Rec-3

5 Kasım 2021 Cuma 13:33

Q1| Find the domain and the range of the function

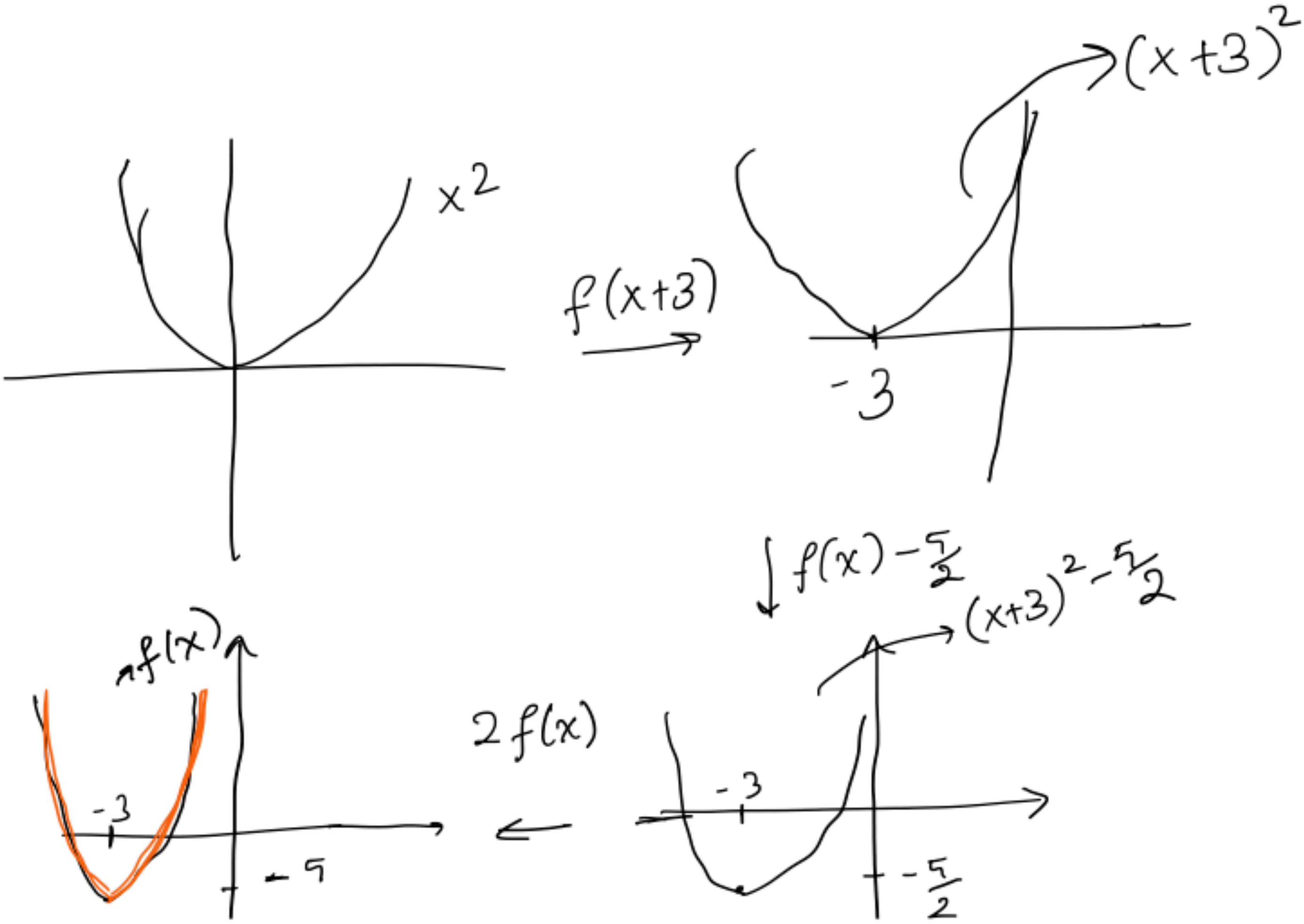
Sketch the graph of $f(x) = 2x^2 + 12x + 13$

$Dom(f) = \mathbb{R}$, $Range(f) = [-5, \infty)$

$$2x^2 + 12x + 13 = 2\left(x^2 + 6x + \frac{13}{2}\right)$$

$$= 2\left(\underbrace{(x+3)^2}_{0} - \frac{5}{2}\right)$$

If $x = -3$ then $f(-3) = -5$



Q2| Find $h \circ s$ and domain where

a) $h(x) = x - \frac{1}{x}$ and $s(x) = \frac{x+5}{x-7}$

Domain of $h \circ s = \{ x \in \mathbb{R} \mid x \in Dom(s) \ \& \ s(x) \in Dom(h) \}$

