SOLUTION KEY

Section: 163

Name & Surname: _____

Math 120 Spring 2017-2018
Quiz no.: 02
Date: 23.03.18
Time Limit: ~15 Minutes

ID Number: _____

Grade:

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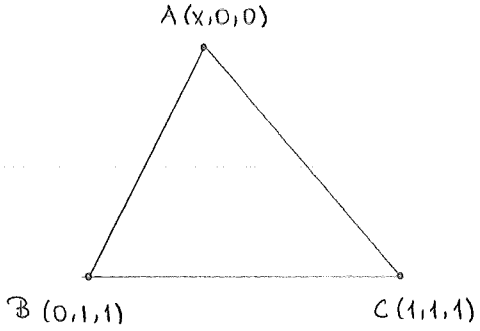
1. Show that for any x, the triangle with vertices $A(x,0,0), B(0,1,1), C(1,1,1)$ has area $\sqrt{2}/2$.



Area of this triangle can be evaluated by

$$\text{Area}(\overset{\Delta}{ABC}) = \frac{1}{2}\,\|\vec{AB} \times \vec{AC}\|$$

let x be an arbitrary real number

$\vec{AB} = (0,1,1) - (x,0,0) = \langle -x, 1, 1\rangle$

$\vec{AC} = (1,1,1) - (x,0,0) = \langle 1-x, 1, 1\rangle$

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} i & j & k \\ -x & 1 & 1 \\ 1-x & 1 & 1 \end{vmatrix} = \langle 0, 1, -1\rangle$$

$$\|\vec{AB} \times \vec{AC}\| = \sqrt{0^2 + 1^2 + (-1)^2} = \sqrt{2}$$

Therefore, $\text{Area}(\overset{\Delta}{ABC}) = \frac{1}{2}\,\|\vec{AB} \times \vec{AC}\| = \frac{\sqrt{2}}{2}$ for all $x \in \mathbb{R}$.