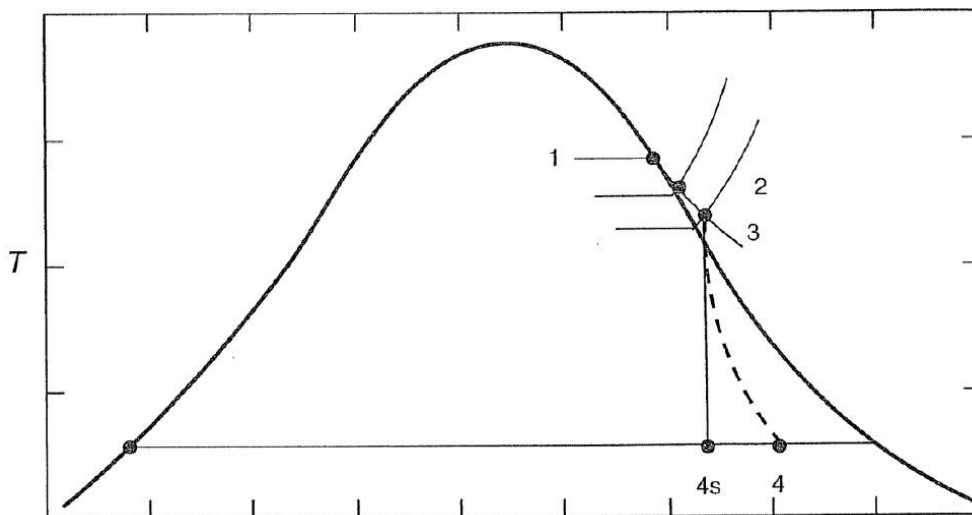
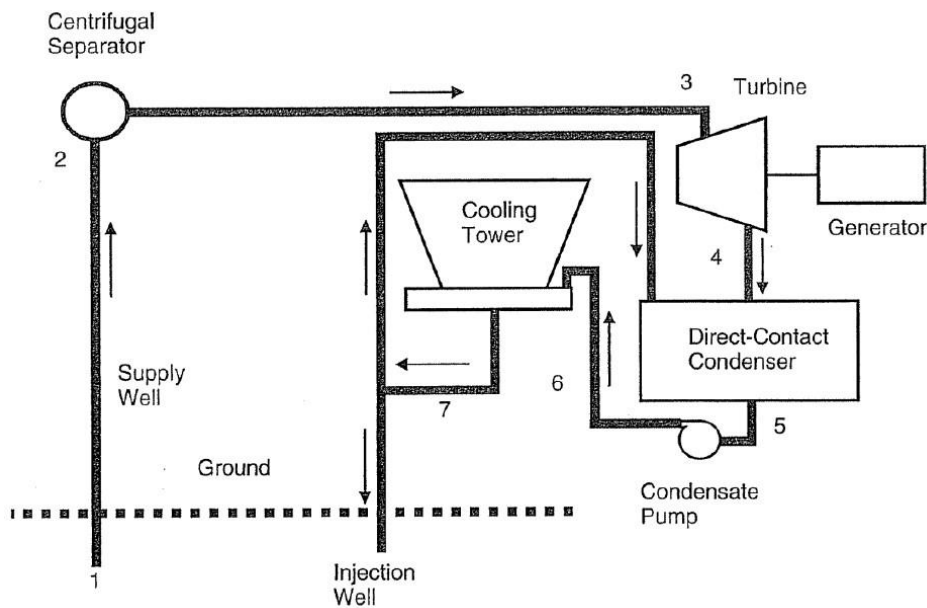


GEOTHERMAL ENERGY

Problem 1

A vapor-dominated geothermal system is supplied with saturated steam at 3 MPa. The steam enters the turbine at 0.5 MPa and exits at 15 kPa. The turbine isentropic efficiency is 82 %, and the electrical generator is 90 % efficient. If re-injection occurs at the cooling tower, analyze the system performance (find the thermal efficiency and the heat flow rate).

What flow rate of steam is required for a power generation of 10 MW?



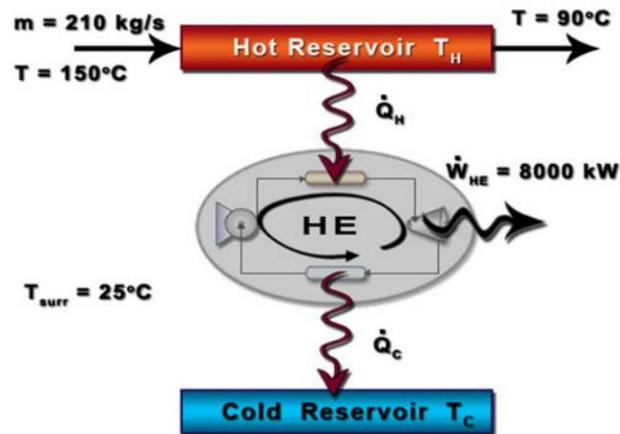
Answer: $\eta_{th} = 20.9 \%$; $m = 18.6 \text{ kg/s}$

Problem 2 (ME436s16m2-3 / ME436s18q8 / ME405s18q8)

A geothermal power plant uses geothermal water (brine) extracted at 150 °C at a rate of 210 kg/s as the heat source and produces 8000 kW of net power. The geothermal water leaves the plant at 90 °C. If the environment temperatures is 25 °C, determine

- the actual thermal efficiency
- the maximum possible thermal efficiency, and
- the actual rate of heat rejection from this power plant.

