac{1}{h_o}$$

The thermal conductivity of 304 stainless steel from Appendix A-2 is 14.9 W/mK.

The heat transfer coefficient requires the Reynolds number:

Water: $V = \dfrac{\dot{m}}{\rho\, A_i} = \dfrac{\dot{m}}{\rho\, \dfrac{\pi}{4}\, D_i^2}$ $\qquad$ $Re = \dfrac{\rho\, V\, D}{\mu} = \dfrac{4\,\rho\,\dot{V}}{\pi\,\mu\, D_i}$

For water from Appendix A-6 with $T_w = 40\ ^\circ C$:

$\mu = 6.34\ 10^{-4}\ N.s/m^2$
$k = 0.631\ W/m.K$
$Pr = 4.19$
$\rho = 992.2\ kg/m^3$

$$Re = \frac{4\ \rho\ \dot{V}}{\pi\ \mu\ D_i} = \frac{4\ (992.2)\ (0.016)}{\pi\ (6.34\ 10^{-4})\ (0.04)} = 797\ 000$$

This is turbulent flow, so using the Dittus-Boelter equation:

$$Nu = 0.023\ Re^{0.8}\ Pr^{0.4} = 0.023\ (797000)^{0.8}\ (4.19)^{0.4} = 2150$$

$$h_w = \frac{Nu\ k}{D_i} = \frac{(2150)\ (0.631)}{(0.04)} = 33\ 900\ W/m^2.K$$

For the oil Reynolds number and Nusselt number, we need to use the hydraulic diameter as the characteristic length:

$$D_h = \frac{4\,A_x}{P_{wetted}} = \frac{4\,\frac{\pi}{4}\left[D_3^2 - (D_t + 2\,t)^2\right]}{\pi\,D_3 + \pi\,(D_t + 2\,t)} = D_3 - (D_t + 2\,t)$$

$$= 0.06 - \left(0.04 + 2\,(0.001)\right) = 0.0180 \text{ m}$$

$$V = \frac{\dot{V}}{A_x} = \frac{\dot{V}}{\frac{\pi}{4}\left[D_3^2 - (D_i + 2\,t)^2\right]} = \frac{0.01}{\frac{\pi}{4}\left[(0.06)^2 - (0.04 + 0.002)^2\right]}$$

$$= 6.93 \text{ m/s}$$

For oil from Appendix A-6 with $T_0 = 147\ ^\circ C$:

$$\left.\right\} \quad \begin{aligned} \mu &= 56.4\ 10^{-4}\ N.s/m^2 \\ k &= 0.133\ W/m.K \\ Pr &= 103 \\ \rho &= 812.1\ kg/m^3 \end{aligned}$$

$$Re = \frac{(812.1)\ (6.93)\ (0.018)}{56.4\ 10^{-4}} = 18\ 000$$

This is turbulent flow, so again using the Dittus-Boelter equation:

$$Nu = 0.023\ Re^{0.8}\ Pr^{0.3} = 0.023\ (18000)^{0.8}\ (103)^{0.3} = 372$$

$$h_o = \frac{Nu\ k}{D_i} = \frac{(772)\ (0.133)}{(0.018)} = 2750\ W/m^2.K$$

The wall resistance is:

$$R_w = \frac{(D_i + 2\,t)\ln\left(\left(D_i + 2\,t\right)/D_i\right)}{2\,k} = \frac{(0.04 + 0.02)\ln\left(0.042/0.04\right)}{2\,(14.9)}$$

$$= 0.000069\ \text{m}^2.\text{K/W}$$

(a) Without fouling the overall heat transfer coefficient is (with consistent units omitted for brevity):

$$\frac{1}{U_o} = \frac{1}{33\,900}\left(\frac{0.042}{0.04}\right) + 0.000069 + \frac{1}{2750}$$

$$= 0.000031 + 0.000069 + 0.000364$$

$$U_o = 2160\ \text{W/m}^2.\text{K}$$

(b) With fouling using fouling factors estimated from Table 13.1 of:

$R''_{oil} \approx 0.000175\ m^2.K/W$ and $R''_{w} \approx 0.0004\ m^2.K/W$

$$\frac{1}{U_o} = 0.000031 + 0.0004\,\frac{0.042}{0.040} + 0.000069 + 0.000175 + 0.000364$$

$$= 0.00106\ m^2.K/W$$

$$U_o = 944\ W/m^2.K$$

The oil side heat transfer coefficient dominates the situation when there is no fouling. Adding fouling has a significant effect on the overall heat transfer coefficient.

11.3 Logarithmic Mean Temperature Difference, LMTD

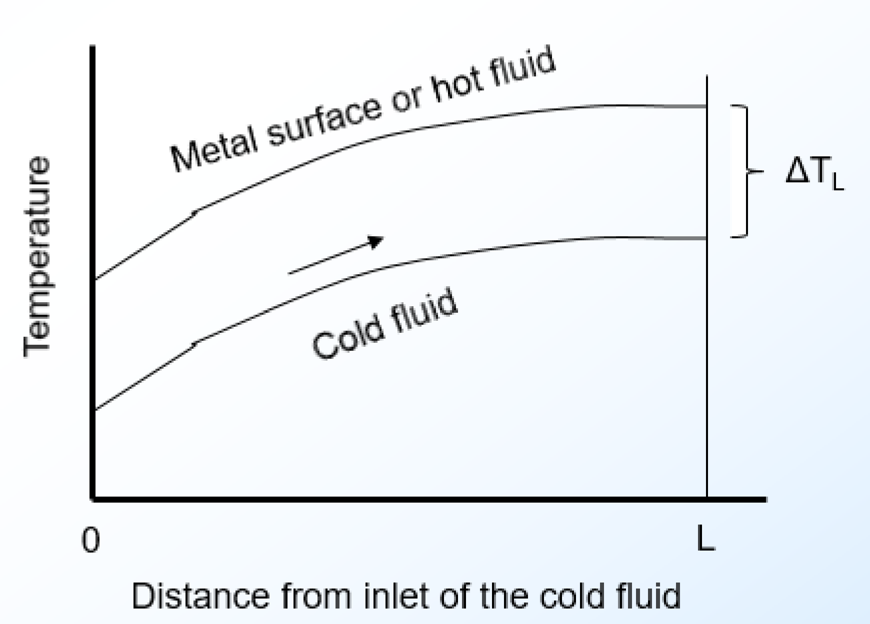
In stationary type heat exchangers, the temperture of the fluids change.

So does the heat flow rate.

Let's plot the temperature changes versus position for several cases

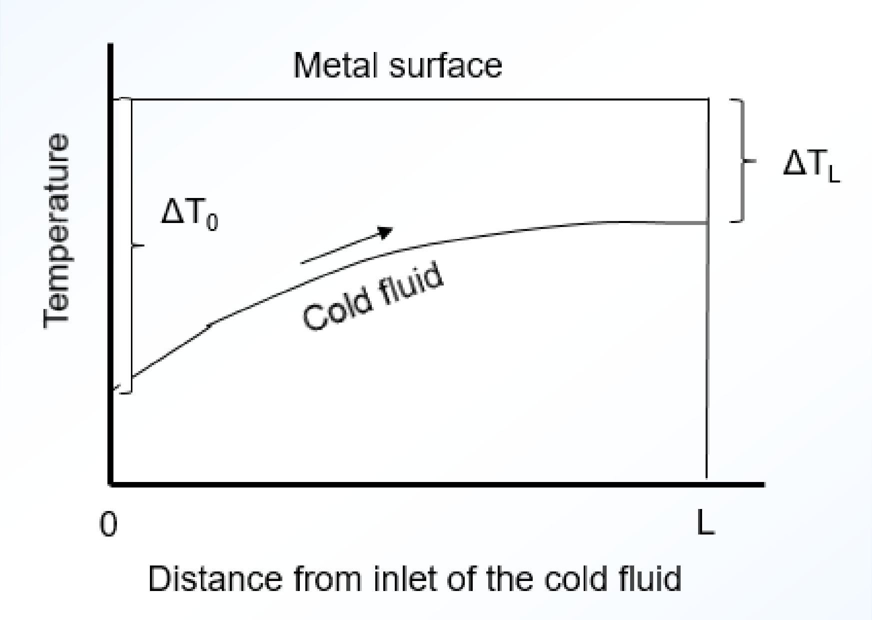
I. Uniform heat flux

(ΔT stays constant)