$h \circ s(x) = \underline{\underline{h}}(\underline{\underline{s(x)}})$

$Dom(s) = \mathbb{R} - \{7\}$

$Dom(h) = \mathbb{R} - \{0\}$

$Dom(h \circ s) = \{ x \in \mathbb{R} \mid x \in \mathbb{R} - \underline{\underline{\{7\}}} \ \& \ \underline{\underline{s(x) \in \mathbb{R} - \{0\}}} \}$

$s(x) = 0 \iff \frac{x+5}{x-7} = 0 \iff x = -5$

$\Rightarrow Dom(h \circ s) = \mathbb{R} - \{7, -5\}$

Domain of $s \circ h = \{ x \in \mathbb{R} \mid x \in Dom(h) \ \& \ h(x) \in Dom(s) \}$

$= \{ x \in \mathbb{R} \mid x \in \mathbb{R} - \{0\} \ \& \ h(x) \in \mathbb{R} - \{7\} \}$

$h(x) = 7 \iff \frac{x^2 - 1}{x} = 7 \iff x^2 - 7x - 1 = 0$

$$\Delta = 49 + 4 \cdot 7 > 0$$

$$x_1, x_2 = \frac{7 \mp \sqrt{53}}{2}$$

Find $h \circ s(x)$?

b) $h(x) = \dfrac{1+2x}{3-4x}$ and $s(x) = \dfrac{1-3x}{-2-4x}$

$$\mathrm{Dom}(h) = \mathbb{R} - \left\{\frac{3}{4}\right\}, \quad \mathrm{Dom}(s) = \mathbb{R} - \left\{-\frac{1}{2}\right\}$$

$$\mathrm{Dom}(h \circ s) = \{x \in \mathbb{R} \mid x \in \mathrm{Dom}(s) \ \& \ s(x) \in \mathrm{Dom}(h)\}$$

$$= \left\{x \in \mathbb{R} \mid x \in \mathbb{R} - \left\{-\frac{1}{2}\right\} \ \& \ \underline{\underline{s(x) \in \mathbb{R} - \left\{\frac{3}{4}\right\}}}\right\}$$

$$s(x) = \frac{3}{4} \iff \frac{1-3x}{-2-4x} = \frac{3}{4} \iff 4 - \cancel{12x} = -6 - \cancel{12x}$$

$$\iff \underline{\underline{4 = -6}}$$

$\Rightarrow$ There is no point so that $s(x) = \frac{3}{4}$

$$\mathrm{Dom}(h \circ s) = \mathbb{R} - \left\{-\frac{1}{2}\right\}$$

$$h \circ s(x) = h(s(x)) = \frac{1 + 2\,s(x)}{3 - 4\,s(x)}$$

$$= \frac{1 + 2\left(\frac{1-3x}{-2-4x}\right)}{3 - 4\left(\frac{1-3x}{-2-4x}\right)}$$

$$= \frac{-\cancel{2}-4x+\cancel{2}-6x}{-6-\cancel{12x}-4+\cancel{12x}} = \frac{-10x}{-10} = x$$

$$h \circ s(x) = id \Rightarrow h = s^{-1} \quad \& \quad s = h^{-1}$$

Q3| Find the inverse functions of the following funs. and domain of inverses if exists, or explain why it does not exist.

a) $f(x) = \dfrac{1}{2x-9}$

! f is invertible (f^{-1} exists) $\Leftrightarrow$ f is one-to-one & onto on its domain & range

$Dom(f) = \mathbb{R} - \{\frac{9}{2}\}$

$Range(f) = \mathbb{R} - \{0\}$

Claim: f is one-to-one

If $f(x_1) = f(x_2)$ $x_1, x_2 \in Dom(f)$ then $x_1 = x_2$.

