

SOLUTION KEY

Section: 34

Name & Surname: _____

Math 120 Spring 2017-2018

Quiz no.: 03

ID Number: _____

Date: 05.04.18

Time Limit: ~15 Minutes

Grade: _____

1. Determine whether the following function f is continuous at $(0,0)$.

$$f(x,y) = \begin{cases} \frac{xy}{\sqrt{x^2+y^2}} & (x,y) \neq (0,0) \\ 1 & (x,y) = (0,0) \end{cases}$$

For all $(x,y) \in \mathbb{R}^2$ except $(0,0)$, the function $f(x,y)$ is continuous, we need to check continuity at the point $(0,0)$:

If $\lim_{(x,y) \rightarrow (0,0)} f(x,y) = f(0,0)$ then the function is continuous at $(0,0)$

Firstly find the limit as (x,y) approaches $(0,0)$:

$$0 \leq \left| \frac{xy}{\sqrt{x^2+y^2}} \right| = \frac{|x| \cdot |y|}{\sqrt{x^2+y^2}} = |x| \frac{\sqrt{y^2}}{\sqrt{y^2+x^2}} = |x| \sqrt{\frac{y^2}{y^2+x^2}} < |x| < 1$$

And since $\lim_{(x,y) \rightarrow (0,0)} 0 = \lim_{(x,y) \rightarrow (0,0)} |x| = 0$, by squeeze Theorem $\lim_{(x,y) \rightarrow (0,0)} \left| \frac{xy}{\sqrt{x^2+y^2}} \right| = 0$

Thus; $\lim_{(x,y) \rightarrow (0,0)} f(x,y) = 0$.

And since $\lim_{(x,y) \rightarrow (0,0)} f(x,y) = 0 \neq f(0,0) = 1$, f is not continuous at $(0,0)$.