SOLUTION KEY

Section: 71

Name & Surname: ____________________

Math 120 Spring 2017-2018
Quiz no.: 01
Date: 09.03.18
Time Limit: ~15 Minutes

ID Number: ____________________

Grade:

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1. Find the radius and interval of convergence of the following power series:

$$\sum_{n=1}^{\infty} \frac{(x+1)^n}{n(n+1)}.$$

Let $a_n = \frac{(x+1)^n}{n(n+1)}$ for $n \geq 1$. Apply Ratio Test;

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left| \frac{(x+1)^{\cancel{n+1}}}{\cancel{(n+1)}.(n+2)} \cdot \frac{n\cancel{(n+1)}}{(x+1)^{\cancel{n}}} \right| = \lim_{n\to\infty} \left| \frac{n}{n+2} \cdot (x+1) \right| = \lim_{n\to\infty} \left| \frac{\cancel{n}}{\cancel{n}(1+\frac{2}{n})^{0}} (x+1) \right|$$

$$= |x+1|$$

Thus, if $|x+1| < 1$ then the series is absolutely convergent.

$|x+1|<1 \Rightarrow -1 < x+1 < 1 \Rightarrow -2<x<0$ Check the convergence at the end points:

Let $x=-2$: $\sum_{n=1}^{\infty} \frac{(-1)^n}{n(n+1)}$ is an alternating series with $b_n = \frac{1}{n(n+1)} > 0$ for $n \geq 1$.

b_n is decreasing and $\lim_{n\to\infty} b_n = 0$, by Alternating Series Test $\sum_{n=1}^{\infty} (-1)^n b_n$ is convergent.

Let $x=0$: $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$. Let $c_n = \frac{1}{n(n+1)}$ and $d_n = \frac{1}{n^2}$, both are positive sequences

$\lim_{n\to\infty} \frac{c_n}{d_n} = \lim_{n\to\infty} \frac{n^2}{n^2+n} = \lim_{n\to\infty} \frac{\cancel{n^2}}{\cancel{n^2}(1+\frac{1}{n})^{0}} = 1$ hence $\sum_{n=1}^{\infty} c_n$ and $\sum_{n=1}^{\infty} d_n$ have the same behaviour. By P-test $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is convergent, so by Limit Comparison Test $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ is also convergent.

Therefore; the interval of convergence: $[-2, 0]$
the Radius of convergence: 1.