Let $x_1, x_2 \in \mathbb{R} - \{\frac{9}{2}\}$ so that $\boxed{f(x_1) = f(x_2)}$

$$\frac{1}{2x_1-9} = \frac{1}{2x_2-9} \Leftrightarrow 2x_2 - 9 = 2x_1 - 9$$

$$\boxed{\begin{array}{l} 2x_1 - 9 \neq 0 \\ 2x_2 - 9 \neq 0 \end{array}}$$

$$\Leftrightarrow 2x_2 = 2x_1 \Leftrightarrow \boxed{x_1 = x_2}$$

Claim: f is onto

there exists $b \in Dom f$ so

For every $a \in Range(f)$, ...

that $f(b)=a$. $\leftarrow$ Def^n of onto function

For every $a \in \underline{\underline{\mathbb{R}-\{0\}}}$, $\exists b \in \mathbb{R}-\{\frac{9}{2}\}$ $f(b)=a$

$$f(b)=\frac{1}{2b-9}=a \iff \frac{1}{a}=2b-9$$

$$\iff \frac{1}{2}\left(\frac{1}{a}+9\right)=b$$

Since f is one-to-one and onto, f^{-1} exists.

$$f^{-1}: \mathbb{R}-\{0\} \to \mathbb{R}-\{\tfrac{9}{2}\}$$

$$x \longmapsto \frac{1}{2}\left(\frac{1}{x}+9\right)$$

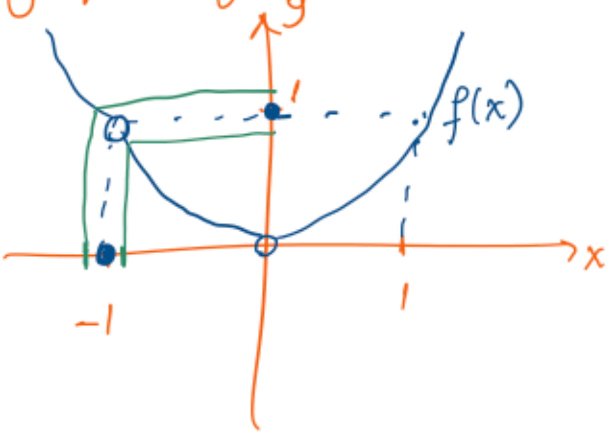
b) $g(x)=x+|x|$

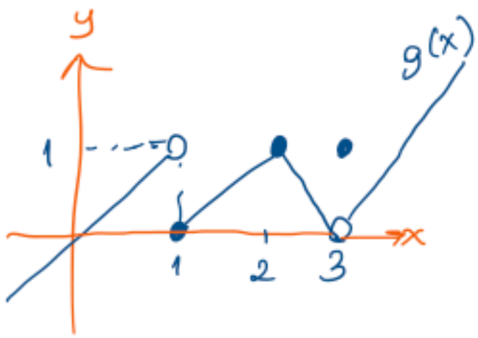
$Dom(g)=\mathbb{R}$, $Range(g)=[0,\infty)$

If $x<0$ then $g(x)=x-x=0$

$\Rightarrow g$ is $\underline{\underline{\text{not}}}$ one-to-one so it is not invertible.

Q4| Find the limits for the following functions whose graphs given below





a) $\lim\limits_{x\to -1} f(x) = \underline{1}$

d) $\lim\limits_{x\to 1} g(x) =$

b) $\lim_{x \to 0} f(x) = 0$

c) $\lim_{x \to 1} f(x) = 1$

$\lim_{x \to 1^+} g(x) = 0$

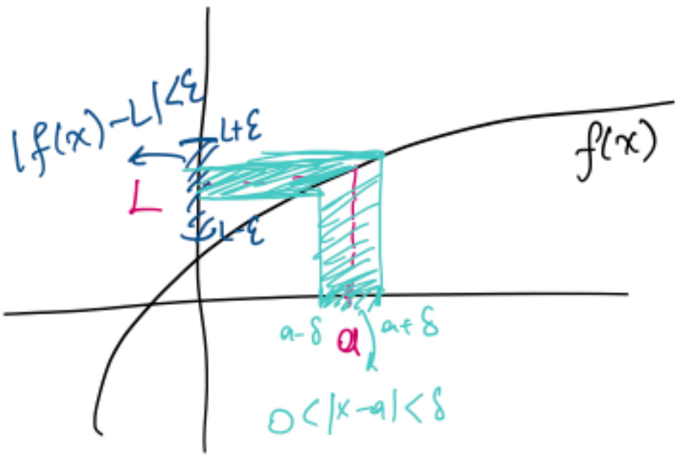
$\neq \Rightarrow \lim_{x \to 1} g(x)$ does not exist

$\lim_{x \to 1^-} g(x) = 1$

e) $\lim_{x \to 2} g(x) = 1$

f) $\lim_{x \to 3} g(x) = 0$

Q5| Find suitable δ-values for following limits.