Diagram :



List all the assumptions first:

Specific heat of brine: $c_p = 3.2 \text{ kJ/kg.K}$

Answer: (a) $\eta_{\text{th}} = 19.8 \%$

(b) $\eta_{\text{Carnot}} = 29.6 \%$

(c) $\dot{Q}_C = 32\,320 \text{ kJ}$

Problem 3 (ME436s20f-3)

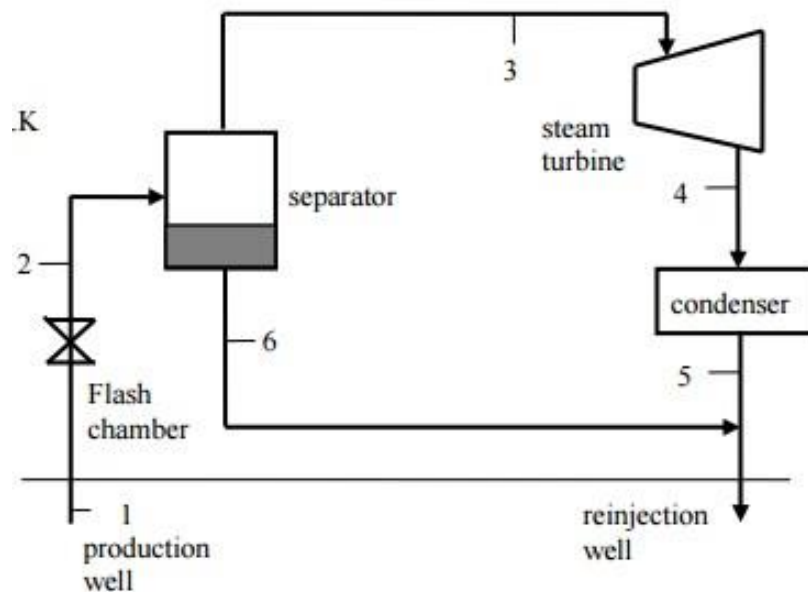
An innovative way of power generation involves the utilization of geothermal energy, the energy of hot water that exists naturally underground (hot springs), as the heat source. If a supply of hot water at 140°C is discovered at a location where the environmental temperature is 20°C, determine the maximum thermal efficiency a geothermal plant built at that location can have. If the power output of the plant is to be 5 MW, what is the minimum mass flow rate of hot water needed?

Problem 4 (ME405f16h5 / ME436s17h9)

A single-flash geothermal power plant uses hot geothermal water at 230°C as the heat source. The power output from the turbine, the thermal efficiency of the plant, the exergy of the geothermal liquid at the exit of the flash chamber, and the exergy destructions and exergy efficiencies for the flash chamber, the turbine, and the entire plant are to be determined. The mass flow rate at production well is 230 kg/s.

Assumptions:

- 1 Steady operating conditions exist.
- 2 Kinetic and potential energy changes are negligible.



Answer: $\dot{W}_T = 10.842 \text{ kW}$; $\dot{X}_6 = 17\,257 \text{ kW}$; $\dot{X}_{\text{dest,FC}} = 5080 \text{ kW}$; $\eta_{\text{II,T}} = 50 \%$

$$\eta_{\text{II,Plant}} = 21.8 \%$$

Problem 5 (ME405f16m2-2 / ME405f16f-3 / ME436s17m2-2 / ME436s17f-3 / ME405f22f-3)

A geothermal power plant uses geothermal water extracted at 160 °C at a rate of 440 kg/s as the heat source and produces 22 MW of net power. The geothermal water leaves the plant at 90 °C. If the environment temperature is 25°C, determine

- (a) the actual thermal efficiency;
- (b) the maximum ideal thermal efficiency;
- (c) the second-law efficiency; and
- (d) the actual rate of heat rejection from this power plant.

List the assumptions first.

Specific heat of brine: $c_p = 3.2 \text{ kJ/kg.K}$

Answer: (a) $\eta_{th} = 22.3 \%$

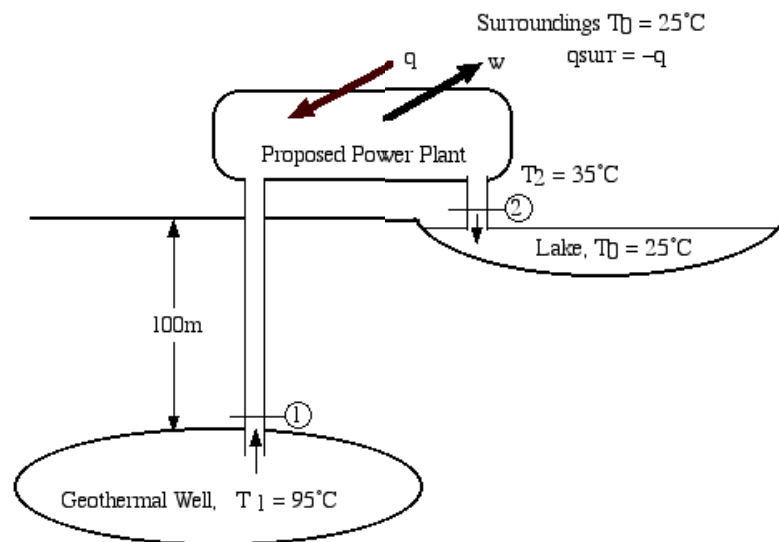
(b) $\eta_{Carnot} = 31.2 \%$

(c) $\eta_{II} = 71.6 \%$

(d) 76560 kW

Problem 6 (ME436s16m2-2 / ME405s18m2-2 / ME436s18m2-2 / ME436s22h10 / ME405s22h9)

Assume that a village has discovered an underground geothermal well containing water at 95°C , 100 m below the surface. They are evaluating the option using the water from this well to provide power to the village. Determine the maximum (ideal) available work in kJ/kg that could be obtained from a power plant that will pump water from the well, extract energy from the water, and discharge the water to a lake at the surrounding temperature $T_0 = 25^\circ\text{C}$. Assume that the water temperature can be reduced to 35°C at the outlet of the power plant.



