

SOLUTION KEY

Section: 34

Name & Surname: _____

Math 120 Spring 2017-2018

Quiz no.: 06

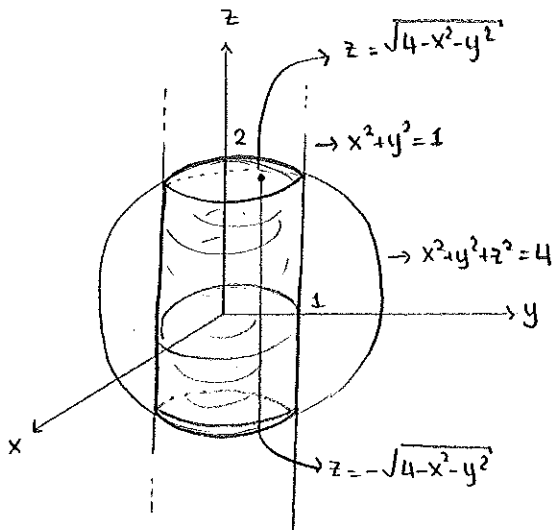
ID Number: _____

Date: 17.05.18

Time Limit: ~15 Minutes

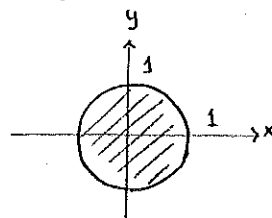
Grade: _____

1. Find the volume of the solid that lies inside both the cylinder $x^2 + y^2 = 1$ and the sphere $x^2 + y^2 + z^2 = 4$.



In cylindrical coordinates:

Projection onto xy-plane:



$$\theta \in [0, 2\pi]$$

$$r \in [0, 1]$$

$$\rightarrow -\sqrt{4-x^2-y^2} \leq z \leq \sqrt{4-x^2-y^2} \quad \text{write cylindrical coordinates;}$$

$$-\sqrt{4-r^2} \leq z \leq \sqrt{4-r^2}$$

$$dV = r \, dz \, dr \, d\theta$$

$$\text{Volume} = \int_0^{2\pi} \int_0^1 \int_{-\sqrt{4-r^2}}^{\sqrt{4-r^2}} 1 \, r \, dz \, dr \, d\theta = \int_0^{2\pi} \int_0^1 R z \Big|_{-\sqrt{4-r^2}}^{\sqrt{4-r^2}} \, dr \, d\theta$$

$$= \int_0^{2\pi} \int_0^1 2R \sqrt{4-r^2} \, dr \, d\theta = - \int_0^{2\pi} \left(\frac{4-r^2}{\frac{3}{2}} \right) \Big|_0^1 \, d\theta = - \frac{2}{3} \left[(4-1)^{3/2} - 4^{3/2} \right] \int_0^{2\pi} d\theta$$

$$= \frac{4\pi}{3} \left[4^{3/2} - 3^{3/2} \right]$$