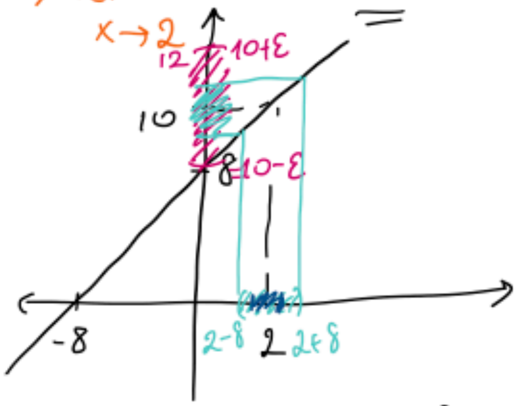


$\lim_{x \to a} f(x) = L$

$\Longleftrightarrow$

For a given $\varepsilon > 0$, there exists $\delta \in \mathbb{R}^0$ so that $0 < |x-a| < \delta$ implies that $|f(x) - L| < \varepsilon$

a) $\lim_{x \to 2} (x+8) = 10$ and $\varepsilon = 2$



$|f(x) - 10| < 2$

$|x + 8 - 10| < 2$

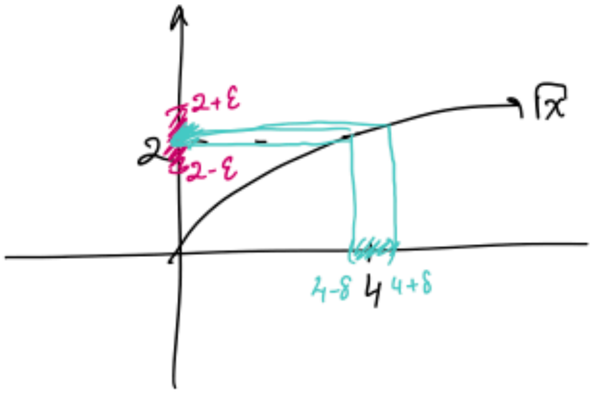
$|x - 2| < 2$

When we choose $\delta = 2$, we have $|f(x) - 10| < 2$

Remark: Any value less than 2 also works.

Remark: Any value ...

b) $\lim_{x \to 4} \sqrt{x} = 2$ and $\varepsilon = 0{,}5$



$|f(x) - 2| < \frac{1}{2}$

$\Leftrightarrow$

$|\sqrt{x} - 2| < \frac{1}{2}$

$\Leftrightarrow$

$-\frac{1}{2} < \sqrt{x} - 2 < \frac{1}{2}$

$\Leftrightarrow$

$\frac{3}{2} < \sqrt{x} < \frac{5}{2}$

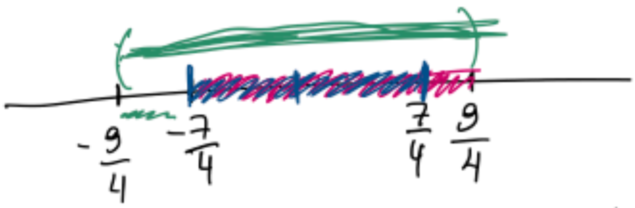
$\boxed{\frac{9}{4} < x < \frac{25}{4}}$

We are looking for $0 < |x-4| < \delta$,

$-\frac{7}{4} < x - 4 < \frac{9}{4}$

We should see $-\delta < x-4 < \delta$

If we choose $\boxed{\delta = \frac{7}{4}}$



If we don't have mirrored boundaries, we should choose minimum between them.

c) $\lim_{x \to 1} (2x^2 + 7) = 9$ and $\varepsilon = 1$

$|f(x) - 9| < 1 \Rightarrow |2x^2 + 7 - 9| < 1$

$\Rightarrow |2x^2 - 2| < 1$

$\Rightarrow |x^2 - 1| < \frac{1}{2}$

$\Rightarrow -\frac{1}{2} < x^2 - 1 < \frac{1}{2}$

$\Rightarrow \frac{1}{2} < x^2 < \frac{3}{2}$

$$\Rightarrow \frac{\quad}{2} < x < \frac{\quad}{2}$$

$$\Rightarrow \frac{1}{\sqrt{2}} < x < \frac{\sqrt{3}}{\sqrt{2}}$$

$$\Rightarrow \frac{1-\sqrt{2}}{\sqrt{2}} < x-1 < \frac{\sqrt{3}-\sqrt{2}}{\sqrt{2}}$$

Similar to part b, $\delta = \min\left\{ \left|\frac{1-\sqrt{2}}{\sqrt{2}}\right|, \left|\frac{\sqrt{3}-\sqrt{2}}{\sqrt{2}}\right| \right\}$

$$= \frac{\sqrt{3}-\sqrt{2}}{\sqrt{2}}$$