SOLUTION KEY

Section: 163

Name & Surname:

Math 120 Spring 2017-2018
Quiz no.: 04
Date: 20.04.18
Time Limit: ~15 Minutes

ID Number:

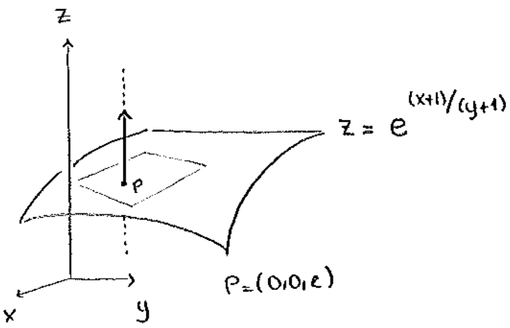
Grade:

---

1. Find an equation of the line normal to the surface

$$e^{(x+1)/(y+1)} - z = 0$$

at the point $(x, y, z) = (0, 0, e)$.



Find the normal vector of the tangent plane to the surface $e^{(x+1)/(y+1)} - z = 0$ at the point $(0,0,e)$.

This normal vector is the direction vector of the line that is perpendicular to the surface at $(0,0,e)$.

Check whether $(0,0,e)$ is on the surface or not:

$e^{(0+1)/(0+1)} - e = 0$ ✓

$(0,0,e)$ is on the surface.

NORMAL VECTOR: Let $f(x,y,z) = e^{(x+1)/(y+1)} - z$

the gradient vector of f at $(0,0,e)$ gives the normal vector.

$$\nabla f(x,y,z) = (f_x, f_y, f_z)$$

$$= \left(\frac{1}{y+1} \cdot e^{(x+1)/(y+1)}, -\frac{(x+1)}{(y+1)^2} e^{(x+1)/(y+1)}, -1 \right)$$

$$\nabla f(0,0,e) = (e, -e, -1)$$

LINE EQUATION: The equation of the line passing through $(0,0,e)$ with direction vector $\vec{d} = (e,-e,-1) = \nabla f(0,0,e)$

$$(x,y,z) = (0,0,e) + t(e,-e,-1), \quad t \in \mathbb{R}$$

$$\left.\begin{aligned} x &= t.e \\ y &= -t.e \\ z &= e - t \end{aligned}\right\} \quad t \in \mathbb{R}.$$