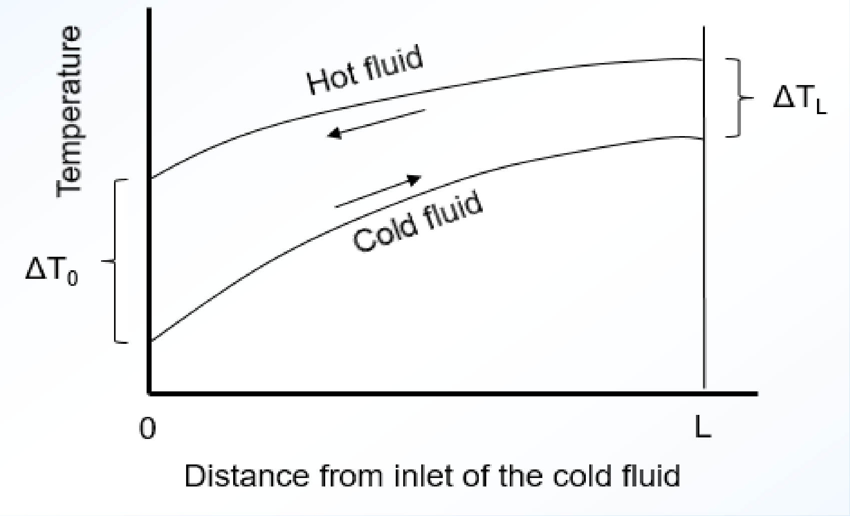
II. Uniform surface temperature

as in a boiler (gas heated)



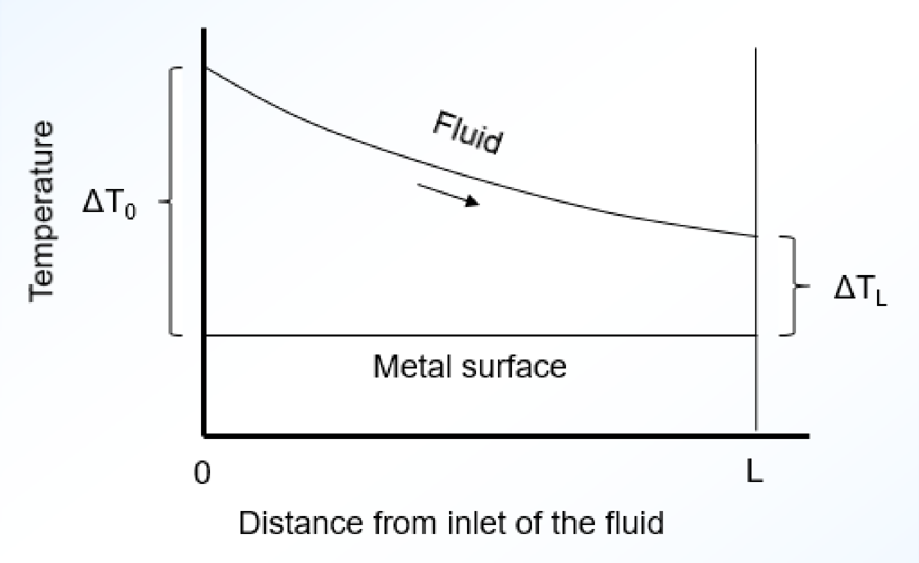
III. Counter flow

heat exchanger

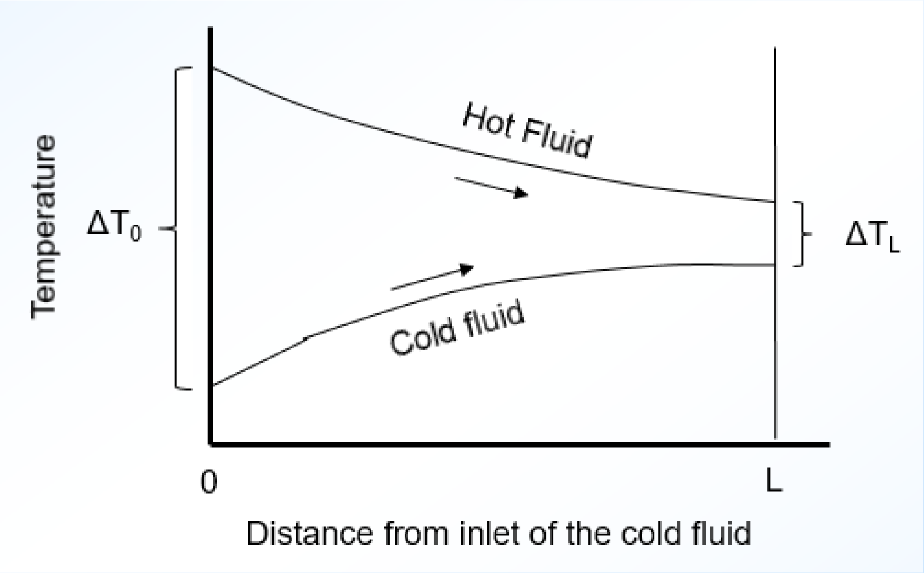


IV. Uniform surface temperature

as in a condensor



V. Parallel flow heat exchanger



LMTD $\dot{Q} = A_{tot}\ U_{mean}\ \Delta T_{mean}$

What is A_{tot}?

What is U_{mean}?

What is ΔT_{mean}?

A_{tot} is the total heat transfer area between the two fluids

U_{mean} is the mean (or average) overall heat transfer coefficient based on A_{tot}

Δt_{mean} is the mean (or the average) temperature difference betwwee the two fluids

Effectiveness $\dot{Q} = \varepsilon\ \dot{Q}_{max}$

What is ε?

What is Q_{max}?

