

- Duration: 15 min (10:15-10:30)
- Write your NAME, SURNAME, ID and SECTION.
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Question:

Evaluate the line integral

$$\int_C \frac{z}{\sqrt{x^2 + y^2}} ds$$

where  $C$  is the curve  $x = 3 \cos t$ ,  $y = 3 \sin t$ ,  $z = 4t$  for  $0 \leq t \leq 2\pi$ .

$$C(t) = (3 \cos t, 3 \sin t, 4t), \quad t \in [0, 2\pi]$$

$$\Rightarrow C'(t) = (-3 \sin t, 3 \cos t, 4)$$

$$\begin{aligned} \Rightarrow ds &= \|C'(t)\| dt = \sqrt{(-3 \sin t)^2 + (3 \cos t)^2 + 4^2} dt \\ &= \sqrt{9 \sin^2 t + 9 \cos^2 t + 16} dt \\ &= \sqrt{9 + 16} dt = 5 dt \end{aligned}$$

$$\begin{aligned} \int_C \frac{z}{\sqrt{x^2 + y^2}} ds &= \int_0^{2\pi} \frac{4t}{\sqrt{9 \sin^2 t + 9 \cos^2 t}} \cdot 5 dt = \int_0^{2\pi} \frac{20t}{3} dt \\ &= \frac{10}{3} t^2 \Big|_0^{2\pi} \\ &= \frac{10}{3} (4\pi^2 - 0) \\ &= \frac{40}{3} \pi^2 \end{aligned}$$