Specific reversible work: $w_{rev} = (h_1 - h_2) - T_0 (s_1 - s_2) + g (z_1 - z_2)$

Change in enthalpy: $\Delta h = h_1 - h_2 = c_p (T_1 - T_2)$

Change in entropy: $\Delta s = s_1 - s_2 = c_p \ln\left(\frac{T_1}{T_2}\right)$

Specific heat: $c_p = 4.18 \text{ kJ/kg.K}$

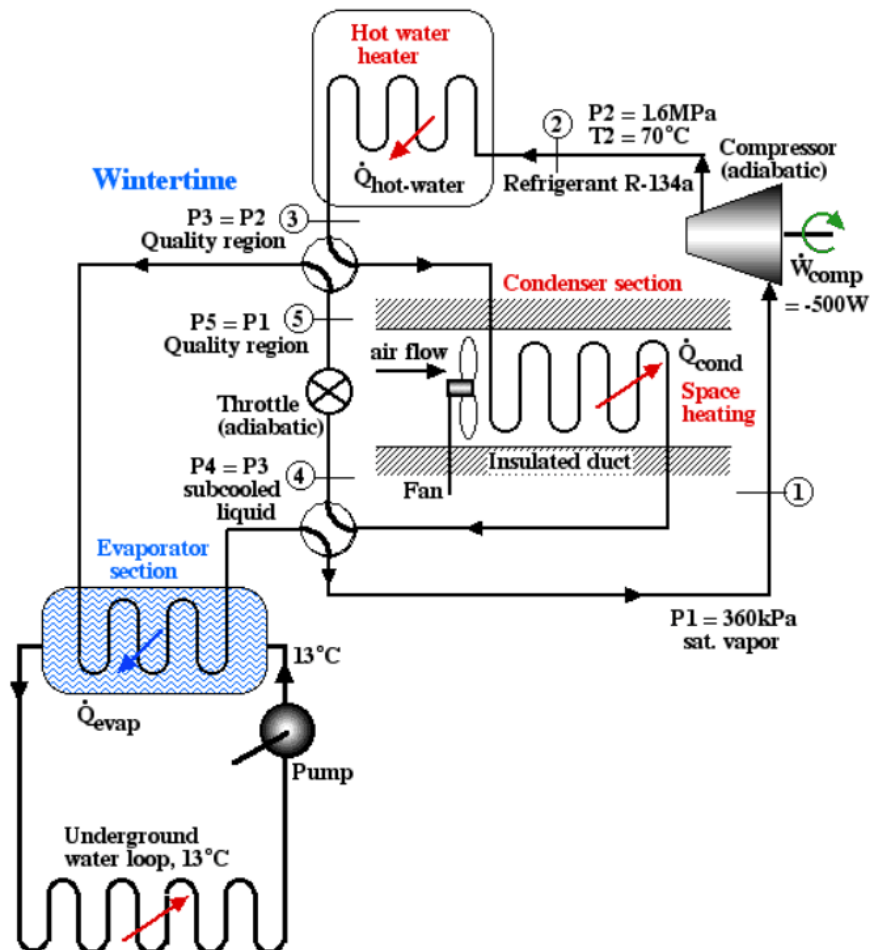
Acceleration of gravity: $g = 9.82 \text{ m/s}^2$