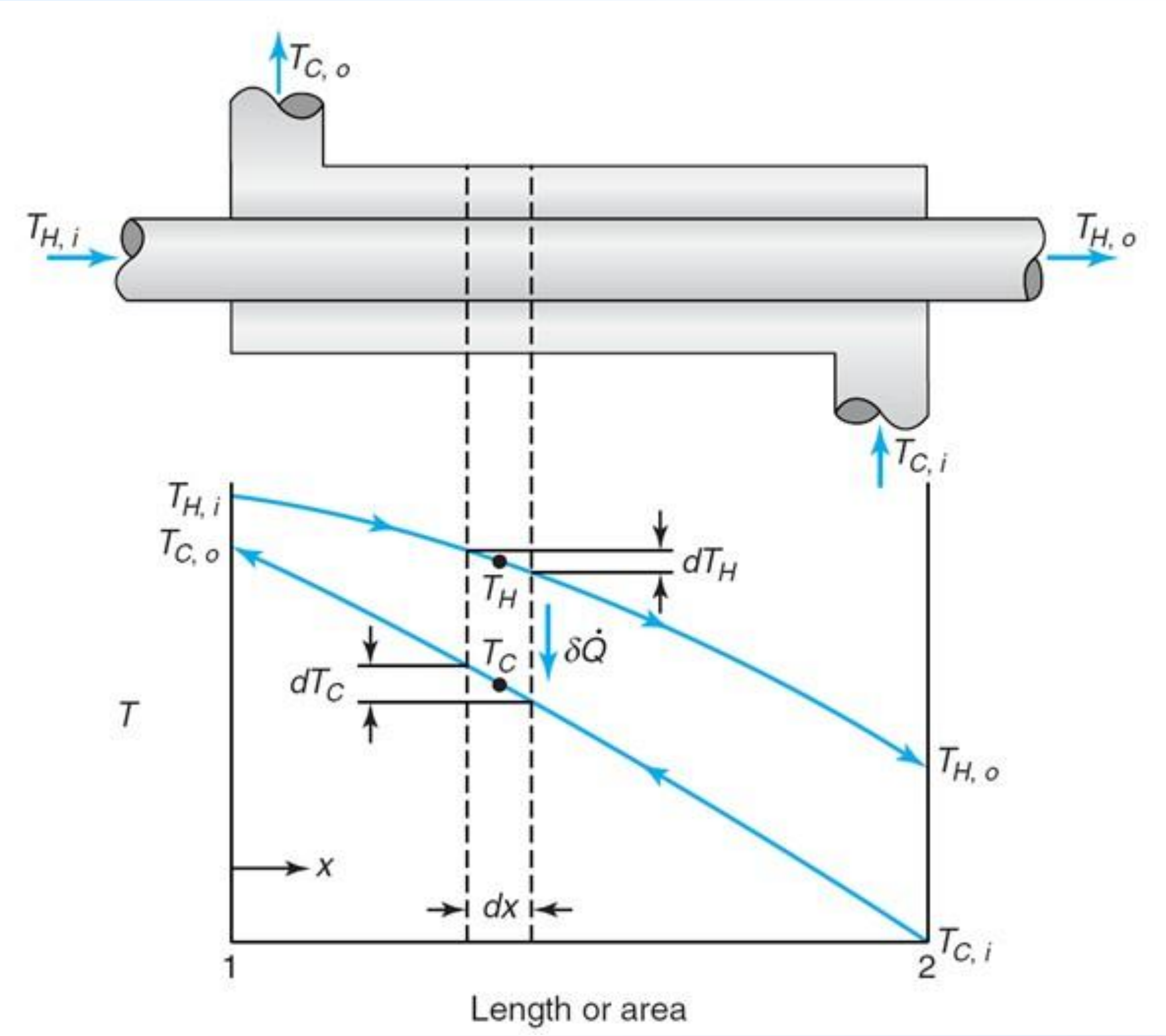
ε_t is the heat exchanger effectiveness

Q_{max} is the maximum possible (hypothetical) heat flow rate

The LMTD Method - Assumptions

1. Steady state
2. There are constant specific heats if the flow is single phase. If there is phase change (boiling or condensation), it occurs at a constant temperature (and pressure)
3. The constant overall heat transfer coefficient applies over the complete heat exchanger
4. If there are multiple tubes, each tube has the same flow rate. Likewise, the flow outside the tubes is evenly distributed across the heat exchanger
5. Temperature and velocities are uniform over all cross-sectional flow areas
6. The two fluids exchange heat only with each other, and there is no shaft work or heat generation. Potential and kinetic energy effects are ignored
7. Axial conduction along the solid surfaces is ignored

The LMTD Method



$$\delta\dot{Q} = U\,(T_H - T_C)\,dA$$

$$\delta\dot{Q} = -\,\dot{m}_H\;c_{p,H}\;dT_H$$

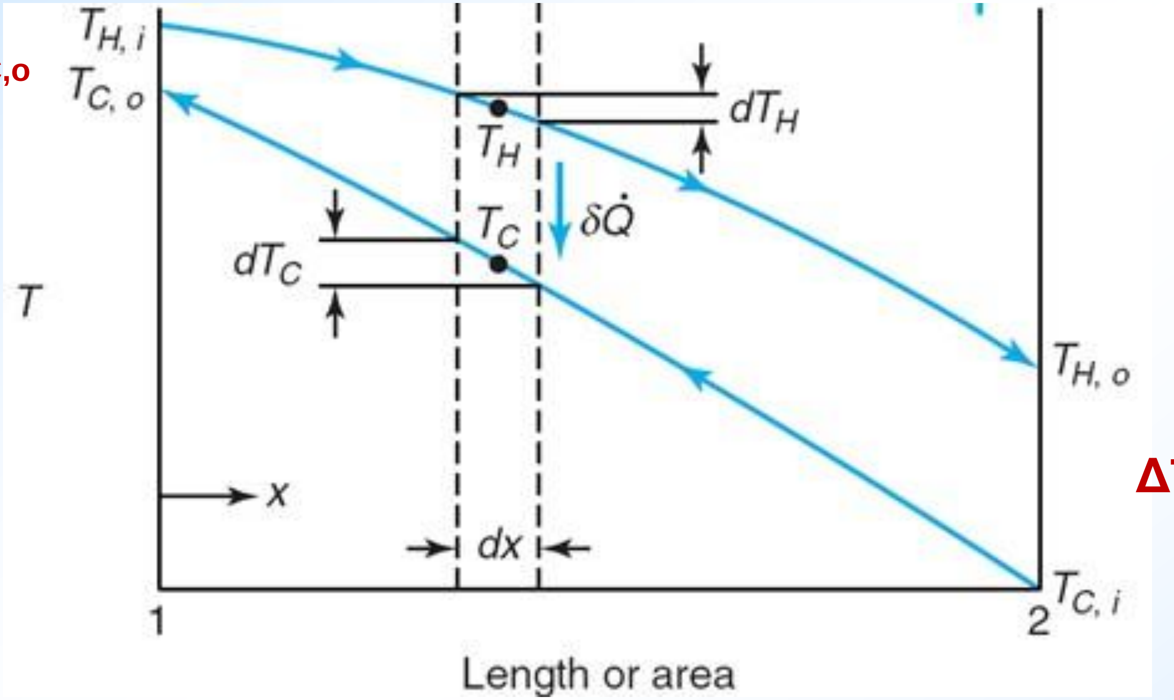
$$\delta\dot{Q} = -\,\dot{m}_C\;c_{p,C}\;dT_C$$

See the derivation

on page 631

$$\dot{Q} = U\,A\,\frac{\Delta T_1 - \Delta T_2}{\ln\left(\dfrac{\Delta T_1}{\Delta T_2}\right)}$$

$\Delta T_1 = T_{H,i} - T_{C,o}$



$\Delta T_2 = T_{H,o} - T_{C,i}$

$$\dot{Q} = U\,A\,\frac{\Delta T_1 - \Delta T_2}{\ln\left(\dfrac{\Delta T_1}{\Delta T_2}\right)} = \dot{m}_H\; c_{p,H}\;(T_{H,i} - T_{H,o}) = \dot{m}_C\; c_{p,C}\;(T_{C,o} - T_{C,i})$$

Note that this is for cross-flow heat exchangers.

For other type of heat exchangers use the correction factor, F

$$\dot{Q} = U\,A\,F\,\frac{\Delta T_1 - \Delta T_2}{\ln\left(\frac{\Delta T_1}{\Delta T_2}\right)} = \dot{m}_H\, c_{p,H}\,(T_{H,i} - T_{H,o}) = \dot{m}_C\, c_{p,C}\,(T_{C,o} - T_{C,i})$$

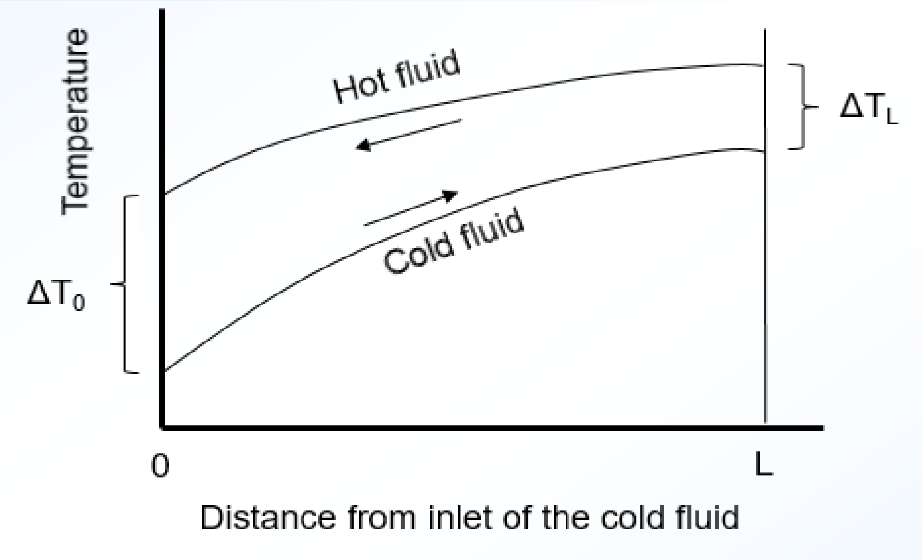
Use the charts in Figure 13.7 for the correction factor, F

Use the steps given in Table 13.2 for designing (finding the required area) or rating (finding U or inlet and/or outlet temperatures) a heat exchanger using the LMTD method.

