MATH 120 2020-2 Thursday 08:40-10:30 - Quiz 5

Duration: 15min

- Write your NAME, SURNAME, ID and SECTION.
- Upload your solutions to Gradescope as a SINGLE JPG or SINGLE PDF PAGE.

Let R be the region bounded by the cone $z^2 = x^2 + y^2$ and the paraboloid $z = x^2 + y^2$ in the first octant. Express $\iiint_R xydV$ as an iterated triple integral in cylindrical coordinates.

Cylindrical coordinates

$x = r\cos\theta \qquad z = z$

$y = r\sin\theta$

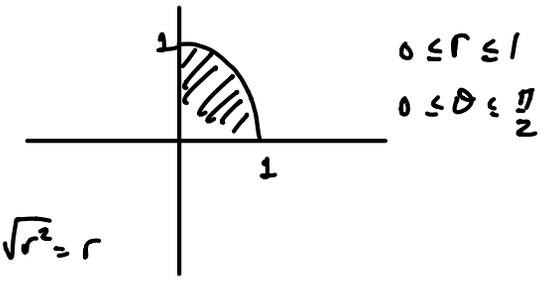
Insert these into $xy = r\cos\theta\, r\sin\theta = r^2\sin\theta\cos\theta$

Volume element $\quad dV = r\,dr\,d\theta\,dz$ (OR in some other orders)

Intersection $\quad z = x^2+y^2 = z \Rightarrow z^2 = z \Rightarrow z(z-1) = 0$

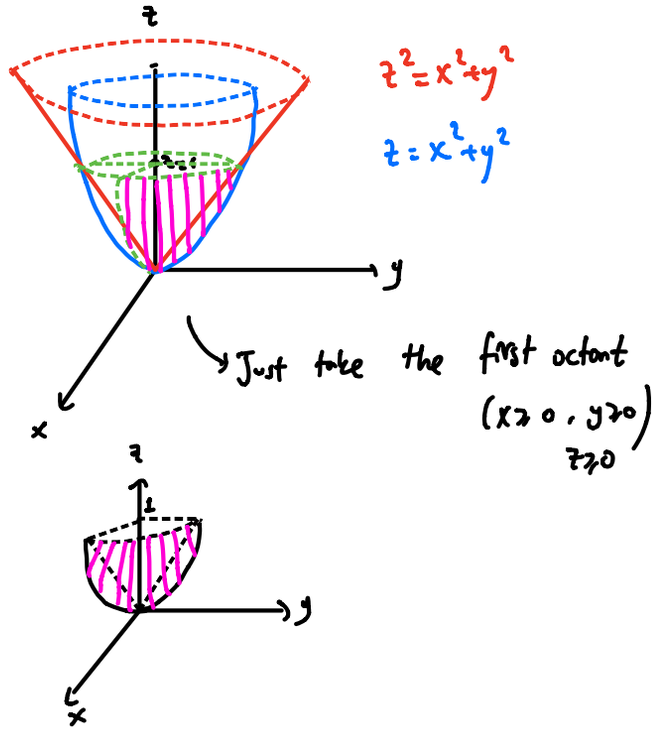
$z = 1$

$1 = z = x^2+y^2$ Projection to xy plane



for z above we have $z^2 = x^2+y^2 \Rightarrow z = \sqrt{x^2+y^2} = \sqrt{r^2} = r$

below " " $z = x^2+y^2 \Rightarrow z = x^2+y^2 = r^2$



$$\int_0^{\frac{\pi}{2}} \int_0^{1} \int_{r^2}^{r} (r^2\sin\theta\cos\theta)\, r\, dz\, dr\, d\theta$$