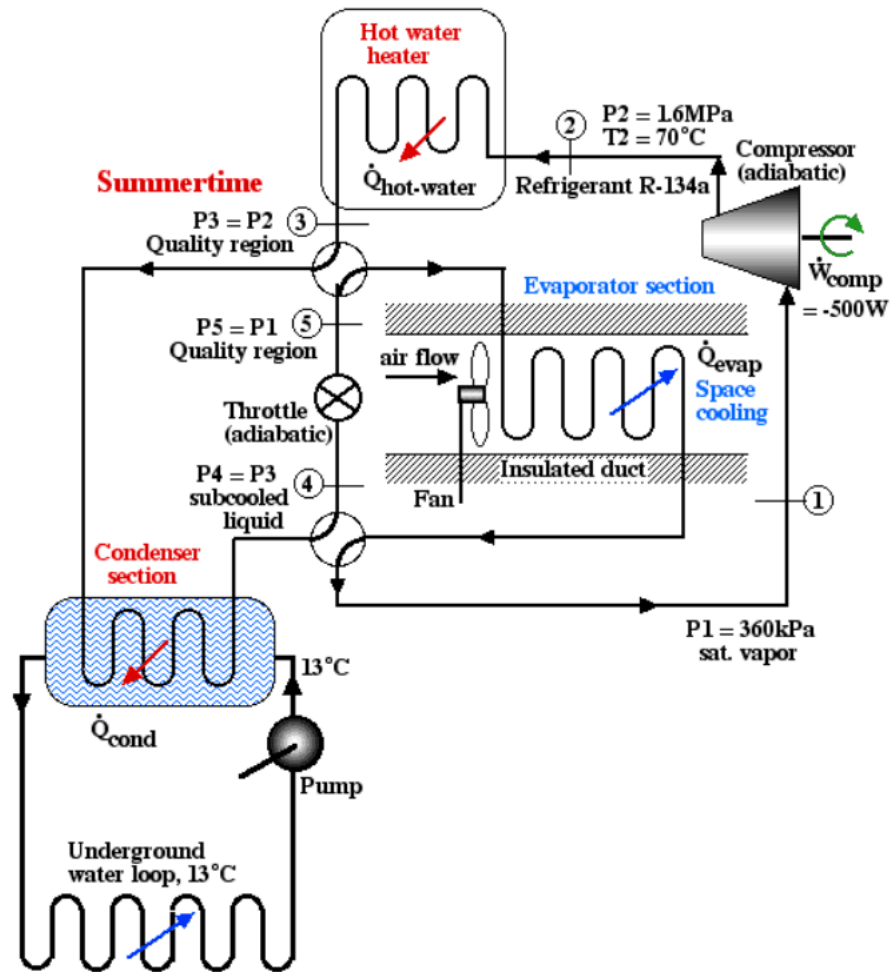
Conversion factors: $1 \text{ N} = 1 \text{ kg}\cdot\text{m}/\text{s}^2$, $1 \text{ kJ} = 1000 \text{ N}\cdot\text{m}$

Answer: $w_{\text{rev}} = 28.1 \text{ kJ}/\text{kg}$

Problem 7 (ME436s19h7 / ME405s19h7 / ME436s20h8)

We wish to do a preliminary thermodynamic analysis of the following home geothermal heat pump system designed for wintertime hot water and space heating. Notice that with suitable valving, this system can be used both in **winter** for space heating and in **summer** for air conditioning, with hot water heating throughout the year.



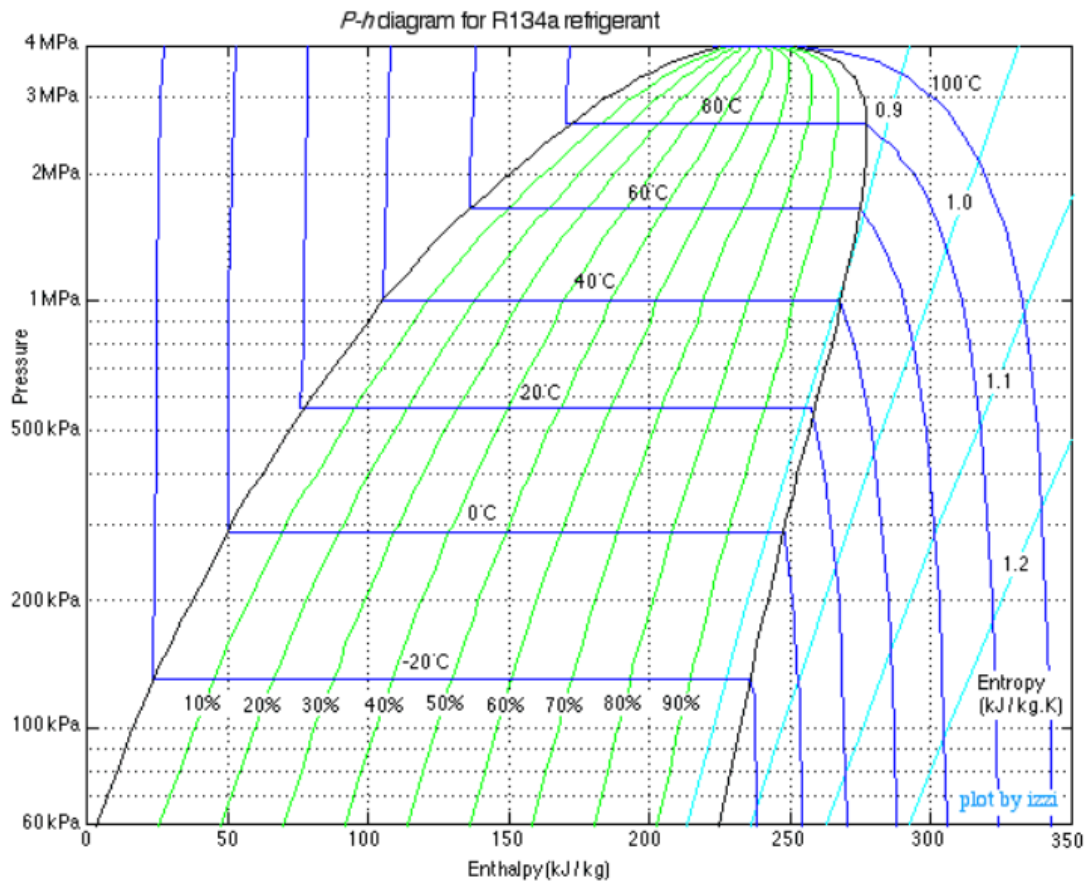


Notice that the condenser section includes both the hot water and space heater and station (3) is specified as being in the Quality region. Assume that 50 °C is a reasonable maximum hot water temperature for home usage, thus at a high pressure of 1.6 MPa, the maximum power available for hot water heating will occur when the refrigerant at station (3) reaches the saturated liquid state. Assume also that the refrigerant at station (4) reaches a subcooled liquid temperature of 20 °C while heating the air.