Example 2

Consider an oil cooler for a large Diesel engine to cool SAE-30 lubricating oil from 65 ºC to 55 ºC using sea water at an inlet temperature 25 ºC with a temperature rise of 10 ºC. The design heat load is Q = 200 kW. Assuming an average U_m = 740 W/m^2.K based on the outer surface area of the tubes, determine the heat transfer surface area required for a single pass (a) counter flow and (b) parallel flow arrangements.

(a) Counter flow case



$$\Delta T_0 = 55 - 25 = 30\ ^\circ C$$

$$\Delta T_L = 65 - 35 = 30\ ^\circ C$$

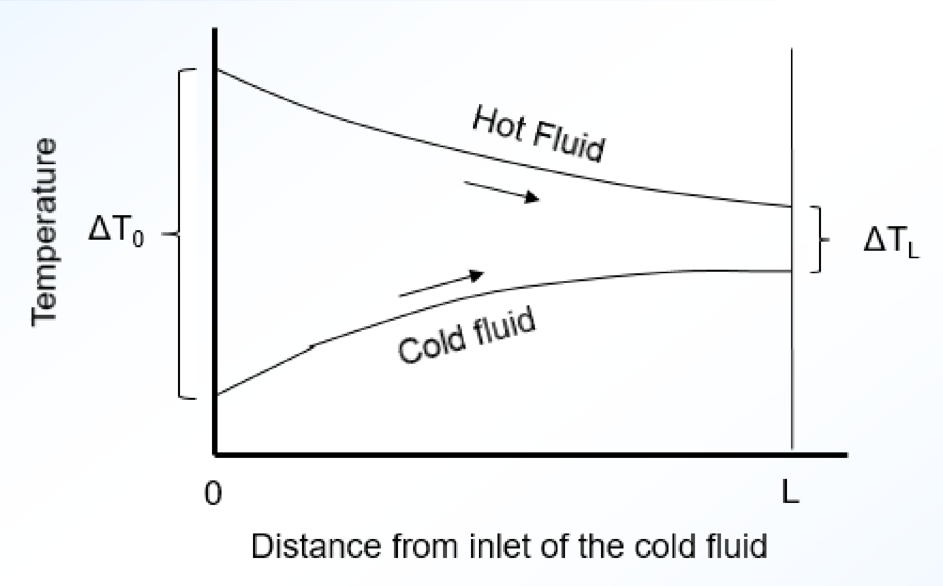
$$LMTD = \Delta T_0 = \Delta T_L = 30\ ^\circ C$$

$$\dot{Q} = A_t\ U_m\ LMTD$$

$$A_t = \frac{\dot{Q}}{U_m\ \Delta T_m} = \frac{200000}{(30)\ (740)}$$

$$A_t \simeq 9\ m^2$$

(a) Parallel flow case



$$\Delta T_0 = 65 - 25 = 40 \ ^\circ C$$

$$\Delta T_L = 55 - 35 = 20 \ ^\circ C$$

$$LMTD = \frac{40 - 20}{\ln\left(\frac{40}{20}\right)} = 28.8 \ ^\circ C$$

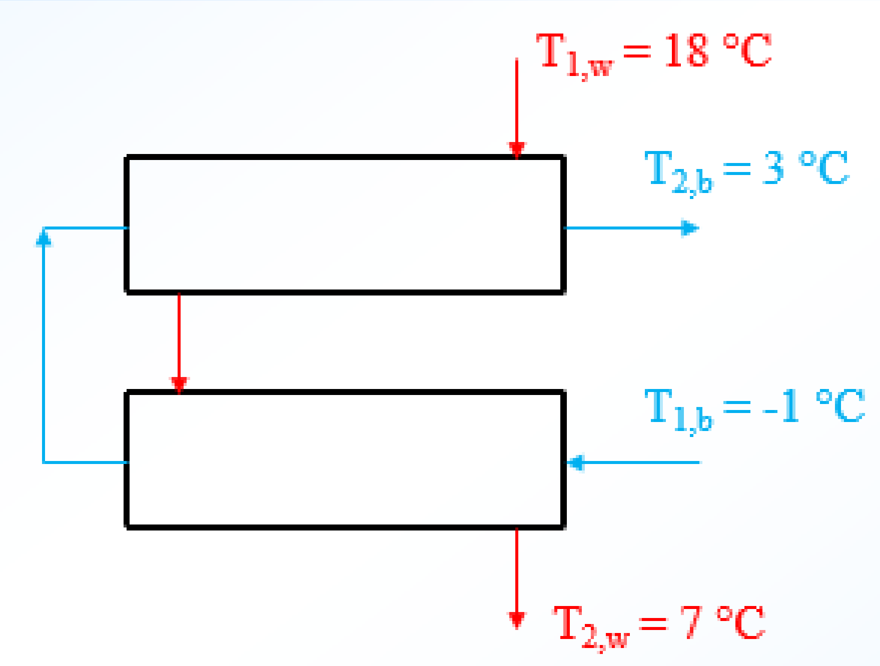
$$\dot{Q} = A_t \ U_m \ LMTD$$

$$A_t = \frac{\dot{Q}}{U_m \ \Delta T_m} = \frac{200000}{(28.8)\ (740)}$$

$$A_t \simeq 9.37 \ m^2$$

For the conter flow arrangement, slightly less area is required.

Example 3



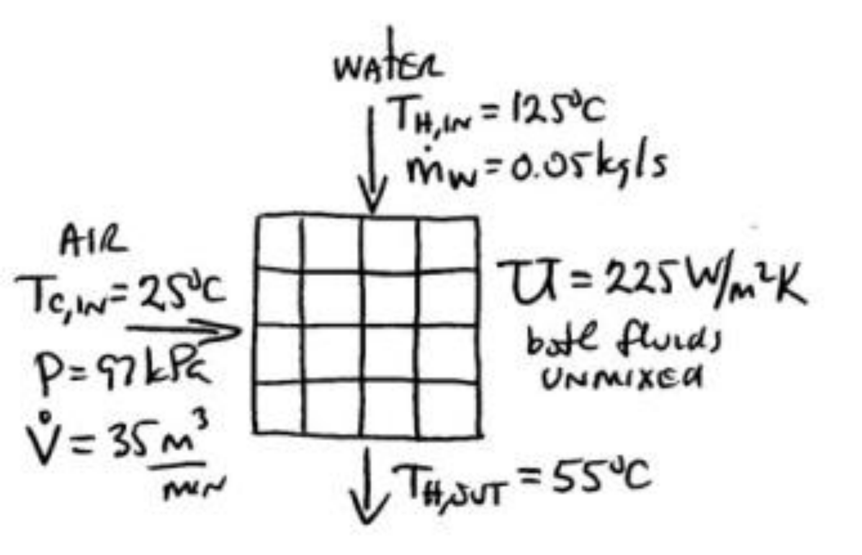
Water is to be cooled from 18 ºC to 7 ºC by using brine at an inlet temperature of -1 ºC with a temperature rise of 4 ºC. The brine and water flows are on the tube and shell sides, respectively. Determine the total heat transfer area required for a cross-flow arrangement shown in the figure by assuming U_m = 850 W/m2.K and a design heat load Q = 6000 W. F = 1

$$\text{LMTD} = \frac{8 - 15}{\ln\left(\dfrac{8}{15}\right)} = 11.135\ ^\circ\text{C}$$

$$A = \frac{\dot{Q}}{U\ F\ \text{LMTD}} = \frac{6000}{(850)\ (1)\ (11.135)} = 0.634\ \text{m}^2$$

Example 4 (Kaminsky P. 13-28)

Car radiators are single-pass cross-flow heat exchangers with both fluids unmixed. Water at 0.05 kg/s enters the tubes at 125 ºC and leaves at 55 ºC. Air enters the heat exchanger at 35 m^3/min, 25 ºC, and 97 kPa. The overall heat transfer coefficient is 225 W/m^2.K. Determine the required heat transfer area (in m^2).



Assumptions:

1. The system is steady.
2. No work is done on or by the control volume.
3. Potential and kinetic energy effects are negligible.
4. Air is an ideal gas.

