SOLUTION KEY

Section: 163

Name & Surname:

Math 120 Spring 2017-2018
Quiz no.: 06
Date: 18.05.18
Time Limit: ~15 Minutes

ID Number:

Grade:

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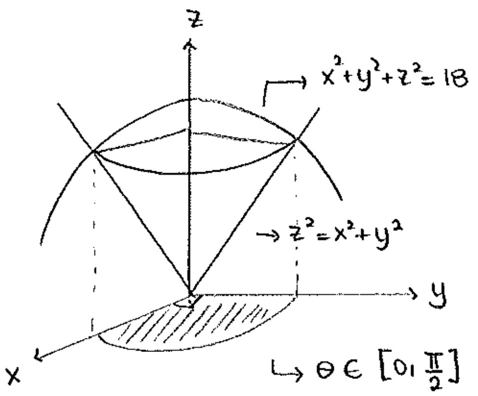
1. Convert $\displaystyle\int_0^3 \int_0^{\sqrt{9-y^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{18-x^2-y^2}} (x^2+y^2+z^2)\,dz\,dx\,dy$ into iterated integrals in spherical coordinates.

From the integral we have ; $0 \leqslant y \leqslant 3$

$0 \leqslant x \leqslant \sqrt{9-y^2}$ that is $x=0$, $x^2+y^2=9$ are boundaries

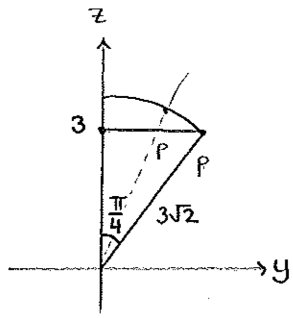
$\sqrt{x^2+y^2} \leqslant z \leqslant \sqrt{18-x^2-y^2}$ that is $z^2=x^2+y^2$, $x^2+y^2+z^2=18$ are boundaries

So the region is ;



Let $x=0$

$\left.\begin{array}{l} x=0 \\ y^2+z^2=18 \\ z^2=y^2 \end{array}\right\} \; z^2=9 \Rightarrow z=3$



$\phi \in [0, \frac{\pi}{4}]$

$\rho \in [0, 3\sqrt{2}]$

$\theta \in [0, \frac{\pi}{2}]$

$x = \rho \sin\phi \cos\theta$
$y = \rho \sin\phi \sin\theta$
$z = \rho \cos\phi$

$$f(x,y,z) = x^2+y^2+z^2 \Rightarrow f(\rho,\phi,\theta) = \rho^2\sin^2\phi\cos^2\theta + \rho^2\sin^2\phi\sin^2\theta + \rho^2\cos^2\phi$$
$$= \rho^2\sin^2\phi(\cos^2\theta+\sin^2\theta) + \rho^2\cos^2\phi = \rho^2(\sin^2\phi+\cos^2\phi) = \rho^2$$

$dV = \rho^2 \sin\phi \, d\phi \, d\rho \, d\theta$

Thus;

$$\int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{4}} \int_0^{3\sqrt{2}} \rho^2 \cdot \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$