Using the conditions shown on the diagram and assuming that station (3) is at the saturated liquid state

- On the P - h diagram provided below carefully plot the five processes of the heat pump together with the following constant temperature lines: 50 °C (hot water), 13 °C (ground loop), and -10 °C (outside air temperature).
- Using the R134a property tables determine the enthalpies at all five stations and verify and indicate their values on the P - h diagram.
- Determine the mass flow rate of the refrigerant R134a.
- Determine the power absorbed by the hot water heater and that absorbed by the space heater.

- Determine the time taken for 100 liters of water at an initial temperature of 20 °C to reach the required hot water temperature of 50 °C.
- Determine the Coefficient of Performance of the hot water heater [COP_{HW}] (defined as the heat absorbed by the hot water divided by the work done on the compressor)
- Determine the Coefficient of Performance of the heat pump [COP_{HP}] (defined as the total heat rejected by the refrigerant in the hot water and space heaters divided by the work done on the compressor).

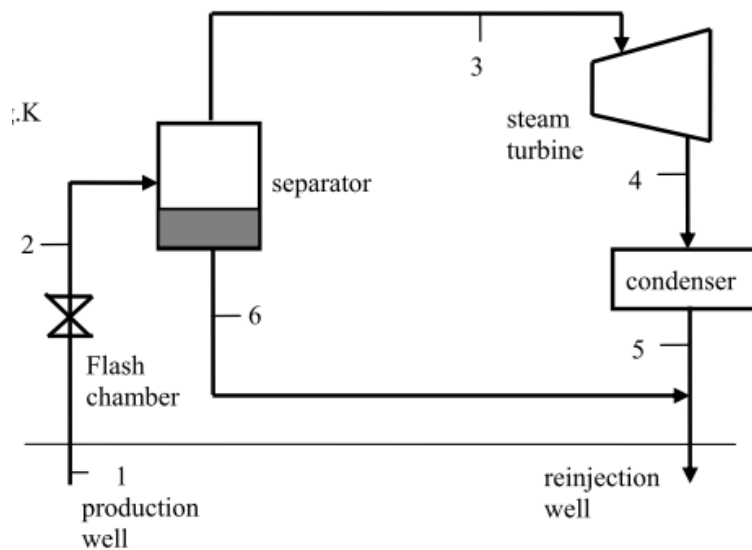


Answers:

- (c) $m = 0.0127 \text{ kg/s}$
- (d) Power absorbed by the hot water heater = 2 kW
- (e) Power absorbed by the space heater = 0.72 kW
- (f) $\Delta t = 105 \text{ minutes}$
- (g) $COP_{HW} = 4.0$
- (h) $COP_{HP} = 5.4$

Problem 8 (ME436s20q9)

A single-flash geothermal power plant uses hot geothermal water at 230 °C as the heat source. Assume that steady operating conditions exist. Mass flow rate at the production well is 230 kg/s.



The following are to be determined:

- 1) The power output from the turbine, W_T ;
- 2) The thermal efficiency of the plant, η_{th} ;
- 3) The exergy of the geothermal liquid at the exit of the flash chamber, \dot{X}_6 ;
- 4) The exergy destructions and exergy efficiencies for the flash chamber, the turbine, and the entire plant.

The following properties are given:

	T, °C	P, kPa	x	h, kJ/kg	s, kJ/kg.K
State 0	25		0	104.83	0.3672
State 1	230		0	990.14	2.61
State 2		500	0.1661	990.14	2.6841
State 3		500	1	2748.1	6.8207
State 4		10	0.95	2464.3	7.7739
State 6		500	0	640.09	1.8604

Assume steady state operation, and neglect changes in KE and PE.

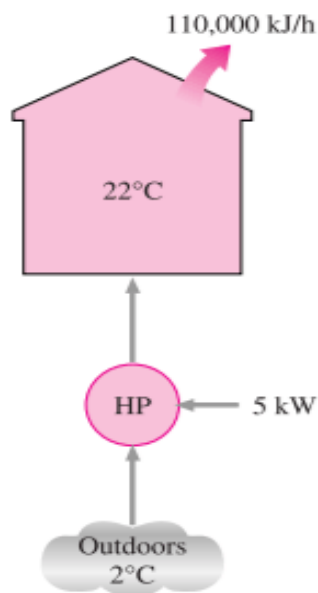
Problem 9 (ME436s21q9 / ME405s21q10)

A geothermal power plant uses geothermal water extracted at 160 °C at a rate of 140 kg/s as the heat source and produce 22 MW of net power. The return temperature of the water is 95 °C . The environment temperature is 25 °C. Determine

- The actual thermal efficiency;
- The maximum possible thermal efficiency; and
- The actual rate of heat rejection from this power plant.

Answers: $\eta_{th} \cong 0.07$; $\eta_C = 0.31$; $Q_{out} \cong 41000$ kW

Problem 10 (ME436s21h9 / ME405s21h10)



A heat pump is used to maintain a house at 22 °C by extracting heat from the outside air on a day when the outside air temperature is 2 °C. The house is estimated to lose heat at a rate of 110 000 kJ/h, and the heat pump consumes 5 kW of electric power when running. Is this heat pump powerful enough to do the job?