The governing equation for the LMTD method is:

$$\dot{Q} = U\ A\ F\ LMTD \quad \Rightarrow \quad A = \frac{\dot{Q}}{U\ F\ LMTD}$$

The heat transfer rate can be obtained from conservation of energy applied to the water flow. Assuming steady, no work, negligible potential and kinetic energy effects, and ideal fluid with constant specific heat so that $\Delta h = c_p\ \Delta T$:

$$\dot{Q} = \dot{m}_w\ c_{p,w}\ (T_{H,in} - T_{H,out})$$

From Appendix A-6 for water at an average temperature of (125 + 55) / 2 = 90 ºC, $c_{p,w} \approx 4.206$ kJ/kg.K:

$$\dot{Q} = (0.05)\ (4.206)\ (125 - 55) = 14.72 \text{ kW}$$

For the LMTD we need $T_{C,out}$. The air outlet temperature is obtained with an energy balance on the air using the same assumptions as used for the water and using the heat transfer rate calculated above. Therefore,

$$\dot{Q} = \dot{m}_a \; c_{p,a} \; (T_{C,out} - T_{C,in}) \quad \Rightarrow \quad T_{C,out} = T_{C,in} + \frac{\dot{Q}}{\dot{m}_a \; c_{p,a}}$$

From Appendix A-7 for air with an estimated $T_{out} \approx 45$ ºC, $T_{avg} = (25 + 45) / 2 = 35$ ºC, $c_p \approx 1.006$ kJ/kg.K:

For the mass flow rate: $\dot{m}_a = \dfrac{\dot{V}}{\nu_a}$ Assuming air is an ideal gas

$$\nu = \frac{R\,T}{P\,M} = \frac{(8.314)\,(25 + 273)}{(97)\,(29.87)} = 0.882 \text{ m}^3/\text{kg}$$

$$\dot{m}_a = \frac{\dot{V}}{\nu_a} = \frac{(35)\,(1\,/\,60)}{0.882} = 0.66 \text{ kg/s}$$

$$T_{C,out} = 25 + \frac{14.7}{(0.66)\,(1.006)} = 47.1\ ^\circ C$$

$$\Delta T_1 = T_{H,in} - T_{C,out} = 125 - 47.1 = 77.9\ ^\circ C$$

$$\Delta T_2 = T_{H,out} - T_{C,in} = 55 - 25 = 30\ ^\circ C$$

$$LMTD = \frac{\Delta T_1 - \Delta T_2}{\ln\left(\frac{\Delta T_1}{\Delta T_2}\right)} = \frac{77.9 - 30}{\ln\left(\frac{77.9}{30}\right)} = 50.2\ ^\circ C$$

Using Figure 13-7c to determine F:

$$R = \frac{25 - 47.1}{55 - 125} = 0.32$$

$$P = \frac{55 - 125}{25 - 125} = 0.70$$

From Figure 13-7c for crossflow with both fluids unmixed, F = 0.99

Total heat transfer area: $A = \dfrac{(14.7)\,(1000)}{(225)\,(0.99)\,(50.2)} = 1.32\ m^2$

11.4 Heat Exchanger Effectiveness

$$\dot{Q} = A\,U_m\ \text{LMTD}$$

If Q is calculated base on this equation, the presumption is that both inlet and outlet temperatures are known.

However, the inlet and outlet temperatures are not known in practical problems.

Define heat exchanger effectiveness as: $\varepsilon = \dfrac{\text{Actual heat transfer rate}}{\text{Maximum possible heat transfer rate}}$

$$\varepsilon = \frac{\dot{Q}_{act}}{\dot{Q}_{max}} = \frac{\dot{Q}_{act}}{(\dot{m}\,c_p)_{min}\left(T_{H,i} - T_{C,i}\right)}$$

$$\dot{Q}_{max} = (\dot{m}\ c_p)_{min}\ \left(T_{H,i} - T_{C,i}\right)$$

This assumes that A_t is infinite and there are no losses

$$\dot{Q} = \varepsilon\ (\dot{m}\ c_p)_{min}\ \left(T_{H,in} - T_{C,in}\right)$$

See the derivation on pages 638 and 639.

$$\varepsilon = \frac{\dot{Q}_{act}}{\dot{Q}_{max}} = \frac{1 - \exp\left[- \left(1 - \dfrac{(\dot{m}\ c_p)_{min}}{(\dot{m}\ c_p)_{max}}\right) \dfrac{U\ A}{(\dot{m}\ c_p)_{min}} \right]}{1 - \dfrac{(\dot{m}\ c_p)_{min}}{(\dot{m}\ c_p)_{max}} \exp\left[- \left(1 - \dfrac{(\dot{m}\ c_p)_{min}}{(\dot{m}\ c_p)_{max}}\right) \dfrac{U\ A}{(\dot{m}\ c_p)_{min}} \right]}$$

$$\varepsilon = \frac{\dot{Q}_{act}}{\dot{Q}_{max}} = \frac{1 - \exp\left[- \left(1 - \frac{(\dot{m}\, c_p)_{min}}{(\dot{m}\, c_p)_{max}} \right) \frac{U\,A}{(\dot{m}\, c_p)_{min}} \right]}{1 - \frac{(\dot{m}\, c_p)_{min}}{(\dot{m}\, c_p)_{max}} \exp\left[- \left(1 - \frac{(\dot{m}\, c_p)_{min}}{(\dot{m}\, c_p)_{max}} \right) \frac{U\,A}{(\dot{m}\, c_p)_{min}} \right]}$$

Define number of heat transfer units: $NTU = \frac{U\,A}{(\dot{m}\, c_p)_{min}}$

Define heat capacity ratio: $C^* = \frac{(\dot{m}\, c_p)_{min}}{(\dot{m}\, c_p)_{max}}$

For evaporators and condensors $C^* = 0$ because $C_{max} \rightarrow \infty$ (Heat capacity rate is considered infinite if fluid remains ata constant temperature.)

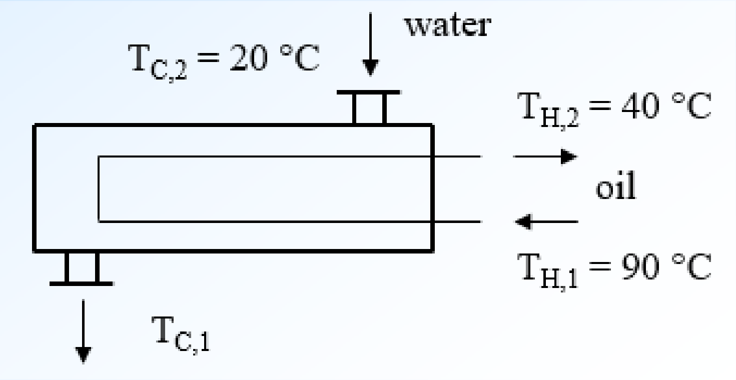
$$\varepsilon = \frac{\dot{Q}_{act}}{\dot{Q}_{max}} = \frac{1 - \exp\left[-\left(1 - \frac{(\dot{m}\, c_p)_{min}}{(\dot{m}\, c_p)_{max}}\right) NTU\right]}{1 - \frac{(\dot{m}\, c_p)_{min}}{(\dot{m}\, c_p)_{max}} \exp\left[-\left(1 - \frac{(\dot{m}\, c_p)_{min}}{(\dot{m}\, c_p)_{max}}\right) NTU\right]}$$

$$\varepsilon = \frac{\dot{Q}_{act}}{\dot{Q}_{max}} = \frac{1 - \exp\left[-NTU\,(1 - C^*)\right]}{1 - C^* \exp\left[-NTU\,(1 - C^*)\right]}$$

If $C^* = 0$ $\varepsilon = \frac{\dot{Q}_{act}}{\dot{Q}_{max}} = 1 - \exp(-NTU)$

See Table 13-3 and Figure 13.8

Example 5



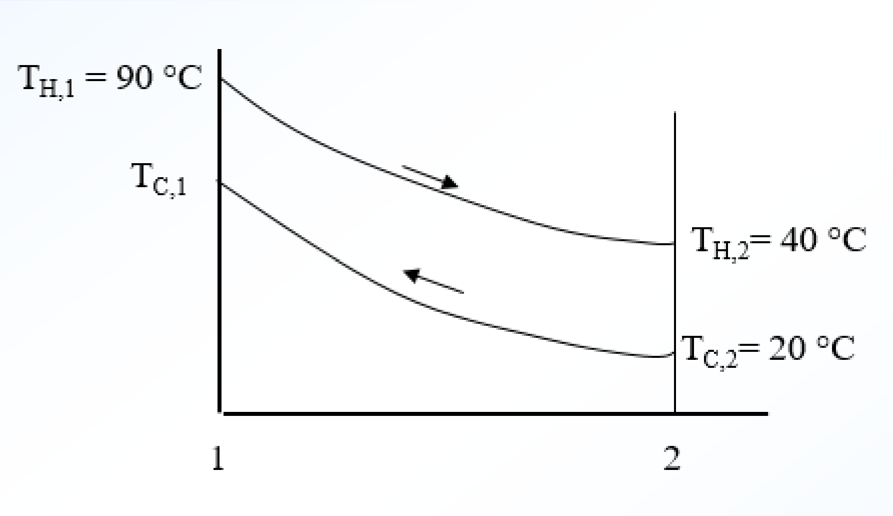
m_w = 1 kg/s ⇨ $C_w = C_C$ = 4.18 kW/K

m_o = 1 kg/s

c_o = 2.1 kJ/kg.K

} $C_o = C_H$ = 2.1 kW/K

U = 400 W/m^2.K Find A



Solution:

$$\dot{Q} = U\,A\,F\,\Delta T_m$$

$$\dot{Q} = C_H\,\Delta T_H = C_C\,\Delta T_C$$

$$\dot{Q} = (4.18)\left(T_{C,1} - 20\right)$$

$$= (2.1)\,(90 - 40) = 105 \text{ kW}$$

$$T_{C,1} = 20 + 25 = 45\ ^\circ\text{C}$$

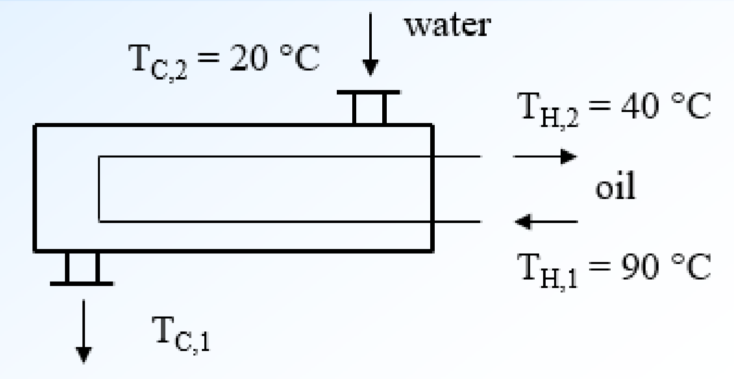
$$\Delta T_m = LMTD = \frac{(90 - 45) - (40 - 20)}{\ln\left(\frac{45}{20}\right)} = \frac{25}{0.811} = 30.8 \simeq 31 \text{ K}$$

Use Figure 10.8

$$\left.\begin{aligned} P &= \frac{50}{70} = 0.714 \\ R &= \frac{25}{50} = 0.5 \end{aligned}\right\} \quad F \cong 0.72$$

$$A = \frac{\dot{Q}}{U\,F\,\Delta T_m} = \frac{105}{(400)(0.72)(31)} = 11.8 \simeq 12 \text{ m}^2$$

Example 6



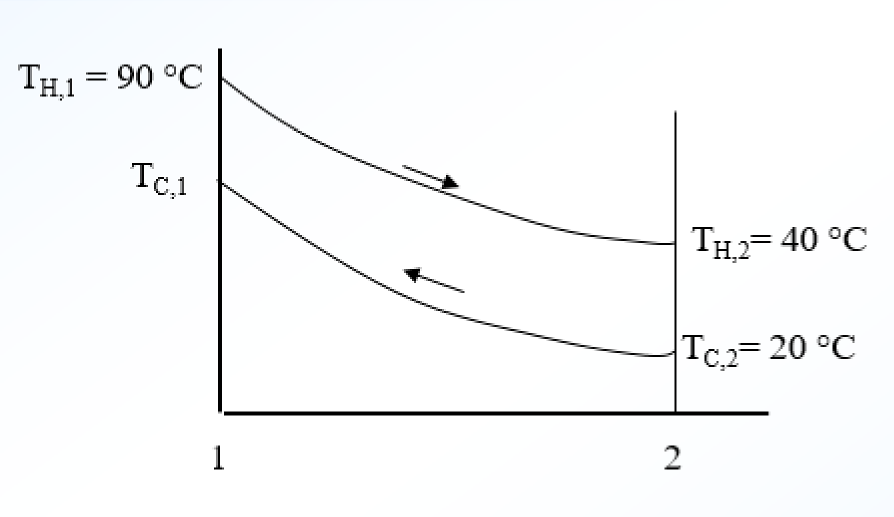
$m_w = 1$ kg/s $\Rightarrow$ $C_w = C_C = 4.18$ kW/K

$m_o = 1$ kg/s

$c_o = 2.1$ kJ/kg.K

$\}$ $C_o = C_H = 2.1$ kW/K

$U = 400$ W/m^2.K $A = 12$ m^2 Find $\dot{Q}$



Solution:

$$\dot{Q} = \varepsilon\ \dot{Q}_{max}$$

$$\dot{Q}_{max} = (2.1)\ (90 - 20) = 147 \text{ kW}$$

Use Figure 10.16

$$\frac{A\,U}{C_{min}} = \frac{(12)\,(400)}{2100} = 2.28$$

$$\frac{C_{min}}{C_{max}} = \frac{2.1}{4.2} = 0.5$$

$$\varepsilon \cong 0.71$$

$$\dot{Q} = \varepsilon\, Q_{max} = (0.71)\,(147) = 104 \text{ kW} \quad \text{OK!}$$

Example 7

A shell-and-tube type heat exchanger is designed to cool 1.5 kg/s oil (c_p = 2100 J/kg.K) from 65 °C to 42 °C by using 1.0 kg/s water (c_p = 4200 J/kg.K) at an inlet temperature of 26 °C. U_m = 680 W/m^2.K. Determine A_t for

(a) Single-shell pass heat exchanger of Fig. 10.16; and

(b) Two-shell pass heat exchanger of Fig. 10.17.

(a)

$$C_h = \dot{m}_h \, c_{p,h} = (1.5)\,(2100) = 3150 \text{ W/K}$$

$$C_c = \dot{m}_c \, c_{p,c} = (1.0)\,(4200) = 4200 \text{ W/K}$$

$$\frac{C_{min}}{C_{max}} = \frac{3150}{4200} = 0.75$$

$$\varepsilon = \frac{T_{h,in} - T_{h,out}}{T_{h,in} - T_{c,in}} = \frac{65 - 42}{65 - 26} \simeq 0.6$$

$$\left.\begin{aligned} &\varepsilon = 0.6 \\ &\frac{C_{min}}{C_{max}} = 0.75 \end{aligned}\right\} \quad \text{From Fig 10.16:} \quad \frac{A_t\, U_m}{C_{min}} = 1.7$$

$$A_t = 1.7\,\frac{C_{min}}{U_m} = 1.7\,\frac{3150}{680} = 7.875\ m^2$$

(b)

$$\left.\begin{aligned} &\varepsilon = 0.6 \\ &\frac{C_{min}}{C_{max}} = 0.75 \end{aligned}\right\} \quad \text{From Fig 10.17:} \quad \frac{A_t\, U_m}{C_{min}} = 1.35$$

$$A_t = 1.35\,\frac{C_{min}}{U_m} = 1.35\,\frac{3150}{680} = 6.25\ m^2$$

11.5 Fouling and Scaling of Heat Exchangers

Deposits during operation will cause additional thermal resistance.

$$R = \frac{1}{A_i\ h_i} + \frac{F_i}{A_i} + \frac{t}{k\ A_m} + \frac{F_o}{A_o} + \frac{1}{A_o\ h_o}$$