Answer: Yes

Figure P6-95

Problem 11 (ME436s21f-4 / ME405s21f-4 / ME405s23f-3)

A geothermal power plant uses R-134a in an ORC (Organic Rankine Cycle). The turbine receives saturated vapor at 50 °C (state 3) and produces 80 kW mechanical power. The fluid exits the turbine at 20 °C (state 4).

- Draw a schematic diagram of the whole power plant and identify devices (components) on the diagram.
- Find the mass flow rate of the organic fluid, assuming ideal turbine operation.

Kinetic and potential energy effects can be ignored.

Properties of R-134a:

State	P, kPa	T, 50 °C	h_f , kJ/kg	h_g , kJ/kg	s_f , kJ/kg.K	s_g , kJ/kg.K
3	1319.15	50		275.28		0.91165
4	571.7	20	79.32	261.60	0.3006	0.9224

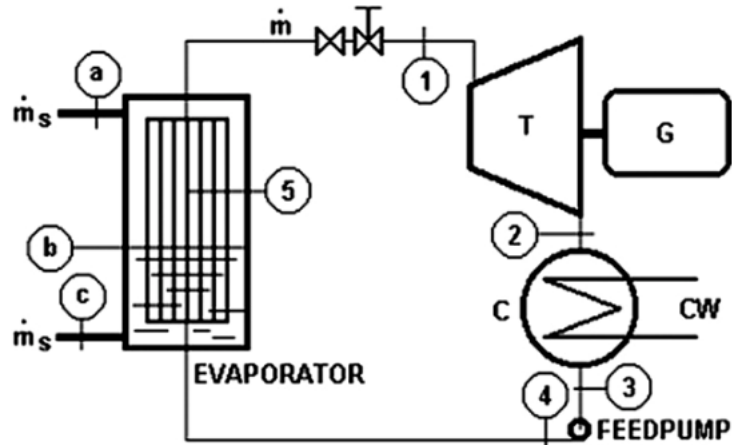
Problem 12 (ME436s22q10 / ME405s22q9)

A hot water reservoir is discovered at a depth of 1000 m. Downhole instrumentation indicates a pressure of 10 MPa and a temperature of 250 °C. A control valve at the surface maintains a constant mass flow rate up the well and keeps the fluid pressure at the wellhead equal to 800 kPa. Assume adiabatic conditions along the well and neglect the effects of friction.

- Calculate the state of the geofluid (brine) at the wellhead under these conditions.
- Sketch the process from the well bottom to the wellhead on a temperature-entropy diagram.
- If flashing occurs in the wellbore, estimate the depth below the surface at which it takes place.

Problem 13 (ME405f22q10)

The original Larderello geothermal power station employed binary-type units in which geothermal steam (saturated vapor) was used to heat and evaporate pure water which circulated in a close, simple Rankine cycle. See the simplified plant schematic below.



The following data are given:

$$P_a = P_b = P_c = 1.0 \text{ MPa} \quad T_c = 130 \text{ }^\circ\text{C (subcooled liquid)}$$

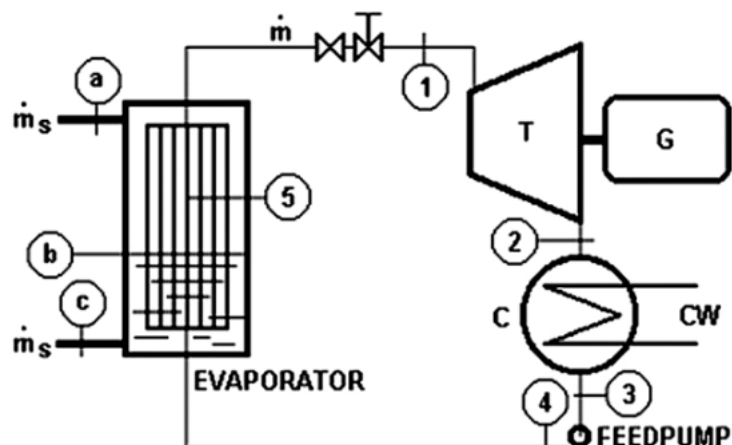
$$\text{Turbine wet efficiency} = 0.77 \quad T_1 = 160 \text{ }^\circ\text{C (saturated vapor)}$$

$$\text{Feed pump efficiency} = 1.00 \quad T_3 = 50 \text{ }^\circ\text{C (saturated liquid)}$$

Sketch the plant processes in a T-s diagram. Label all states using the notation given in the schematic.

Problem 14 (ME405f22h10)

The original Larderello geothermal power station employed binary-type units in which geothermal steam (saturated vapor) was used to heat and evaporate pure water which circulated in a close, simple Rankine cycle. See the simplified plant schematic below.



The following data are given:

$$P_a = P_b = P_c = 1.0 \text{ MPa} \quad T_c = 130 \text{ }^\circ\text{C (subcooled liquid)}$$

$$\text{Turbine wet efficiency} = 0.77 \quad T_1 = 160 \text{ }^\circ\text{C (saturated vapor)}$$

Feed pump efficiency = 1.00 $T_3 = 50\text{ }^\circ\text{C}$ (saturated liquid)

(a) Calculate (in kJ/kg for pure water):

1. Turbine work, w_T .
2. Feed pump work, w_p .
3. Net cycle work, w_{net} .

(b) Calculate the net cycle thermal efficiency, η_{th} .

(c) If the plant is to produce 10 000 kW of net power, calculate:

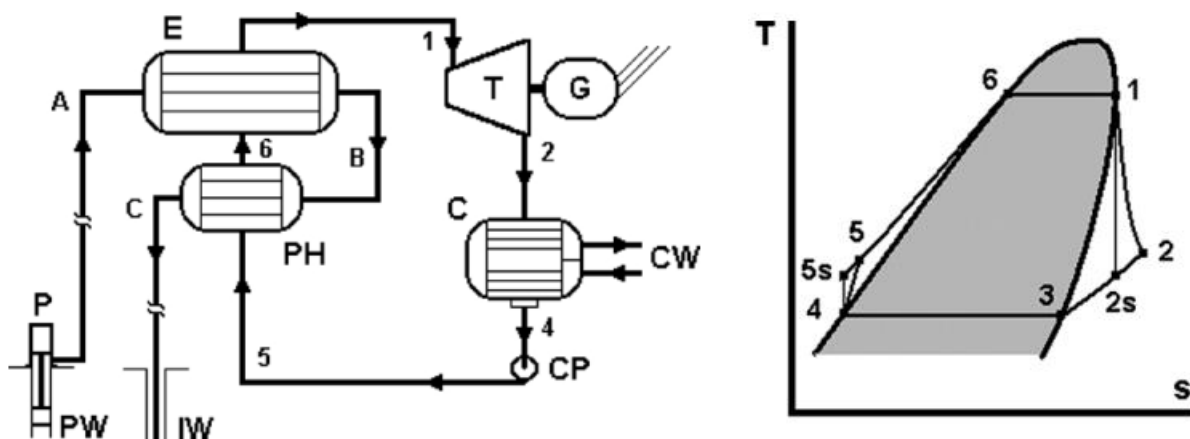
1. Mass flow rate of pure water, in kg/s.
2. Mass flow rate of geothermal steam, in kg/s.

(d) For a dead-state temperature of 25C, find the Second Law resource utilization efficiency,

η_u .

Problem 15 (ME405s23h9)

A basic binary-cycle geothermal power plant is shown in the figure. The working fluid is isopentane, $i\text{-C}_5\text{H}_{12}$, and that the cycle has a subcritical boiler pressure. The net cycle power is 1,200 kW, a typical value for this type of plant. Pressure losses in all heat exchangers and piping will be assumed negligible.



The cycle specifications are as follows:

- Brine inlet temperature, $T_A = 440\text{ K}$
- Brine specific heat, $c_b = 4.19\text{ kJ/kg}\cdot\text{K} = \text{constant}$
- Brine density 900 kg/m^3
- Pinch-point temperature difference = 5 K
- Preheater-evaporator pressure, $P_5 = P_6 = P_1 = 2.0\text{ MPa}$
- Condensing temperature, $T_4 = 320\text{ K}$

- Turbine isentropic efficiency = 85 %
- Feed pump isentropic efficiency = 75 %.

Some of the properties of isopentane:

- State 1: Saturated vapor at 2 MPa: $s_1 = 2.2022 \text{ kJ/kg.K}$, $h_1 = 741.08 \text{ kJ/kg}$.
- State 2s: Isentropic turbine outlet state; $s_{2s} = s_1$, $P_2 = P_{\text{sat}} = 186.6 \text{ kPa}$ for $T_3 = 320 \text{ K}$.
By interpolation, $h_{2s} = 649.66 \text{ kJ/kg}$.
- State 3: Saturated vapor at $T_3 = 320 \text{ K}$: $s_3 = 1.9887 \text{ kJ/kg.K}$, $h_3 = 578.16 \text{ kJ/kg}$.
- State 4: Saturated liquid at $T_3 = 320 \text{ K}$: $v_4 = 0.001686 \text{ m}^3/\text{kg}$, $h_4 = 249.50 \text{ kJ/kg}$.
- State 5: The definition of the pump efficiency is similar to that for the turbine.
- State 6: Saturated liquid at 2 MPa: $h_6 = 5545.73 \text{ kJ/kg}$.

Determine the following:

- Specific work of the turbine: w_T
- Heat rejected to the cooling water: q_c
- Specific work of the feed pump: w_P
- Heat transferred to the working fluid: q_{in}
- Cycle thermal efficiency: η_{th}
- Mass flow rate of i-C₅H₁₂: m_{i-C5}

Answer: $w_T = 77.7 \text{ kJ/kg}$; $q_c = 414 \text{ kJ/kg}$; $w_P = - 4 \text{ kJ/kg}$; $q_{in} = 487.50 \text{ kJ/kg}$
 $\eta_{th} = 15.1\%$. ; $m_{i2C5} = 16.3 \text{ kg/s}$.

Problem 16 (ME405f23q9)

A counterflow, double-pipe heat exchanger is used in a geothermal binary plant to heat isobutane, i-C₄H₁₀, from 20 to 70 °C. The i-C₄H₁₀ flows at 100 kg/s and has a $c_p = 2.3 \text{ kJ/kg.K}$; the brine enters at 100 °C and leaves at 45 °C. The overall heat transfer coefficient is $U = 568 \text{ W/m}^2.\text{K}$.

Calculate:

- Required brine mass flow rate in kg/s.
- Required heat transfer surface area in m².
- Second Law efficiency in %, if $T_o = 15 \text{ °C}$.