R: Thermal resistance

A_o, A_i: Outside and inside surface area of the tube

$$A_m = \frac{A_o - A_i}{\ln\left(A_o / A_i\right)}$$

: Logarithmic mean area

F_o, F_i: Unit fouling resistances, $m^2.K/W$

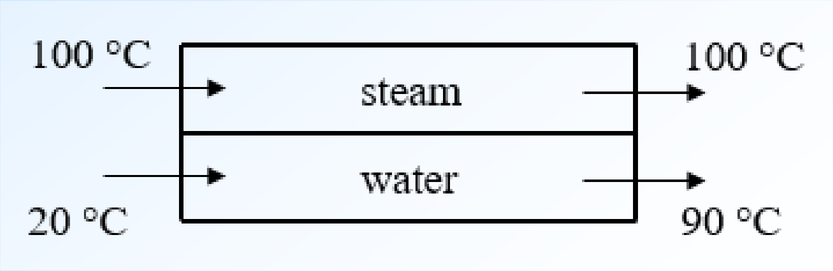
h_o, h_i: Heat transfer coefficients on the outside and inside

t, k: Thickness and conductivity of the tube

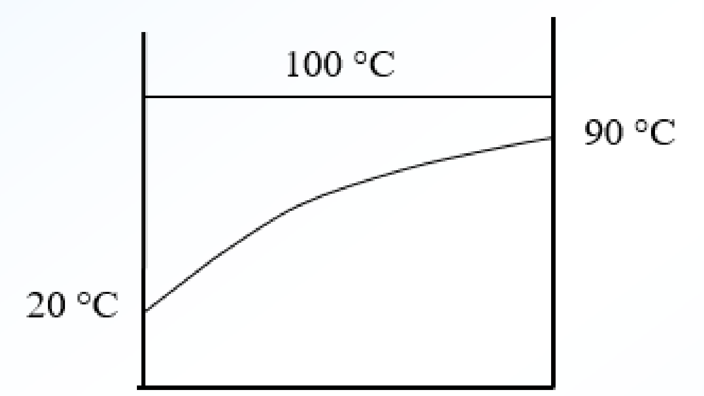
$$U_o = \frac{1}{A_o\ R}$$

based on the outside surface

Example 8



Multiple pipe - single pass condenser



$\dot{m}_H = 1.0$ kg/s   Rate of condensation

$U = 4000$ W/m^2.K

Find m_c and A

$$\dot{Q} = \dot{m}_H\, h_{fg} = (1)\,(2257) = 2257 \text{ kW}$$

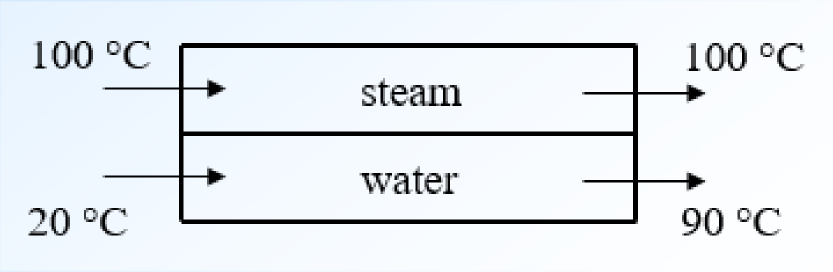
$$\dot{m}_C = \frac{\dot{Q}}{c_C\,(\Delta T)_C} = \frac{2257}{(4.2)\,(70)} = 7.7 \text{ kg/s}$$

$$(\Delta T)_m = \text{LMTD} = \frac{80 - 10}{\ln\left(\frac{80}{10}\right)} = 33.7 \text{ K}$$

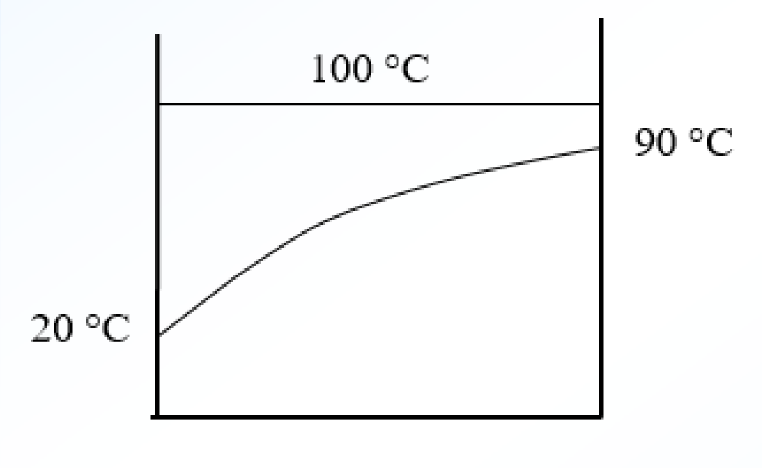
$$A = \frac{\dot{Q}}{U\,(\Delta T)_m} = \frac{2257}{(4)\,(33.7)} = 16.8 \text{ m}^2$$

Note that LMTD is very different from the average, (80+10)/2 = 45 K and parallel or cross flow makes no difference

Example 9



Multiple pipe - single pass condenser



Same heat exchanger as in Example 8

$\dot{m}_C = 7.7$ kg/s $T_{C,in} = 20\ ^\circ C$

Find the rate of condensation, $\dot{m}_H$

Think of steam as fluid with $c \rightarrow \infty$

$$\dot{Q}_{max} = (7.7)\,(4.2)\,(80) = 2590 \text{ kW}$$

$$\frac{A\,U}{C_{min}} = \frac{(16.8)\,(4.0)}{(7.7)\,(4.2)} = 2.08$$

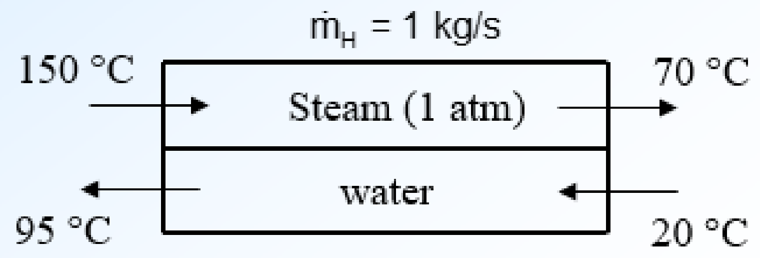
$$\frac{C_{min}}{C_{max}} = 0$$

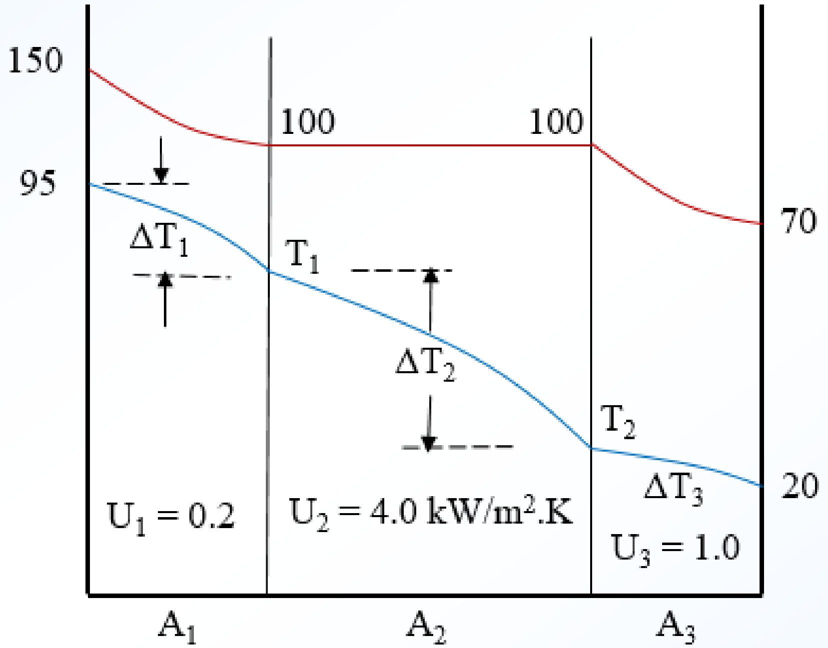
Fig. 10.12 or 10.13 → $\varepsilon \cong 0.87$

$$\dot{Q} = (0.87)\,(2590) \cong 2250 \text{ kW}$$

$$\dot{m}_H = 2250 / 2260 \simeq 1.0 \text{ kg/s}$$

Example 10





Find the total heat transfer area.