Data: Specific heat of brine is about 4 kJ/kg.K

$$\text{Heat Flow Rate} = \dot{Q} = (\dot{m} c_p \Delta T)_{\text{hot}} = (\dot{m} c_p \Delta T)_{\text{cold}} = A U \Delta T_m$$

$$\Delta T_m = \text{LMTD} = \frac{\Delta T_1 - \Delta T_2}{\ln \frac{\Delta T_1}{\Delta T_2}}, \quad \Delta T_1 = T_{\text{hot,in}} - T_{\text{cold,out}}, \quad \Delta T_2 = T_{\text{hot,out}} - T_{\text{cold,in}}$$

$$\text{Heat exchanger efficiency} = \eta = \frac{\dot{Q}}{\dot{Q}_{\text{max}}} = \frac{(\dot{m} c_p \Delta T)}{C_{\text{min}} (T_{\text{hot,in}} - T_{\text{cold,in}})}$$

Answers: (a) $m_b = 52 \text{ kg/s}$ (b) $A = 738.3 \text{ m}^2$

Problem 17 (ME405f23h9)

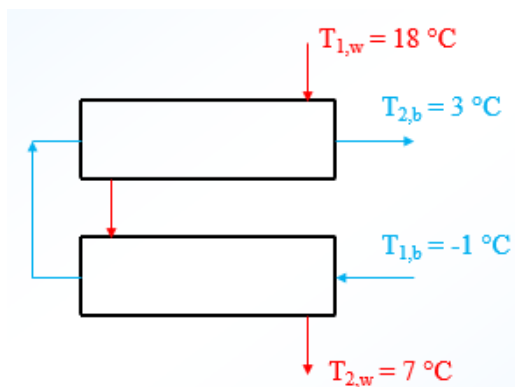
Consider a simple geothermal binary plant consisting of a heat exchanger to vaporize the working fluid (R-12), a turbine ($\eta_t = 85\%$), a condenser, and a simple feed pump ($\eta_p = 75\%$). The turbine inlet conditions are $P_1 = 2.7 \text{ MPa}$ and $T_1 = 140^\circ\text{C}$; the turbine exhaust pressure is $P_2 = 700 \text{ kPa}$. The condenser outlet condition is $P_3 = 700 \text{ kPa}$, saturated liquid. The pump outlet and heat exchanger inlet pressure is $P_4 = 3.1 \text{ MPa}$.

Calculate the following with the aid of R-12 property tables:

- Specific work of the turbine, w_t , kJ/kg.
- Specific work of the pump, w_p , kJ/kg.
- Specific heat added, q_a , kJ/kg.
- Specific heat rejected, q_c , kJ/kg.
- Cycle thermal efficiency (First Law), η_{th} , %.

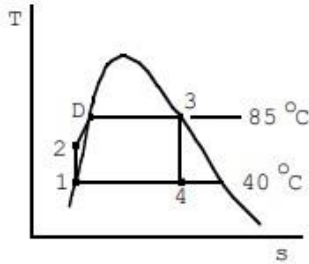
Answers: (a) $w_T = 28.7 \text{ kJ/kg}$ (b) $w_p = -1.42 \text{ kJ/kg}$

Problem 18



Water is to be cooled from 18°C to 7°C by using brine at an inlet temperature of -1°C with a temperature rise of 4°C . The brine and water flows are on the tube and shell sides, respectively. Determine the total heat transfer area required for a cross-flow arrangement shown in the figure by assuming $U_m = 850 \text{ W/m}^2\cdot\text{K}$ and a design heat load $Q = 6000 \text{ W}$.

Problem 19 (ME405f24h11)



A supply of geothermal hot water is to be used as the energy source in an ideal Rankine cycle, with R-134a as the cycle working fluid. Saturated vapor R-134a leaves the boiler at a temperature of 85 °C, and the condenser temperature is 40 °C. Calculate the thermal efficiency of this cycle and comment.

Answer: $\eta_{th} = 0.102$

Problem 20 (ME405f24f-5)

A counterflow, double-pipe heat exchanger is used in a geothermal binary power plant to heat isobutane, $i-C_4H_{10}$, from 20 to 70 °C. The $i-C_4H_{10}$ flows at 100 kg/s and has a $c_p = 2.3$ kJ/kg.K; The brine enters at 100 °C and leaves at 45 °C. The overall heat transfer coefficient is $U = 568$ W/m².K. Calculate:

- (a) Required brine mass flow rate in kg/s;
- (b) Required heat transfer surface area in m²;
- (c) First law efficiency of the heat exchanger;
- (d) How would you define the second-law efficiency of the heat exchanger if $T_o = 15$ °C.

Discuss in words but do not calculate.

Data: Specific heat of brine is about 4 kJ/kg.K

$$\text{Heat Flow Rate} = \dot{Q} = (\dot{m} c_p \Delta T)_{hot} = (\dot{m} c_p \Delta T)_{cold} = A U \Delta T_m$$

$$\Delta T_m = \text{LMTD} = \frac{\Delta T_1 - \Delta T_2}{\ln \frac{\Delta T_1}{\Delta T_2}}, \quad \Delta T_1 = T_{hot,in} - T_{cold,out}, \quad \Delta T_2 = T_{hot,out} - T_{cold,in}$$

$$\text{Heat exchanger efficiency (also called effectiveness)} = \eta = \frac{\dot{Q}}{\dot{Q}_{max}} = \frac{(\dot{m} c_p \Delta T)}{C_{min} (T_{hot,in} - T_{cold,in})}$$

Answers: (a) $m_b = 52$ kg/s ; (b) $A = 738.3$ m² ; (c) $\eta = 0.69$