$$\dot{Q}_1 = (2.0)\,(50)\,(1) = 100 \text{ kW}$$

$$\dot{Q}_2 = (2257)\,(1) = 2257 \text{ kW}$$

$$\dot{Q}_3 = (4.2)\,(3.0)\,(1) = 126 \text{ kW}$$

Total = 2483 kW

$$C_C = \frac{2453}{75} = 33.1 \text{ kW/K}$$

$$\dot{m} = \frac{33.1}{4.2} = 7.88 \text{ kg/s}$$

$$\Delta T_1 = \frac{100}{33.1} = 3.02 \text{ K} \qquad (T_1 = 92\ ^\circ C)$$

$$\Delta T_2 = \frac{2257}{33.1} = 68.19 \text{ K} \qquad (T_2 = 23.8\ ^\circ C)$$

$$\Delta T_3 = \frac{126}{33.1} = 3.81 \text{ K}$$

Check: 75:01 K

$$(\Delta T_m)_1 = \frac{55 - 8}{\ln(55/8)} = 24.4 \text{ K}$$

$$(\Delta T_m)_2 = \frac{76.2 - 8}{\ln(76.2/8)} = 30.2 \text{ K}$$

$$(\Delta T_m)_3 = \frac{76.2 - 50}{\ln(76.2/50)} = 62.2 \text{ K}$$

$$A_1 = \frac{100}{(0.2)\,(24.4)} = 20.5 \text{ m}^2$$

$$A_2 = \frac{2257}{(4.0)\,(30.2)} = 18.7 \text{ m}^2$$

$$A_3 = \frac{126}{(1.0)\,(62.2)} = 2.0 \text{ m}^2$$

$A = 41.2 \text{ m}^2$

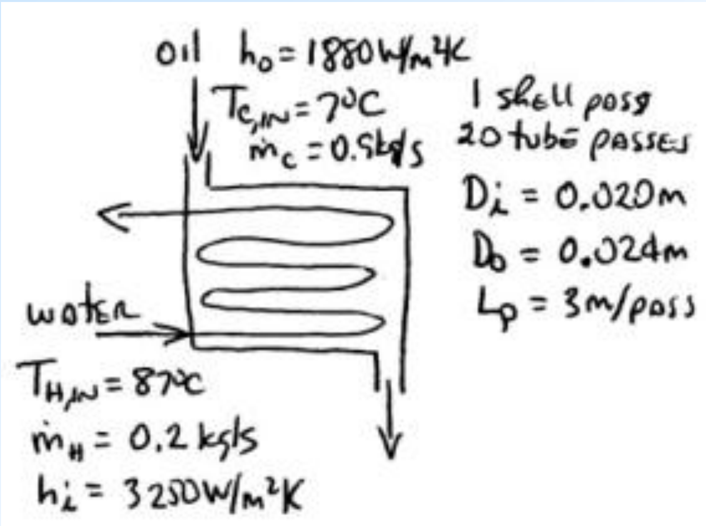
Note the relative values

Example 11, Kaminsky P.13-46

A shell-and-tube heat exchanger with 1 shell pass and 20 tube passes uses hot water on the tube side to heat unused engine oil on the shell side. The single 304 stainless-steel tube has inner and outer diameters of 20 and 24 mm, respectively, and a length per pass of 3 m. The water enters at 87 ºC and 0.2 kg/s. The oil enters at 7 ºC and 0.9 kg/s. The shell-side (oil) heat transfer coefficient is 1880 $W/m^2.K$, and the tube-side (water) heat transfer coefficient is 3250 $W/m^2.K$.

(a) Determine the outlet temperature of the oil (in ºC).

(b) Determine the new outlet temperature of the oil, if, over time, the oil fouls the surface such that a fouling factor of 0.003 $m^2.K/W$ can be assumed (in ºC).



A The outlet temperature is sought, which is equivalent to determining the heat transfer rate. This means that this is a rating problem, and the preferred approach is the ε-NTU method. The overall heat transfer coefficient must be evaluated from the given information.

Assumptions

1. The overall heat transfer coefficient is uniform over the heat exchanger.
2. The water side flow is fully developed.
3. The system is steady.
4. No work is done or by the control volume.
5. Potential and kinetic energy effects are negligible.
6. Oil is an ideal liquid with constant specific heat.

(a)

The governing equation for the ε-NTU method is: $\dot{Q} = \varepsilon\,\dot{Q}_{max} = \varepsilon\,(\dot{m}\,c_p)_{min}\,(T_{H,in} - T_{C,in})$

If we determine $(\dot{m}\,c_p)_{min}$ $\frac{C_{min}}{C_{max}}$ and $NTU = \frac{U\,A}{C_{min}}$ then we can evaluate ε.

Once we have Q , we apply conservation of energy to the oil flow to obtain the outlet temperature.

Assume steady, no work, negligible potential and kinetic energy, and an ideal liquid with constant specific heat so that $\Delta h = c_p\,\Delta T$:

$$\dot{Q} = m_C\;c_{p,C}\;(T_{C,out} - T_{C,in})$$

Equating the energy equation and the rate equation, and solving:

$$T_{C,out} = T_{C,in} + \frac{\varepsilon\,\dot{Q}_{max}}{\dot{m}\,c_{p,C}}$$

We evaluate the oil specific heat at its average temperature.

We estimate the oil outlet temperature to be 33 ºC, and $T_{avg} = \frac{7 + 33}{2} = 20\ ^{\circ}C$

so that from Appendix A-6 c_p = 1.880 kJ/kg.K.

Likewise, estimate for water, T_{avg} = 55 ºC and c_p = 4.179 kJ/kg.K

The heat capacity rates are: C_H = (0.2 kg) (4.179) = 0.835 kW.K and

C_C = (0.9) (1.880) = 1.692 kW.K

Therefore, $C_H = C_{min} \rightarrow C_{min} / C_{max} = 0.835 / 1.692 = 0.494$

$$\dot{Q}_{max} = (\dot{m}\ c_p)_{min}\ (T_{H,in} - T_{C,in}) = (0.835)\ (87 - 7) = 66.8 \text{ kW}$$

The thermal conductivity of 304 stainless steel from Appendix A-2 is k = 14.9 W/m.K

$$\frac{1}{U\,A} = \frac{1}{h_i\;A_i} + \frac{\ln\left(D_o/D_i\right)}{2\,\pi\,k\,N\,L} + \frac{1}{h_o\;A_o}$$

$$= \frac{1}{(3250)\;\pi\;(0.02)\;(20)\;(3)} + \frac{\ln\left(0.024/0.020\right)}{2\;\pi\;(14.9)\;(20)\;(3)} + \frac{1}{(1880)\;\pi\;(0.024)\;(20)\;(3)}$$

$$\frac{1}{U\,A} = 0.0000816 + 0.0000325 + 0.000118$$

$$U\,A = 4320\ \text{W/K} \qquad NTU = \frac{4320}{835} = 5.17$$

From Figure 13-8c, ε ≈ 0.76.

$$T_{C,out} = T_{C,in} + \frac{\varepsilon\;\dot{Q}_{max}}{\dot{m}\;c_{p,C}} = 7 + \frac{(0.60)\;(66.8)}{1.692} = 37\ ^\circ C$$

(b)

With the addition of fouling, the overall heat transfer coefficient changes. Therefore,

$$\frac{1}{U\,A} = 0.0000816 + 0.0000325 + 0.000118 + \frac{0.003}{\pi(0.024)\,(20)(3)}$$

$$U\,A = 1120 \text{ W/K} \qquad NTU = \frac{1120}{835} = 1.34$$

$$T_{C,out} = T_{C,in} + \frac{\varepsilon\,\dot{Q}_{max}}{\dot{m}\,c_{p,C}} = 7 + \frac{(0.60)\,(66.8)}{1.692} = 30.7\ ^{\circ}C$$

As calculated, fouling can have a significant detrimental effect on the heat transfer in a heat exchanger.